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The Collateralizability Premium

Non-Technical Summary

When firms are constrained in their ability to obtain credit, the impact of a general negative shock to the economy is potentially much stronger than in the case when firms can still obtain outside funding. One way for a firm to relax such a credit constraint is to hold a relatively large share of assets, which can be used as collateral in a debt transaction, i.e., which can be pledged against the loan or the bond. Assumingly, firms with a larger share of such collateralizable assets are less risky since they are less exposed to economic shocks. Since equity holders are the first to be hit by such shocks, a lower exposure means that equity is less risky, which in turn means lower stock returns in equilibrium. This is the main hypothesis that we investigate in this paper, both theoretically and empirically.

To examine the relationship between the degree of asset collateralizability and expected returns, we construct a measure of firms' asset collateralizability as the value-weighted average of the collateralizability of the different types of assets owned by the firm. We then sort stocks into portfolios according to this collateralizability measure and document that the spread between the low and the high collateralizability portfolio is on average close to 8 percent per year within the subset of financially constrained firms. The difference in returns remains significant after controlling for the usual factors including the market, size, value, momentum, and profitability. We also develop a theoretical model with heterogeneous firms and financial constraints, and the model generates results that are well in line with the data.

Our analysis shows that the composition of a firm's assets can be very relevant when it comes to assessing its riskiness. Investors recognize the added value of assets that can be pledged against bonds or loans to relax credit constraints, especially in economically bad times, and they require a lower risk premium when a firm has a higher share of collateralizable capital.

The Collateralizability Premium

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Abstract

A common prediction of macroeconomic models of credit market frictions is that the tightness of financial constraints is countercyclical. As a result, theory implies a negative collateralizability premium; that is, capital that can be used as collateral to relax financial constraints provides insurance against aggregate shocks and commands a lower risk compensation compared with non-collateralizable assets. We show that a long-short portfolio constructed using a novel measure of asset collateralizability generates an average excess return of around 8% per year. We develop a general equilibrium model with heterogeneous firms and financial constraints to quantitatively account for the collateralizability premium.

JEL Codes: E2, E3, G12

Keywords: Cross-Section of Returns, Financial Frictions, Collateral Constraint

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1   Introduction

A large literature in economics and finance emphasizes the importance of credit market frictions for the overall impact of macroeconomic fluctuations.[1] Although models differ in details, a common prediction is that financial constraints exacerbate economic downturns because they are more binding in recessions. As a result, theories of financial frictions predict that assets relaxing financial constraints should provide insurance against aggregate shocks. We evaluate the implications of this mechanism for the cross-section of equity returns.

From an asset pricing perspective, when financial constraints are binding, the value of collateralizable capital includes not only the dividends it generates but also the present value of the Lagrangian multipliers of the collateral constraints it relaxes. If financial constraints are tighter in recessions, then a firm holding more collateralizable capital should require a lower expected return in equilibrium, since the collateralizability of its assets provides a hedge against the risk of becoming financially constrained in recessions, making the firm less risky overall.

To examine the relationship between asset collateralizability and expected returns, we first construct a measure of firms' asset collateralizability. Guided by the corporate finance theory linking firms' capital structure decisions to collateral constraints (e.g., Rampini and Viswanathan (2013)), we measure asset collateralizability as the value-weighted average of the collateralizability of the different types of assets owned by the firm. Our measure can be interpreted as the fraction of firm value that can be attributed to the collateralizability of its assets.

We sort stocks into portfolios according to this collateralizability measure and document that the spread between the low and the high collateralizability portfolio is on average close to 8% per year within the subset of financially constrained firms. The difference in returns remains significant after controlling for factors such as the market, size, value, momentum, and profitability.

To quantify the effect of asset collateralizability on the cross-section of expected returns,

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[1]Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012) provide comprehensive reviews of this literature.

we develop a general equilibrium model with heterogeneous firms and financial constraints. In our model, firms are operated by entrepreneurs who experience idiosyncratic productivity shocks. As in Kiyotaki and Moore (1997, 2012), lending contracts cannot be fully enforced and therefore require collateral. Firms with high productivity and low net worth have higher financing needs; therefore, in equilibrium, they acquire more collateralizable assets in order to borrow. In the constrained efficient allocation in our model, heterogeneity in productivity and net worth translate into heterogeneity in the collateralizability of firm assets. In this setup, we show that, at the aggregate level, collateralizable capital requires lower expected returns in equilibrium, and, in the cross-section, firms with high asset collateralizability earn low risk premiums.

In our model, assets with different levels of collateralizability are traded, and firms with higher financing needs endogenously acquire more collateralizable assets. Because owners of collateralizable assets rationally expect that potential buyers will be able to use the assets as collateral to relax their borrowing constraint, the price of collateralizable assets must contain the present value of the appropriately normalized Lagrangian multipliers associated with financial constraints. The countercyclicality of these Lagrangian multipliers, therefore, in equilibrium, translates into the cross-sectional dispersion of expected returns across collateralizable-sorted portfolios.

We show that our model, when calibrated to match the conventional macroeconomic quantity dynamics and asset pricing moments, is able to generate a significant collateralizability spread. As in the data, firms with more asset collateralizability have higher financial leverage. Despite the higher leverage, a higher degree of asset collateralizability is associated with lower average returns. Quantitatively, our model matches the empirical relationship between asset collateralizability, leverage, and expected returns in the data quite well.

Related Literature This paper builds on the large macroeconomics literature studying the role of credit market frictions in generating fluctuations across the business cycle (see Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012) for recent reviews). The papers most closely related to ours are those emphasizing the importance of borrowing constraints and contract enforcements, such as Kiyotaki and Moore (1997, 2012), Gertler

and Kiyotaki (2010), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Elenev, Landvoigt, and Van Nieuwerburgh (2018). Gomes, Yamarthy, and Yaron (2015) study the asset pricing implications of credit market frictions in a production economy. A common prediction of the papers in this literature is that the tightness of borrowing constraints is countercyclical. We study the implications of this prediction for the cross-section of expected returns.

Several papers in this literature, in particular, develop models in which asset prices include not only the present value of the cash flow but also the present value of the Lagrangian multipliers for the relevant borrowing constraint it relaxes. Examples include, Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Elenev, Landvoigt, and Van Nieuwerburgh (2018). More recently, Liu, Wang, and Zha (2013) and Miao and Wang (2018) study the implications of related setups where the non-cash flow component of asset prices can be interpreted as bubbles. None of the above papers, however, focus on the implications of the model for the cross section of expected returns.

Our paper is also related to the corporate finance literature that emphasizes the importance of asset collateralizability for the capital structure decisions of firms. Albuquerque and Hopenhayn (2004) study dynamic financing with limited commitment, Rampini and Viswanathan (2010, 2013) develop a joint theory of capital structure and risk management based on asset collateralizability, and Schmid (2008) considers the quantitative implications of dynamic financing with collateral constraints. Falato, Kadyrzhanova, and Sim (2013) provide empirical evidence for the link between asset collateralizability and leverage in aggregate time series and in the cross section.

Our paper further belongs to the literature on production-based asset pricing, for which Kogan and Papanikolaou (2012) provide an excellent survey. From a methodological point of view, our general equilibrium model allows for a cross section of firms with heterogeneous productivity and is related to previous work including Gomes, Kogan, and Zhang (2003), Gârleanu, Kogan, and Panageas (2012), Ai and Kiku (2013), and Kogan, Papanikolaou, and Stoffman (2017). Compared with these papers, our model incorporates financial frictions. In addition, our aggregation result is novel in the sense that despite heterogeneity in productivity and the presence of aggregate shocks, the equilibrium in our model can be solved for without

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having to use any distribution as a state variable.

Finally, our paper is connected to the broader literature linking investment to the cross-section of expected returns. Berk, Green, and Naik (1999) discuss how optimal investment decisions affect the valuation of growth options and assets in place. Zhang (2005) provides an investment-based explanation for the value premium. Li (2011) and Lin (2012) focus on the relationship between R&D investment and expected stock returns. Eisfeldt and Papanikolaou (2013) develop a model of organizational capital and expected returns. Belo, Lin, and Yang (2017b) study implications of equity financing frictions on the cross-section of stock returns.

The rest of the paper is organized as follows. We summarize our empirical results on the relationship between asset collateralizability and expected returns in Section 2. We describe a general equilibrium model with collateral constraints in Section 4 and analyze its asset pricing implications in Section 5. In Section 6, we provide a quantitative analysis of our model. Section 7 concludes. A detailed presentation of our empirical analysis is provided in Section B of the appendix.

2 Empirical Facts

2.1 Measuring collateralizability

To empirically examine the link between asset collateralizability and expected returns, we first construct a measure of asset collateralizability at the firm level. Models with financial frictions typically feature a collateral constraint that takes the following general form:

$$B_{i,t} \leq \sum_{j=1}^{J} \zeta_j q_{j,t} K_{i,j,t+1}, \tag{1}$$

where $B_{i,t}$ denotes the total amount of borrowing by firm i at time t, and where $q_{j,t}$ is the price of type-j capital at time t.[2] The amount $K_{i,j,t+1}$ of type-j capital used by firm i at time $t+1$ is determined at time t (i.e., we assume a one-period time to build, as in standard real

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[2]In the model, we assume firms can only issue one-period bonds. A firm has to repay all the debt in order to borrow new debt. Under this assumption, the current one-period bond indeed represents the total debt of a firm.

business cycle models).

Different types of capital differ with respect to their degree of collateralizability. The parameter $\zeta_j \in [0,1]$ in (1) measures the degree to which type-j capital is collateralizable. The expression $\zeta_j = 1$ implies that type-j capital can be fully collateralized, whereas $\zeta_j = 0$ means that this type of capital cannot be collateralized at all. Equation (1) thus says that total borrowing by the firm is constrained by the total collateral it can provide.

Our collateralizability measure is a value-weighted average of the collateralizability of different types of firm assets. Specifically, the overall collateralizability of firm i's assets at time t, $\bar{\zeta}_{i,t}$, is defined as

$$\bar{\zeta}_{i,t} \equiv \sum_{j=1}^{J} \zeta_j \frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}}, \tag{2}$$

where $V_{i,t}$ denotes the total value of firm i's assets. In models with collateral constraints, the value of the collateralizable capital typically includes the present value of both the cash flows it generates and of the Lagrangian multipliers of the collateral constraint. These represent the marginal value of relaxing the constraint through the use of collateralizable capital. In Section 6.4 we show that, in our model, the firm-level collateralizability measure $\bar{\zeta}_{i,t}$ can be intuitively interpreted as the relative weight of the present value of the Lagrangian multipliers in the total value of the firm's assets. As a result, it summarizes the heterogeneity in firms' risk exposure arising from the asset collateralizability.

To empirically construct the collateralizability measure $\bar{\zeta}_{i,t}$ for each firm, we follow a two-step procedure. First, we use a regression-based approach to estimate the collateralizability parameters ζ_j for each type of capital. Motivated by previous work (e.g., Rampini and Viswanathan (2013, 2017)), we classify assets into three broad categories based on their collateralizability: structure, equipment, and intangible capital. Focusing on the subset of financially constrained firms for which the constraint (1) holds with equality, we divide both sides of the equation by the total value of a firm's assets at time t, $V_{i,t}$, and obtain

$$\frac{B_{i,t}}{V_{i,t}} = \sum_{j=1}^{J} \zeta_j \frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}}.$$

The above equation links firm i's leverage ratio, $\frac{B_{i,t}}{V_{i,t}}$ to its value-weighted collateralizability

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measure. Empirically, we run a panel regression of firm leverage, $\frac{B_{i,t}}{V_{i,t}}$, on the value weights of the different types of capital, $\frac{q_{j,t}K_{i,j,t+1}}{V_{i,t}}$, to estimate the collateralizability parameters ζ_j for structure and equipment, respectively.[3] The results for the estimates of the collateralizability parameters are shown in Table 1.

We run this leverage regression for the full sample as well as for subsets of financially constrained firms. We use four different measures to determine whether a firm is considered financially constrained: the WW index (Whited and Wu (2006)), the SA index (Hadlock and Pierce (2010)), an indicator of whether the firm has paid dividends or not in a given year, and all three measures combined.

As we can see in all of the specifications, there is a significant difference between structure and equipment capital, in which the former contributes more to a firm's debt capacity than the latter, represented by a significantly greater regression coefficient. This result is in line with Campello and Giambona (2013).

The firm-specific "collateralizability score" at time t, denoted by $\bar{\zeta}_{i,t}$, is then computed as a value-weighted average of the collateralizability coefficients across different types of assets; that is,

$$\bar{\zeta}_{i,t} = \sum_{j=1}^{J} \widehat{\zeta}_j \frac{q_{j,t}K_{i,j,t+1}}{V_{i,t}},$$

where $\widehat{\zeta}_j$ is the coefficient estimated from the panel regression described above. We provide further details regarding the construction of the collateralizability measure in Appendix D.2.

2.2 Collateralizability and expected returns

Equipped with the time series of the collateralizability measure for each firm, we follow the standard procedure and construct collateralizability-sorted portfolios. Consistent with our theory, we focus on the subset of financially constrained firms, whose asset valuations contain a non-zero Lagrangian multiplier component.

Table 2 reports average annualized excess returns, t-statistics, volatilities, Sharpe ratios

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[3]We impose the restriction that $\zeta = 0$ for intangible capital, both because previous work typically argues that intangible capital cannot be used as collateral, and because its empirical estimate is slightly negative in unrestricted regressions.

and market betas for the five collateralizability-sorted portfolios. We consider the same three basic indicators for whether a firm is financially constrained as above.

The top panel shows that, based on the WW index, the average equity return for firms with low collateralizability (quintile 1) is 7.86% higher on an annualized basis than that of a typical high collateralizability firm (quintile 5). We call this return spread the (negative) collateralizability premium. The return difference is statistically significant with a t-value of 2.53, and its Sharpe ratio is 0.44. The premium is robust with respect to the way we measure whether a firm is financially constrained, as can be seen from the middle and bottom panels of Table 2.[4]

In sum, the evidence on the collateralizability spread in the group of financially constrained firms strongly supports our theoretical prediction that collateralizable assets are less risky and therefore are expected to earn a lower return. In the next section, we develop a general equilibrium model with heterogeneous firms and financial constraints to formalize the above intuition and to quantitatively account for the negative collateralizability premium.

3   A Three-Period Model

To illustrate the basic premise of our theory, in this section, we develop a simple three-period model with closed-form solutions to illustrate the importance of Lagrangian multipliers for asset prices in models with financial constraints. In the next section, we then extend this model to a fully dynamic setup with heterogeneous firms.

In our setup, the household maximizes utility subject to standard intertemporal budget

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[4]The negative collateralizability premium is also present for financially unconstrained firms, but not as significantly as for constrained firms. It amounts to about 1% per year in our sample period. This result is consistent with our theory. As we show in our model, the collateralizability premium applies to unconstrained firms because the value of their assets includes the present value of the Lagrangian multipliers of all constraints that potentially become binding in the future. The collateralizability premium for these firms should be lower, because for currently unconstrained firms, the Lagrangian multiplier represents only a small fraction of the firm's value.

constraints:

$$\max_{\{C_t, B_t, \omega_t\}} E\left[\sum_{t=0}^{2} \beta^t \ln C_t\right] \tag{3}$$

$$C_0 + B_0 + \omega_0 V_0 = W_0 + \Pi_0,$$

$$C_1 + B_1 + \omega_1 V_1 = \omega_0 (V_1 + D_1) + B_0 r_0 + \Pi_1,$$

$$C_2 = \omega_1 (V_2 + D_2) + B_1 r_1 + \Pi_2.$$

The economy has three periods. The household has log utility with discount rate $\beta \in (0, 1]$, and its initial wealth is W_0. It also receives profits from the capital producer, $\{\Pi_t\}$. The household trades two assets: equity from the consumption goods producing firm (which pays dividends D_t and is valued at V_t at time t) and debt (a one-period bond which pays a gross rate of interest of r_{t-1} at time t).[5] This implies that in equilibrium, the stochastic discount factor (SDF) implied by household consumption will price both the equity and the bond.[6] We denote the holdings of shares and bonds by ω_t and B_t, respectively.

We denote the stochastic discount factor by M_t and specify the profit maximization problem of the representative firm as

$$\max_{\{D_t, B_t, K_{t+1}, N_{t+1}\}} E\left[\sum_{t=0}^{2} M_t D_t\right] \tag{4}$$

$$D_t + q_t K_{t+1} = N_t + B_t, \quad (t = 0, 1, 2) \tag{5}$$

$$N_{t+1} = A_{t+1} K_{t+1} + (1 - \delta) q_{t+1} K_{t+1} - r_t B_t \quad (t = 0, 1) \tag{6}$$

$$B_t \leq \zeta q_t K_{t+1}, \quad (t = 0, 1) \tag{7}$$

$$D_t \geq 0 \quad (t = 0, 1, 2). \tag{8}$$

Here, q_t denotes the price of capital at time t, K_{t+1} is the amount of capital at time $t + 1$ (determined at time t), N_t is the firm's net worth at time t, A_t denotes productivity, and δ is the rate of capital depreciation.

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[5]In what follows, we simply refer to the consumption goods producing firm as "the firm." As in neoclassical models, the capital goods producer does not make intertemporal decisions.

[6]As in standard neoclassical models, allowing for a full set of Arrow-Debreu securities will pin down a unique stochastic discount factor without affecting other equilibrium prices and quantities. Our model is one in which the financial market is complete in the sense of Debreu (1959). Here, we do not explicitly specify a full set of Arrow-Debreu securities for the household to trade, just to save notation.

The firm's problem is where our model departs from the frictionless neoclassical setup. The key constraint for borrowing is $B_t \leq \zeta q_t K_{t+1}$. Without this collateral constraint, our model reduces to the frictionless neoclassical model. Below we show that whenever this constraint is binding, the price of capital contains not only the present value of the cash flow but also the present value of the Lagrangian multiplier (appropriately normalized) on this constraint.

To complete the specification of the model, the producer of capital goods maximizes profit, given the price of capital q_t:

$$\Pi_t = \max \{q_t K_{t+1} - (1 - \delta) q_t K_t - G(I_t, K_t)\}$$
$$K_{t+1} = (1 - \delta) K_t + I_t,$$

where $G(I, K) = I + \frac{1}{2}\tau \left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ is the total cost of investment, including a quadratic capital adjustment cost with parameter τ.[7] Finally, market clearing requires $\omega_t = 1$ for all t; that is, the household fully owns the equity of the consumption goods producer. The aggregate resource constraint is $C_t + G(I_t, K_t) = A_t K_t$; that is, output is used for consumption and investment.

Our model should be interpreted as one in which households can trade a full menu of Arrow-Debreu securities. In this sense, the market is complete. However, households cannot trade capital directly; only firms can. As we will see in the following analysis, whenever firms are constrained, the price of capital may not equal the value of its replicating portfolio (constructed from Arrow-Debreu securities). In particular, the price of capital may contain a Lagrangian multiplier component. Firms cannot exploit this as an arbitrage opportunity because they are constrained. Our assumptions capture the fact that households can buy and sell stocks and bonds on public financial markets, but they basically never trade physical assets such as equipment and structures directly because they are less efficient than firms in deploying these assets.

The key to understanding the asset pricing implications of our model is the firm's optimization problem. To keep the notation consistent with the fully dynamic model presented

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[7]Here we use the no-arbitrage condition to impose that the price of one unit of K_t must be $(1 - \delta) q_t$, as capital depreciates at rate δ.

below, we focus on the recursive formulation of the firm's optimization problem and write the value function as $V_t(N_t) = \mu_t N_t$, for $t = 0, 1, 2$. Given prices, the firm's maximization problem is a linear programming problem, and therefore the value function must be linear. We denote the Lagrangian multiplier for the collateral constraint (7) as $\frac{\eta_t}{q_t}$ and that for the dividend constraint (8) as $\bar{\mu}_t$. For $t = 0, 1$, we can write the Lagrangian multiplier for the firm's optimization problem as

$$\mu_t N_t = \max_{K_{t+1}, B_t} \left\{ \begin{array}{l} E\left[M_{t+1}\mu_{t+1}\left\{A_{t+1}K_{t+1} + (1-\delta)q_t K_{t+1} - r_t B_t\right\}\right] \\ -\frac{\eta_t}{q_t}\left[B_t - \zeta q_t K_{t+1}\right] + \bar{\mu}_t\left[N_t + B_t - q_t K_{t+1}\right]. \end{array} \right\}. \tag{9}$$

In the above formulation, we assume all dividends are paid in period 2 without loss of generality. We provide details of the first-order conditions and the envelope condition in Appendix A. To save notation, we use lowercase letters to denote quantities normalized by current period capital, for example, $i_t = \frac{I_t}{K_t}$ and $c_t = \frac{C_t}{K_t}$. We summarize our main asset pricing result in the following proposition.

Proposition 1. *Suppose there exists an equilibrium in which the firm's collateral constraints are binding in both $t = 0$ and $t = 1$, then the price of capital in period 1 is given by*

$$q_1 = \frac{1}{\mu(n_1, A_1)}\beta E\left[\frac{A_1 - i(n_1) - \frac{1}{2}\tau(i(n_1) - \delta)^2}{(1-\delta) + i(n_1)}\right] + \zeta\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)}, \tag{10}$$

where the functional form of $i(n_1)$ is given in equation (A8) in Appendix A. The net worth $n_1 = n(A_1 | n_0)$ is a function of A that satisfies the following functional equation:

$$n(A | n_0) = A + (1-\delta)\left[1 + \tau(i(n(A | n_0)) - \delta)\right] - \zeta h(i(n_0) - \delta)\frac{1}{\beta}\frac{i(n_0) + 1 - \delta}{c(A_0, n_0)}\frac{1}{E\left[\frac{1}{c(A, n(A | n_0))}\right]}. \tag{11}$$

The Lagrangian multiplier component of the asset price is positive (i.e., $\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)} > 0$).

Proof. See Appendix A.1. $\square$

Proposition 1 formally establishes that the price of capital contains a Lagrangian multiplier component. In our setup, households are not constrained, and equity and bond prices must satisfy the standard intertemporal Euler equation $E[M_{t+1}R_{t+1}] = 1$. Because

households are not constrained, asset valuations do not directly contain the Lagrangian multipliers of any financial constraint. However, firms are constrained, and they trade capital goods among themselves.[8] Because capital goods can be used to relax a firm's financial constraint, the price of capital, q_t, has a Lagrangian multiplier component, as shown in (10). Firm valuation is indirectly affected by the Lagrangian multipliers, since the equity return is given as $R_{t+1} = \frac{D_{t+1}+V_{t+1}}{V_t}$, and the equity value $V_{t+1} = \mu_{t+1} N_{t+1} = \mu_{t+1} \{A_{t+1} K_{t+1} + (1-\delta) q_t K_{t+1} - r_t B_t\}$ depends on the price of capital.[9]

To maintain tractability, we assume that there is only one type of capital to highlight how asset prices are affected by the Lagrangian multipliers. Our fully dynamic model features two types of assets that differ in the collateralizability parameter ζ, which allows heterogeneity in asset composition to affect firms' equity returns.

To obtain closed-form solutions and provide a sharper characterization of the equilibrium, we make several assumptions on the parameter values of the model. First, we assume $\beta = \delta = 1$ to simplify the derivations. Second, we assume that ζ is bounded from above and below, in particular, $\zeta \in \left(\frac{1}{4}, \frac{1}{2}\right)$. In our model, the parameter ζ governs the tightness of the firm's collateral constraint. Assuming that ζ is bounded from above and below allows us to focus on equilibria where the firm is constrained in both periods. A large ζ would imply that the collateral constraint (7) will not bind in the current period. A small ζ, however, would imply that firms borrow very little in the current period and thus carry very little debt over to the next period. As a result, constraint (7) will not bind in the next period. In the fully dynamic model, this constraint (7) will be binding around the steady state for a wide range of values for ζ. However, in the simple three-period model, it is convenient to impose the above condition on ζ to ensure that the collateral constraint is binding for all realizations of the productivity shocks.

Finally, we assume $A_1 > \frac{1-\zeta}{1-2\zeta}$. As we will show in Appendix A.2, this assumption guarantees that in equilibrium, consumption and investment are both increasing functions

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[8]The representative firm can be interpreted as a continuum of identical competitive firms, as in standard neoclassical economies.

[9]In neoclassical models, it does not matter whether households trade capital directly or not. In our model, this fact is important, and it leads to q_t having a Lagrangian multiplier component. That households do not trade physical capital directly is a reasonable assumption, since they are less efficient than firms in deploying these assets.

of productivity. The next proposition provides a sufficient condition for the existence of constrained equilibria and for the countercyclicality of the Lagrangian multiplier component of asset prices.

Proposition 2. *There exists n^* and $\hat{n}$ such that for all $n_0 \in (n^*, \hat{n})$, the unique equilibrium is one in which the firm is constrained in both periods. In addition, the Lagrangian multiplier component of asset prices is counter-cyclical; that is,*

$$Cov\left(\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)}, A_1\right) < 0. \tag{12}$$

Proof. See Appendix A.2. □

This proposition implies that a sufficient condition for constrained equilibria to obtain is that the firm's net worth has to be bounded above and below by $(n^*, \hat{n})$, where the expressions for n^* and $\hat{n}$ are provided in (A33) and (A34) in Appendix A.2. Intuitively, when the initial net worth is high enough, firms will not be constrained. In our model, because borrowing capacity is limited by n_0, when n_0 is too low and productivity in period 1 is high, the firms despite being constrained in period 0, carries very little debt over to the next period and will not be constrained in the second period. Assuming that $n_0 \in (n^*, \hat{n})$ guarantees that the firm is constrained in both periods. Condition (12) is a formal statement of the collateral constraint being tighter in bad times (i.e., it tends to be high when aggregate productivity is low), which is the key mechanism for the asset pricing implications of our model.

We plot the Lagrangian multiplier component of the price of capital as a function of productivity A_1 in Figure 1. Note that by (11), n_1 depends on the realization of A_1. Therefore, for a fixed n_0, the Lagrangian multipliers, $\eta(n_1, A_1)$ and $\mu(n_1, A_1)$, are only functions of A_1. As A_1 increases, $\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)}$ monotonically decreases to zero, which is reached at the point where constraint (7) stops to bind. It is clear from the above discussion that the countercyclicality of the Lagrangian multipliers is an endogenous equilibrium outcome, so that a general equilibrium setup is needed to study its asset pricing implications. We now introduce a fully dynamic general equilibrium model with heterogeneous firms.

Figure 1: Monotonicity of the Lagrangian Multiplier

This figure shows a numerical example that $\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)}$ is a monotonically decreasing function of TFP shock A_1. We assume the time discount rate $\beta = 1$, the capital depreciation rate $\delta = 1$, and the capital adjustment parameter $\tau = 1$, as in the Proposition 2. We set the productivity in period zero to $A_0 = 3.2$. The productivity shock in period 1, A_1, is assumed to follow a uniform distribution, with a mean of A_0 and a standard deviation of 0.05. The collateralizability parameter ζ is set to 0.4. Initial net worth is $n_0 = 1.93$.
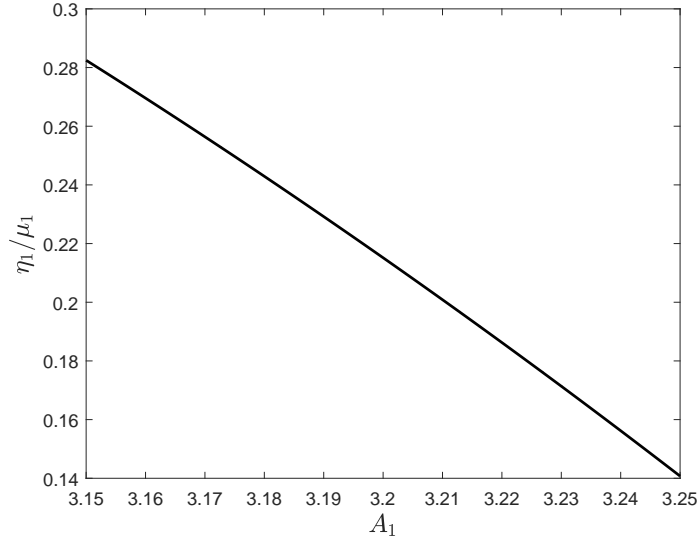
4 The Dynamic Model with Heterogeneous Firms

In this section, we describe the ingredients of our quantitative model of the collateralizability spread. The general spirit of the model is the same as that of the three-period model presented in the previous section. The key additional elements are i) heterogeneous firms with idiosyncratic productivity shocks; ii) two types of capital that differ with respect to their collateralizability; and iii) firm entry and exit. These features allow us to generate quantitatively plausible firm dynamics and heterogeneity in the firms' capital stocks in order to study the implications of collateralizability constraints for the cross-section of equity returns.

4.1 Households

Time is infinite and discrete. We assume that the representative household has a recursive preference with risk aversion γ and intertemporal elasticity of substitution (IES) ψ, as in

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Epstein and Zin (1989):

$$U_t = \left\{ (1 - \beta) C_t^{1 - \frac{1}{\psi}} + \beta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}},$$

where U_t is time-t utility and C_t is time-t consumption. As we will show later in the paper, the recursive preferences in our model, together with the endogenous equilibrium long run risk, generate a volatile pricing kernel and a significant equity premium, as in Bansal and Yaron (2004). The household receives labor and capital income and trade equities and bonds of firms. In every period t, the household purchases $B_{i,t}$ units of the risk-free bond and $\omega_{i,t}$ shares of equity from firm i. It receives a risk-free interest rate $R_{f,t}$ on the bonds and capital income plus a dividend payment $V_{i,t+1} + D_{i,t+1}$ in the next period. We assume that the labor market is frictionless, and labor income is therefore $W_t L_t$. The household budget constraint at time t can therefore be written as

$$C_t + \int \omega_{i,t} V_{i,t} di + \int B_{i,t} di = W_t L_t + R_{f,t-1} \int B_{i,t-1} di + \int \omega_{i,t-1} \left(V_{i,t} + D_{i,t} \right) di.$$

4.2 Firms' profit maximization

There is a continuum of firms in our economy indexed by $i \in [0, 1]$. A firm that starts at time 0 draws an idea with initial productivity $\bar{z}$ and begins the operation with initial net worth N_0. Under our convention, N_0 is also the total net worth of all firms at time 0 because the total measure of all firms is normalized to one.

Let $N_{i,t}$ denote firm i's net worth at time t, and let $B_{i,t}$ denote the total amount of risk-free bonds the firm issues to households at time t. Then the time-t budget constraint for the firm is

$$q_{K,t} K_{i,t+1} + q_{H,t} H_{i,t+1} = N_{i,t} + B_{i,t}. \tag{13}$$

In (13), we assume that there are two types of capital, K and H, that differ in their collateralizability, and we use $q_{K,t}$ and $q_{H,t}$ to denote their prices at time t. Let $K_{i,t+1}$ and $H_{i,t+1}$ denote the amount of capital that firm i purchases at time t, which can be used for production over the period from t to $t + 1$. At time t, the firm is assumed to have an opportunity

to default on its debt contract and abscond with all of the type-H capital and a fraction of $1 - \zeta$ of the type-K capital. Because lenders can retrieve a ζ fraction of the type-K capital upon default, borrowing is limited by

$$B_{i,t} \leq \zeta q_{K,t} K_{i,t+1}. \tag{14}$$

Type-K capital can therefore be interpreted as collateralizable, whereas type-H capital cannot be used as collateral.

From time t to $t+1$, the productivity of entrepreneur i evolves according to the law of motion

$$z_{i,t+1} = z_{i,t} e^{\varepsilon_{i,t+1}}, \tag{15}$$

where $\varepsilon_{i,t+1}$ is a Gaussian shock with mean μ_ε and variance σ_ε^2, assumed to be i.i.d. across agents i and over time. We use $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ to denote firm i's equilibrium profit at time $t+1$, where $\bar{A}_{t+1}$ is aggregate productivity in period $t+1$, and $z_{i,t+1}$ denotes firm i's idiosyncratic productivity. The specification of the aggregate productivity processes will be provided in Section 6.1.

In each period, after production, the firm experiences a liquidation shock with probability λ, upon which it loses its idea and needs to liquidate its net worth to return it to the household.[10] If the liquidation shock happens, the firm restarts with a draw of a new idea with initial productivity $\bar{z}$ and an initial net worth of χN_t in period $t+1$, where N_t is the total (average) net worth of the economy in period t, and $\chi \in (0,1)$ is a parameter that determines the ratio of the initial net worth of entrepreneurs relative to that of the economy-wide average. Conditional on no liquidation shock, the net worth $N_{i,t+1}$ of firm i at time $t+1$ is determined as

$$
\begin{aligned}
N_{i,t+1} &= \pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta) q_{K,t+1} K_{i,t+1} \\
&\quad + (1-\delta) q_{H,t+1} H_{i,t+1} - R_{f,t} B_{i,t}.
\end{aligned} \tag{16}
$$

The interpretation is that the firm receives the profit $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ from pro-

---

[10]This assumption effectively makes firms less patient than the household and prevents them from saving their way out of the financial constraint in the long run.

duction. Its capital holdings depreciate at rate δ, and it needs to pay back the debt borrowed from last period plus interest, amounting to $R_{f,t}B_{i,t}$.

We use V_t^i to denote the value of firm i. Because firms submit their net worth to the household whenever a liquidity shock occurs, firms' profit maximization problem can be specified recursively as

$$V_t^i = \max_{\{K_{i,t+1}, H_{i,t+1}, N_{i,t+1}, B_{i,t}\}} E_t \left[M_{t+1} \{ \lambda N_{i,t+1} + (1-\lambda) V_{t+1}^i \} \right], \tag{17}$$

subject to the budget constraint (13), the collateral constraint (14), and the law of motion of N_i given by (16). In the above equation, $M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{1/(1-\gamma)}} \right)^{\frac{1}{\psi}-\gamma}$ is the SDF implied by the household's maximization problem.

We use variables without a subscript i to denote economy-wide aggregate quantities. The aggregate net worth in the firm sector satisfies

$$N_{t+1} = (1-\lambda) \left[\begin{array}{c} \pi\left(\bar{A}_{t+1}, K_{t+1}, H_{t+1}\right) + (1-\delta)\, q_{K,t+1} K_{t+1} \\ + (1-\delta)\, q_{H,t+1} H_{t+1} - R_{f,t} B_t \end{array} \right] + \lambda \chi N_t, \tag{18}$$

where $\pi\left(\bar{A}_{t+1}, K_{t+1}, H_{t+1}\right)$ denotes the aggregate profit of all firms.

4.3 Production decisions

Final output With $z_{i,t}$ denoting the idiosyncratic productivity for firm i at time t, output $y_{i,t}$ of firm i at time t is assumed to be generated through the following production technology:

$$y_{i,t} = \bar{A}_t \left[z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t} \right)^{\nu} \right]^{\alpha} L_{i,t}^{1-\alpha}. \tag{19}$$

Here, α denotes the capital share, and ν is the span of control parameter, as in Atkeson and Kehoe (2005). Note that collateralizable and non-collateralizable capital are perfect substitutes in production. This assumption is made for tractability.

Firm i's profit at time t, $\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right)$, is given as

$$
\begin{aligned}
\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) &= \max_{L_{i,t}} y_{i,t} - W_t L_{i,t} \\
&= \max_{L_{i,t}} \bar{A}_t \left[z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t}\right)^{\nu} \right]^{\alpha} L_{i,t}^{1-\alpha} - W_t L_{i,t},
\end{aligned}
\tag{20}
$$

where W_t is the equilibrium wage rate, and $L_{i,t}$ is the amount of labor hired by firm i at time t.

It is convenient to write the profit function explicitly by maximizing out labor in equation (20) and using the labor market clearing condition $\int L_{i,t} di = 1$ to get

$$
L_{i,t} = \frac{z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t}\right)^{\nu}}{\int z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t}\right)^{\nu} di},
\tag{21}
$$

so that firm i's profit function becomes

$$
\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha \bar{A}_t z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t}\right)^{\nu} \left[\int z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t}\right)^{\nu} di \right]^{\alpha-1}.
\tag{22}
$$

Given the output $y_{i,t}$ of firm i at time t from equation (19), the total output of the economy is given as

$$
\begin{aligned}
Y_t &= \int y_{i,t} di, \\
&= \bar{A}_t \left[\int z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t}\right)^{\nu} di \right]^{\alpha}.
\end{aligned}
\tag{23}
$$

Capital goods We assume that capital goods are produced from a constant returns to scale and convex adjustment cost function $G(I, K + H)$; that is, one unit of the investment good costs $G(I, K + H)$ units of consumption goods. Therefore, the aggregate resource constraint is

$$
C_t + G(I_t, K_t + H_t) = Y_t.
\tag{24}
$$

We further assume that the fractions ϕ and $1 - \phi$ of the new investment goods can be used for type-K and type-H capital, respectively. This is a simplifying assumption which implies that, at the aggregate level, the ratio of type-K to type-H capital is always equal to $\phi/(1-\phi)$,

and thus the total capital stock of the economy can be summarized by a single state variable. The aggregate stocks of type-H and type-K capital follow the law of motion[11]

$$
\begin{aligned}
H_{t+1} &= (1-\delta) H_t + (1-\phi) I_t \\
K_{t+1} &= (1-\delta) K_t + \phi I_t.
\end{aligned}
\tag{25}
$$

5 Equilibrium Asset Pricing

5.1 Aggregation and construction of equilibrium

Our economy is one with both aggregate and idiosyncratic productivity shocks. In general, we would have to use the joint distribution of capital and net worth as an infinite-dimensional state variable in order to characterize the equilibrium recursively. In this section, we present a novel aggregation result and show that the aggregate quantities and prices of our model can be characterized without any reference to distributions. Given aggregate quantities and prices, quantities and shadow prices at the individual firm level can be computed using equilibrium conditions.

Distribution of idiosyncratic productivity In our model, the law of motion of idiosyncratic productivity shocks in (15) is time invariant, implying that the cross-sectional distribution of $z_{i,t}$ will eventually converge to a stationary distribution.[12] At the macro level, the heterogeneity of idiosyncratic productivity can be conveniently summarized by the simple statistic $Z_t \equiv \int z_{i,t}\, di$, which is useful when computing this integral explicitly.

Given the law of motion of z_i from equation (15) and the fact that entrepreneurs receive a liquidation shock with probability λ, we have

$$
Z_{t+1} = (1-\lambda) \int z_{i,t} e^{\varepsilon_{i,t+1}} di + \lambda \bar{z}.
$$

---

[11]For tractability, we have assumed that type-K and type-H capital are perfect substitutes in production. Under this assumption, if investors can determine which capital to produce, they will always choose not to produce type-H capital, as it can be perfectly substituted by type-K capital in production and is strictly dominated by type-K capital when used as collateral. Our assumption that capital production is Leontief in type-K and type-H ensures that both types of capital are produced in equilibrium.

[12]In fact, the stationary distribution of $z_{i,t}$ is a double-sided Pareto distribution. Our model is therefore consistent with the empirical evidence regarding the power law distribution of firm size.

The interpretation is that only a fraction $(1 - \lambda)$ of entrepreneurs will survive until the next period, and the rest will restart with a productivity of $\bar{z}$. Note that based on the assumption that $\varepsilon_{i,t+1}$ is independent of $z_{i,t}$, we can integrate out $\varepsilon_{i,t+1}$ and rewrite the above equation as [13]

$$
\begin{aligned}
Z_{t+1} &= (1 - \lambda) \int z_{i,t} E\left[e^{\varepsilon_{i,t+1}}\right] di \ + \lambda \bar{z}, \\
&= (1 - \lambda) Z_t e^{\mu_\varepsilon + \frac{1}{2}\sigma_\varepsilon^2} + \lambda \bar{z},
\end{aligned}
\tag{26}
$$

where the last equality follows from the fact that $\varepsilon_{i,t+1}$ is normally distributed. It is straightforward to see that if we choose the normalization $\bar{z} = \frac{1}{\lambda}\left[1 - (1 - \lambda)\, e^{\mu_\varepsilon + \frac{1}{2}\sigma_\varepsilon^2}\right]$ and initialize the economy by setting $Z_0 = 1$, then $Z_t = 1$ for all t. We will maintain this assumption throughout the rest of the paper.

Firm profits   We assume that $\varepsilon_{i,t+1}$ is observed at the end of period t when the entrepreneurs plan next period's capital. As we show in Appendix A.3, this implies that entrepreneur i will choose $K_{i,t+t} + H_{i,t+1}$ to be proportional to $z_{i,t+1}$ in equilibrium. Additionally, since $\int z_{i,t+1}\, di = 1$, we must have

$$
K_{i,t+1} + H_{i,t+1} = z_{i,t+1} \left(K_{t+1} + H_{t+1}\right),
\tag{27}
$$

where K_{t+1} and H_{t+1} are the aggregate quantities of type-K and type-H capital, respectively.

The assumption that capital is chosen after $z_{i,t+1}$ is observed rules out capital misallocation and implies that total output does not depend on the joint distribution of idiosyncratic productivity and capital. This is because, given idiosyncratic shocks, all entrepreneurs choose the optimal level of capital such that the marginal productivity of capital is the same across all entrepreneurs. This fact allows us to write $Y_t = \bar{A}_t \left(K_{t+1} + H_{t+1}\right)^{\alpha\nu} \int z_{i,t} di = \bar{A}_t \left(K_{t+1} + H_{t+1}\right)^{\alpha\nu}$. It also implies that the profit at the firm level is proportional to aggre-

---

[13]The first line requires us to define the set of firms and the notion of integration in a mathematically careful way. Rather than going into the technical details, we refer readers to Feldman and Gilles (1985) and Judd (1985). Constantinides and Duffie (1996) use a similar construction in the context of heterogeneous consumers. See footnote 5 in Constantinides and Duffie (1996) for a more careful discussion on possible constructions of an appropriate measurable space under which the integration is valid.

gate productivity, that is,

$$\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha \bar{A}_t z_{i,t} \left(K_t + H_t\right)^{\alpha\nu}, \tag{28}$$

and the marginal product of capital are equalized across firms for the two types of capital:[14]

$$\frac{\partial}{\partial K_{i,t}}\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \frac{\partial}{\partial H_{i,t}}\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha\nu\bar{A}_t\left(K_t + H_t\right)^{\alpha\nu-1}. \tag{29}$$

Recursive construction of the equilibrium   As in the three-period model, linearity implies that firms' value function is linear in net worth N_t^i and productivity $z_{i,t+1}$, which we denote as $V_t^i\left(N_{i,t}, z_{i,t+1}\right) = \mu_t^i N_{i,t} + \Theta_t^i z_{i,t+1}$. Let $\frac{\eta_{i,t}}{q_{K,t}}$ be the Lagrangian multiplier on the collateral constraint (14) and $\bar{\mu}_{i,t}$ be the Lagrangian multiplier on the budget constraint (13). To simplify notation, we denote $\tilde{M}_{t+1}^i = M_{t+1}\left[\lambda + (1-\lambda)\mu_{t+1}^i\right]$ and write the Lagrangian for the firms' optimization problem as

$$
\begin{aligned}
\mathcal{L} = {} & E_t\left[\tilde{M}_{t+1}^i\left\{\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta_K)q_{K,t+1}K_{i,t+1} + (1-\delta_H)q_{H,t+1}H_{i,t+1} - B_{i,t}R_{f,t}\right\}\right] \\
& - \frac{\eta_{i,t}}{q_{K,t}}\left[B_{i,t} - \zeta q_{K,t}K_{i,t+1}\right] + \bar{\mu}_t^i\left[N_{i,t} + B_{i,t} - q_{K,t}K_{i,t+1} - q_{H,t}H_{i,t+1}\right] + (1-\lambda)z_{i,t+1}E_t\left[M_{t+1}\Theta_{t+1}^i\right],
\end{aligned}
$$

where we use the fact that $z_{i,t+2}$ is independent of aggregate quantities and $E_t\left[z_{i,t+2}\right] = z_{i,t+1}$. As in the three-period model, the envelope condition implies that $\mu_t^i = \bar{\mu}_t^i$ for all i and all t. In general, the Lagrangian multipliers μ_t^i and η_t^i depend on the history of firm-specific shocks. As a result, the equilibrium would have to be constructed using the distribution of Lagrangian multipliers as an infinite-dimensional state variable, which would make the problem numerically difficult to solve. In our setup, thanks to the assumptions that type-K and type-H capital are perfect substitutes in production and that the idiosyncratic shock $z_{i,t+1}$ is observed before the decisions on $K_{i,t+1}$ and $H_{i,t+1}$ are made, we can construct an equilibrium in which μ_t^i and η_t^i are equalized across all firms and equilibrium prices and quantities do not depend on distributions. This will be shown in Proposition 3.

Intuitively, because type-K and type-H capital are perfect substitutes, firms' marginal product of capital depends only on the sum of the two types of capital (and not on the

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[14]To prove (29), we take derivatives of firm i's output function (19) with respect to $K_{i,t}$ and $H_{i,t}$, and then impose the optimality conditions (21) and (27).

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Electronic copy available at: https://ssrn.com/abstract=3474975

explicit composition of total capital), and because the next-period productivity shock can be perfectly observed, firms will trade both types of capital until their marginal product and the tightness of borrowing constraints are equalized. Note that $z_{i,t+1}$ can be observed one-period ahead is a fairly weak assumption, especially when the unit of time is small.[15]

Finally, we assume that aggregate productivity is given by $\bar{A}_t = A_t (K_t + H_t)^{1-\alpha\nu}$, where $\{A_t\}_{t=0}^{\infty}$ is an exogenous Markov productivity process. This assumption follows Frankel (1962) and Romer (1986) and is a parsimonious way to generate endogenous growth. Combined with recursive preferences, however, this assumption increases the volatility of the pricing kernel, as in the stream of the long-run risk models (see, e.g., Bansal and Yaron (2004) and Kung and Schmid (2015)). From a technical point of view, it further simplifies the construction of the equilibrium, as it implies that equilibrium quantities are homogeneous of degree one in the total capital stock, $K_t + H_t$, and equilibrium prices do not depend on $K_t + H_t$.

Under the above assumptions, it is convenient to work with normalized quantities. For a generic variable X_t, we use lowercase $x_t = \frac{X_t}{K_t + H_t}$ to denote X_t normalized by the total capital stock of the economy. To facilitate a recursive procedure in constructing the equilibrium, we use X for current-period quantities and X' for next-period quantities. In the following proposition, we show that a recursive equilibrium can be constructed where aggregate quantities are functions of two Markov state variables, productivity and normalized net worth: (A, n).

Formally, an equilibrium in our model consists of a set of aggregate quantities, $\{C_t, B_t, \Pi_t, K_t, H_t, I_t, N_t\}$, individual entrepreneur choices, $\{K_{i,t}, H_{i,t}, L_{i,t}, B_{i,t}, N_{i,t}\}$, and prices $\left\{M_t, \tilde{M}_t, W_t, q_{K,t}, q_{H,t}, \mu_t, \eta_t, R_{f,t}\right\}$ such that, given prices, quantities satisfy the household's and the entrepreneurs' optimality conditions, the market clearing conditions, and the relevant resource constraints. Below, we present a procedure for constructing a Markov equilibrium, where all prices and quantities are functions of the state variables (A, n). For simplicity, we assume that the initial idiosyncratic productivity across all firms satisfies $\int z_{i,1} di = 1$, the initial aggregate net worth is N_0, aggregate capital holdings start with $\frac{K_1}{H_1} = \frac{\phi}{1-\phi}$, and firms' initial net worth satisfies $n_{i,0} = z_{i,1} N_0$ for all i.

Proposition 3. *(Markov equilibrium)*

Suppose there exists a set of equilibrium functionals $\{c(A, n), i(A, n), \mu(A, n), \eta(A, n), q_K(A, n),$

---

[15]If we were to write a continuous-time model, this assumption would not be needed.

$q_H(A, n), R_f(A, n)\}$ *satisfying the following set of functional equations:*

$$E[M'|A] R_f(A, n) = 1, \tag{30}$$

$$\mu(A, n) = E\left[\left.\tilde{M}'\right| A\right] R_f(A, n) + \frac{\eta(A, n)}{q_K(A, n)}, \tag{31}$$

$$\mu(A, n) = E\left[\left.\tilde{M}'\frac{\alpha\nu A' + (1 - \delta) q_K(A', n')}{q_K(A, n)}\right| A\right] + \zeta\frac{\eta(A, n)}{q_K(A, n)}, \tag{32}$$

$$\mu(A, n) = E\left[\left.\tilde{M}'\frac{\alpha\nu A' + (1 - \delta) q_H(A', n')}{q_H(A, n)}\right| A\right], \tag{33}$$

$$\frac{n}{\Gamma(A, n)} = (1 - \zeta)\phi q_K(A, n) + (1 - \phi) q_H(A, n), \tag{34}$$

$$G'(i(A, n)) = \phi q_K(A, n) + (1 - \phi) q_H(A, n), \tag{35}$$

$$c(A, n) + i(A, n) + g(i(A, n)) = A, \tag{36}$$

where the law of motion of n is given by (A38), the stochastic discount factors M' and $\tilde{M}'$ are defined in (A39) and (A40), and the function $\Gamma(A, n)$ is defined in Equation (34). Then the equilibrium prices and quantities can be constructed as follows, and they constitute a Markov equilibrium:

1. *Given the sequence of exogenous shocks $\{A_t\}$, the sequence of n_t can be constructed using the law of motion in (A38), firms' value function is of the form $V_t^i(N_{i,t}, z_{i,t+1}) = \mu(A_t, n_t) N_{i,t} + \theta(A_t, n_t)(K_t + H_t) z_{i,t+1}$, and the normalized policy functions $c(A, n)$, $i(A, n)$, $\mu(A, n)$, $\eta(A, n)$, $q_K(A, n)$, $q_H(A, n)$, and $R_f(A, n)$ are jointly determined by equations (30)-(36). The normalized value function $\theta(A_t, n_t)$ is given in equation (A50) in Appendix A.3.*

2. *Given the sequence of normalized quantities, aggregate quantities are constructed as follows*

$$\begin{aligned} H_{t+1} &= H_t[1 - \delta + i_t], \quad K_{t+1} = K_t[1 - \delta + i_t] \\ X_t &= x_t[H_t + K_t] \end{aligned}$$

for $x = c, i, b, n, \theta$, $X = C, I, B, N, \Theta$, and all t.

3. Given the aggregate quantities, the individual entrepreneurs' net worth follows from (16). Given the sequences $\{N_{i,t}\}$, the quantities $B_{i,t}$, $K_{i,t}$ and $H_{i,t}$ are jointly determined by equations (13), (14), and (27). Finally, $L_{i,t} = z_{i,t}$ for all i, t.

Proof. See Appendix A.3. □

The above proposition implies that we can solve for aggregate quantities first and then use the firm-level budget constraint and the law of motion of idiosyncratic productivity to construct the cross-section of net worth and capital holdings. Note that our construction of the equilibrium allows $\eta(A, n) = 0$ for some values of (A, n). That is, our general setup allows occasionally binding constraints. Numerically, we use a local approximation method to solve the model by assuming that the constraint is always binding.

In our model, firm value function, $V(N_{i,t}, z_{i,t+1}) = \mu(A_t, n_t) N_{i,t} + \theta(A_t, n_t)(K_t + H_t) z_{i,t+1}$ has two components: $\mu(A_t, n_t) N_{i,t}$ is the present value of net worth and $\theta(A_t, n_t)(K_t + H_t) z_{i,t+1}$ is the present value of profit. In the special case of constant returns to scale, $\theta(A_t, n_t) = 0$ because firms do not make any profit. The general expression for $\theta(A, n)$ is provided in Appendix A.3. By the above proposition, other equilibrium quantities are jointly determined by conditions (30)-(36) independent of the functional form of $\theta(A, n)$. This is because $z_{i,t+1}$ is exogenously given and does not affect the determination of equilibrium optimality conditions.

The above conditions have intuitive interpretations. Equation (30) is the household's intertemporal Euler equation with respect to the choice of the risk-free asset. Equation (31) is the firm's optimality condition for the choice of debt. Equations (32) and (33) are the firm's first-order conditions with respect to the choice of type-K and type-H capital. Equation (34) is the binding budget constraint of firms, Equation (35) is the optimality condition for capital goods production, and Equation (36) is the aggregate resource constraint. Proposition 3 implies that conditions (30)-(36) are not only necessary but also sufficient for the construction of the equilibrium quantities.

In our model, because type-H capital can perfectly substitute for type-K capital in production and both types of capital are freely traded on the market, the marginal product of type-K capital must be equalized across firms. The trading of type-K capital therefore equalizes the Lagrangian multiplier of the financial constraints across firms. This is the key

feature of our model that allows us to construct a Markov equilibrium without having to include the distribution of capital as a state variable.[16]

5.2   The collateralizability spread

To understand the asset pricing implications of our model, note that the return on a firm's equity can be written as

$$
\begin{aligned}
R_{i,t+1} &= \frac{\lambda N_{i,t+1} + (1-\lambda)\left[\mu_{t+1}N_{i,t+1} + \Theta_{t+1}z_{i,t+2}\right]}{\mu_t N_i + \Theta_t z_{i,t+1}}, \\
&= \frac{\mu_t N_i}{\mu_t N_i + \Theta_t z_{i,t+1}}\frac{\left[\lambda + (1-\lambda)\mu_{t+1}\right]N_{i,t+1}}{\mu_t N_i} + \frac{\Theta_t z_{i,t+1}}{\mu_t N_i + \Theta_t z_{i,t+1}}\frac{(1-\lambda)\Theta_{t+1}z_{i,t+2}}{\Theta_t z_{i,t+1}}. \quad (37)
\end{aligned}
$$

Because $E_t[z_{i,t+2}] = z_{i,t+1}$, the expected return can be written as:

$$
E_t[R_{i,t+1}] = \frac{\mu_t N_i}{\mu_t N_i + \Theta_t z_{i,t+1}}E_t\left[\frac{\left[\lambda + (1-\lambda)\mu_{t+1}\right]N_{i,t+1}}{\mu_t N_i}\right] + \frac{(1-\lambda)\Theta_t z_{i,t+1}}{\mu_t N_i + \Theta_t z_{i,t+1}}E_t\left[\frac{\Theta_{t+1}}{\Theta_t}\right].
$$

That is, the return of firms' equity is a weighted average of the return on net worth and that of the return on the claim to future profit. In our calibrated model, ν is close to one and the profit component is much smaller than the net worth component.[17] We therefore focus on the return on net worth component of the return and write

$$
R_{i,t+1} \approx \frac{\lambda + (1-\lambda)\mu_{t+1}}{\mu_t}\frac{N_{i,t+1}}{N_{i,t}}, \quad (38)
$$

where we use the fact that $\mu_{i,t}$ is equalized across all firms. Therefore, the return on firm i's equity can be decomposed into two parts: changes in net worth, $\frac{N_{i,t+1}}{N_{i,t}}$, and changes in the marginal value of net worth, $\frac{\lambda + (1-\lambda)\mu_{t+1}}{\mu_t}$. Using the law of motion of net worth (16), we can write changes in net worth as

$$
\begin{aligned}
\frac{N_{i,t+1}}{N_{i,t}} &= \frac{\alpha\nu A_{t+1}(K_{i,t+1} + H_{i,t+1}) + (1-\delta)q_{K,t+1}K_{i,t+1} + (1-\delta)q_{H,t+1}H_{i,t+1} - R_{f,t}B_{i,t}}{q_{K,t}K_{i,t+1} + q_{H,t}H_{i,t+1} - B_{i,t}} \\
&= \frac{q_{K,t}K_{i,t+1}}{N_{i,t}}\bar{R}_{K,t+1} + \frac{q_{H,t}H_{i,t+1}}{N_{i,t}}\bar{R}_{H,t+1} - \frac{B_{i,t}}{N_{i,t}}R_{f,t}, \quad (39)
\end{aligned}
$$

---

[16]Because of these simplifying assumptions, our model is silent on why some firms are constrained and others are not.

[17]Recall that $\theta_t = 0$ for all t if $\nu = 1$.

where we define $\bar{R}_{H,t+1}$ and $\bar{R}_{K,t+1}$ as the return on type-H and type-K capital, respectively, with

$$\bar{R}_{j,t+1} = \frac{\alpha \nu A_{t+1} + (1-\delta) q_{j,t+1}}{q_{j,t}} \tag{40}$$

for $j = H, K$. In Equation (40), we use an upper bar to denote returns measured in net worth units (and not consumption units).

Using the above notation, the return on firm i's equity in (38) can be written as

$$R_{i,t+1} \approx \frac{\lambda + (1-\lambda)\mu_{t+1}}{\mu_t} \left[\frac{q_{K,t} K_{i,t+1}}{N_{i,t}} \bar{R}_{K,t+1} + \frac{q_{H,t} H_{i,t+1}}{N_{i,t}} \bar{R}_{H,t+1} - \frac{B_{i,t}}{N_{i,t}} R_{f,t} \right]. \tag{41}$$

From the perspective of the cross-section of equity returns, the change in net worth component (the first term on the right-hand side of (41)) is the same for all firms, and differences in expected returns must be due to the term in square brackets, which represents the weighted average of the returns on type-K and type-H capital, minus the return on the risk-free bond. Because type-K and type-H capital have the same marginal product, differences between $\bar{R}_{K,t+1}$ and $\bar{R}_{H,t+1}$ can only be due to differences in $q_{K,t+1}$ and $q_{H,t+1}$. We iterate equation (32) forward to obtain

$$q_{K,t} = E \left[\sum_{j=1}^{\infty} \tilde{M}_{t,t+j} \left\{ (1-\delta)^{j-1} \left[\alpha \nu A_{t+j} + (1-\delta) \zeta \frac{\eta_{t+j}}{\mu_{t+j}} \right] \right\} \right] + \zeta \frac{\eta_t}{\mu_t} \tag{42}$$

and do the same for equation (33) to obtain

$$q_{H,t} = E \left[\sum_{j=1}^{\infty} \tilde{M}_{t,t+j} \left\{ (1-\delta)^{j-1} \alpha \nu A_{t+j} \right\} \right]. \tag{43}$$

Here $\tilde{M}_{t,t+j} \equiv \prod_{k=0}^{j-1} \frac{M_{t+k+1} \left[\lambda + (1-\lambda)\mu_{t+k+1} \right]}{\mu_{t+k}}$ can be interpreted as firms' stochastic discount factor.

Equations (42) and (43) are different from what one would see in standard consumption-based asset pricing models, since type-K capital and type-H capital have identical cash flows but different prices because of the Lagrangian multiplier component of asset prices. Note that $q_{K,t}$, $q_{H,t}$, $\bar{R}_{K,t}$, and $\bar{R}_{H,t}$ are all measured in net worth units and not consumption units. They are prices and returns received by firms and not households. Because firms are

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constrained, asset prices contain a Lagrangian multiplier component. Type-K and type-H capital, from the firms' perspective, have different prices and expected returns even though they generate identical cash flows.[18]

Clearly, the key to understanding the difference in the expected returns of type-K capital and type-H capital is the Lagrangian multiplier component of the asset prices. As we illustrated in the three-period model, the Lagrangian multipliers are countercyclical and act as a hedge against aggregate shocks. As a result, $q_{K,t}$ will be less sensitive to aggregate shocks and less cyclical.

These asset pricing implications of our infinite-horizon model are best illustrated with impulse-response functions. In Figure 2, we plot the responses of quantities and prices to a one-standard-deviation negative productivity shock. We make two observations. First, a negative productivity shock lowers output and investment (second and third graph on the left side) as in standard macro models. In addition, as shown in the bottom graph on the left, entrepreneur net worth drops sharply (third graph on the right side).

Second, because entrepreneur net worth drops sharply, the price of type-H capital also goes down substantially. The decrease in the price of collateralizable type-K capital, on the other hand, is much smaller. This is because the Lagrangian multiplier η on the collateral constraint (first graph on the right side) increases on impact and offsets the effect of a negative productivity shock on the price of this type of capital. As a result, the return of type-K capital responds much less to negative productivity shocks than that of type-H capital (bottom graph on the right side), as can be seen from a comparison between the solid black and the dashed lines. This finding implies that collateralizable capital is indeed less risky than non-collateralizable capital in our model.

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[18]It is important to note, however, that households are not constrained, and they hold the firms' equity and debt. As a result, from the household's perspective, differences in the expected returns on stocks and bonds must be reflected in differences in the cash flow properties of these assets. We thank an anonymous referee for pointing this out to us.

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Figure 2: Impulse-response functions for a negative aggregate productivity shock

The graphs in this figure represent log-deviations from the steady state for quantities (left side) and prices (right side) induced by a one-standard-deviation negative shock to aggregate productivity. The parameters are shown in Table 3. The horizontal axis represents time in months.
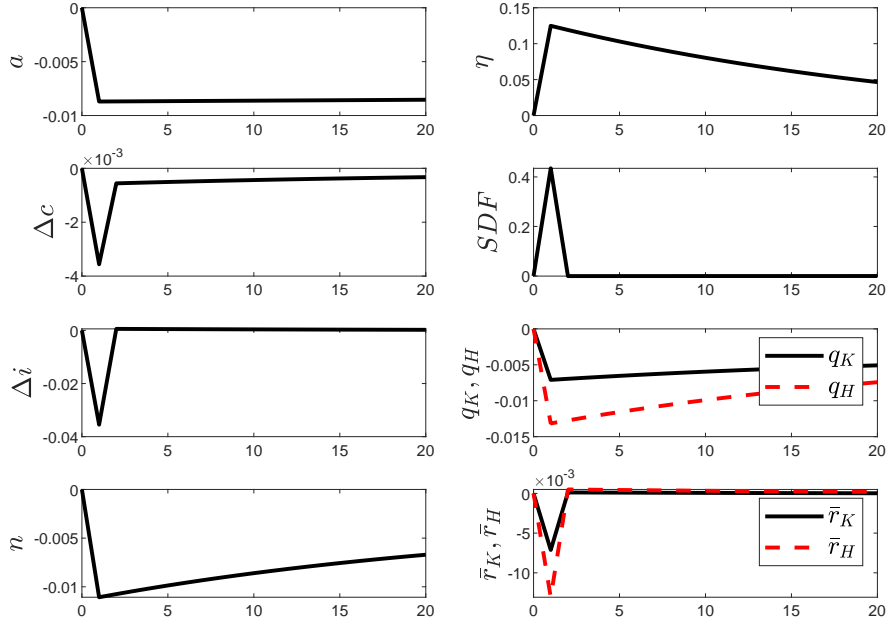
6   Quantitative Model Predictions

In this section, we calibrate our model and evaluate its ability to replicate key moments of both macroeconomic quantities and asset prices at the aggregate level. More importantly, we investigate its performance in terms of quantitatively accounting for key features of firm characteristics and producing a collateralizability premium in the cross-section. For macroeconomic quantities, we focus on a long sample of U.S. annual data from 1930 to 2016. All macroeconomic variables are real and per capita. Consumption, output, and physical investment data are from the Bureau of Economic Analysis (BEA).

To obtain the time series for the total amount of tangible and intangible assets, we firstly aggregate the respective total amount of intangible and tangible capital across all U.S. Compustat firms in a given year. The time series of the aggregate intangible-to-tangible asset ratio is then obtained by dividing the first by the second series element wise.

For the purpose of cross-sectional analyses, we make use of several data sources at the micro level, including (1) firm-level balance sheet data from the CRSP/Compustat Merged

Fundamentals Annual Files, (2) monthly stock returns from CRSP, and (3) industry level non-residential capital stock data from the BEA table "Fixed Assets by Industry." Appendix D.3 provides more details concerning our data sources at the firm and industry level.

6.1 Specification of aggregate shocks

We first formalize the specification of the exogenous aggregate shocks in this economy. First, log aggregate productivity $a \equiv \log(A)$ follows the process

$$a_{t+1} = a_{ss}(1 - \rho_A) + \rho_A a_t + \sigma_A \varepsilon_{A,t+1}, \tag{44}$$

where a_{ss} denotes the steady-state value of a. Second, we also introduce the shocks to entrepreneurs' liquidation probability λ. As is well known in the literature of macroeconomic models with financial frictions, the aggregate productivity shock alone does not create quantitatively enough volatility in capital prices and entrepreneurs' net worth. Additional source of shocks, for example, a capital quality shock as in Gertler and Kiyotaki (2010) and Elenev, Landvoigt, and Van Nieuwerburgh (2018), is needed to generate a higher volatility in net worth. In our model, because a shock to λ affects the entrepreneurs' discount rate and therefore their net worth, without directly affecting the real production, we interpret it as a financial shock, in a spirit similar to Jermann and Quadrini (2012). Importantly, our general model intuition that collateralizable assets provide a hedge against aggregate shocks holds for both productivity and financial shocks.

To technically maintain $\lambda \in (0, 1)$ in a parsimonious way, we set

$$\lambda_t = \frac{\exp(x_t)}{\exp(x_t) + \exp(-x_t)},$$

where x_t follows the process

$$x_{t+1} = x_{ss}(1 - \rho_x) + \rho_x x_t + \sigma_x \varepsilon_{x,t+1},$$

with x_{ss} again denoting the steady-state value. We assume the innovations to a and x have

the following structure:

$$
\begin{bmatrix} \varepsilon_{A,t+1} \\ \varepsilon_{x,t+1} \end{bmatrix} \sim Normal \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{A,x} \\ \rho_{A,x} & 1 \end{bmatrix} \right),
$$

in which the parameter $\rho_{A,x}$ captures the correlation between the two shocks. In the benchmark calibration, we assume $\rho_{A,x} = -1$. First, a negative correlation indicates that a negative productivity shock is associated with a positive discount rate shock. This assumption is necessary to generate a positive correlation between consumption and investment growth, consistent with the data. When only the financial shock ε_x is present, contemporaneous consumption and investment will be affected, but not output. In this case, the resource constraint (24) implies a counterfactually negative correlation between consumption and investment growth. Second, the assumption of a perfectly negative correlation is for parsimony, and it effectively implies that there is only one aggregate shock in this economy. We relax this assumption and provide sensitivity analyses by varying the shock correlation parameter $\rho_{A,x}$ to be different from -1 in Appendix C.

6.2   Calibration

We calibrate our model at the monthly frequency and present the parameters in Table 3. The first group of parameters are those that can be determined based on the literature. In particular, we set the relative risk aversion γ to 20 and the intertemporal elasticity of substitution ψ to 2.3. These parameter values are in line with papers in the long-run risks literature, most notably Bansal and Yaron (2004). The capital share parameter α is set to 0.33, as in the standard real business cycles literature (Kydland and Prescott (1982)). The span of control parameter ν is set to 0.85, consistent with Atkeson and Kehoe (2005).

The parameters in the second group are determined by matching a set of first moments of quantities and prices. We set the long-term average economy-wide productivity growth rate $e^{a_{ss}}$ to match a value for the U.S. economy of 2% per year. The time discount factor β is set to match the average real risk-free rate of 1.10% per year. The share of type-K capital investment ϕ is set to 0.54 to match an intangible-to-tangible-asset ratio of 53% for

the average U.S. Compustat firm.[19] The capital depreciation rate is set to be 11% per year, consistent with the RBC literature (Kydland and Prescott (1982)). For parsimony, we assume the same depreciation rate for both types of capital. The parameter x_{ss} is set to match an average exit probability $\overline{\lambda}$ of 0.01, targeting an average corporate duration of 10 years of U.S. Compustat firms. We calibrate the remaining two parameters related to financial frictions, the collateralizability parameter ζ and the transfer to entering entrepreneurs χ, to generate an average non-financial corporate sector leverage ratio equal to 0.32 and an average investment-to-output ratio of 17%. These values are broadly in line with the data, where leverage is measured by the median lease capital adjusted leverage ratio of U.S. non-financial firms in Compustat.

The parameters in the third group are determined by second moments in the data. The persistence parameter of the TFP shocks ρ_A is set to 0.999 to roughly match the autocorrelation of output growth. We set the persistence parameter of the financial shock ρ_x equal to 0.961 to reproduce the persistence of the corporate leverage ratio that we find in the data. As discussed above, we impose a perfectly negative correlation between productivity and financial shocks; that is, we set $\rho_{A,x} = -1$. The standard deviations of the shock to the exit probability λ, σ_x, and to productivity, σ_A, are jointly calibrated to match the volatilities of consumption growth and the correlation between consumption and investment growth. For the capital adjustment cost function we choose a standard quadratic form, that is,

$$
g\left(\frac{I_t}{K_t + H_t}\right) = \frac{I_t}{K_t + H_t} + \frac{\tau}{2}\left(\frac{I_t}{K_t + H_t} - \frac{I_{ss}}{K_{ss} + H_{ss}}\right)^2,
$$

where X_{ss} denotes the steady-state values for $X \in \{I, K, H\}$. The elasticity parameter of the adjustment cost function, τ, is set to 25 to allow our model to achieve a sufficiently high volatility of investment, broadly in line with the data.

The last group contains the parameters related to the idiosyncratic productivity shocks, μ_z and σ_z. We calibrate them to match the annualized mean (10%) and the annualized volatility (25%) of idiosyncratic productivity growth in the cross-section of U.S. non-financial firms in Compustat. In Appendix C, we present sensitivity analyses to assess the robustness of the quantitative implications of our model with respect to variations in parameter values.

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[19]The construction of intangible capital is explained in detail in Appendix D.3.

6.3 Aggregate moments

We now turn to the quantitative performance of the model at the aggregate level. We solve and simulate our model at the monthly frequency and aggregate the model-generated data to compute annual moments.[20] We show that our model is broadly consistent with the key empirical features of macroeconomic quantities and asset prices. More importantly, it produces a sizable negative collateralizability spread at the aggregate level, i.e., the expected return on collateralizable capital is substantially lower than that on non-collateralizable capital.

Table 4 reports the model-simulated moments of macroeconomic quantities and asset returns and compares them to their counterparts in the data.

In terms of aggregate moments for macro quantities (top panel), our calibration features a low volatility of consumption growth (2.9%) and a relatively high volatility of investment (9.75%). Thanks to the negative correlation between productivity and financial shocks, our model can reproduce a positive consumption-investment correlation (51%), consistent with the data. The model also generates a persistence of output growth in line with aggregate data and an average intangible-to-tangible-capital ratio of 54%, a value broadly consistent with the average ratio across U.S. Compustat firms. The investment-to-output ratio is 17%, which is close to the value of 20% in the data. In summary, our model inherits the success of real business cycle models with respect to the quantity side of the economy.

Turning the attention to the asset pricing moments (bottom panel), our model produces a low average risk-free rate (0.9%) and a high equity premium (5.7%), comparable to key empirical moments for aggregate asset markets. Because of the endogenous leverage implied by the model, our setup is able to generate a high market equity premium compared with standard general equilibrium asset pricing models with production, such as, e.g., Croce (2014) and Ai et al. (2012). Overall, our model performs quite well in terms of standard macro and asset pricing moments at the aggregate level.

Moreover, in our model, the difference in average returns between type-K and type-H

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[20]Because the limited commitment constraint is binding in the steady state, we solve the model using a second-order local approximation around the steady state using the `Dynare` package. We have also solved versions of our model using the global method developed in Ai et al. (2016) and verified the accuracy of the local approximation.

capital, or, equivalently, the difference between their risk premia is negative and large (-7.5% annually).

6.4 The cross section of collateralizability and equity returns

Equation (41) makes it clear that the cross-section of expected returns in our model is driven by the difference in the expected returns on type-K and type-H capital. To understand the implications of this difference, it is convenient to write equation (41) as

$$R_{i,t+1} \approx \frac{\lambda + (1-\lambda)\,\mu_{t+1}}{\mu_t}\left[(1-\omega_{i,t})\,\bar{R}^{Lev}_{K,t+1} + \omega_{i,t}\bar{R}_{H,t+1}\right],\qquad(45)$$

where $\omega_{i,t} = \frac{q_{H,t}H_{i,t+1}}{N_{i,t}}$ is the fraction of the firm's net worth invested in type-H capital, and $\bar{R}^{Lev}_{K,t+1}$ is the levered return on type-K capital, defined as

$$\bar{R}^{Lev}_{K,t+1} = \frac{q_{K,t}K_{i,t+1}}{N_{i,t}}\bar{R}_{K,t+1} - \frac{B_{i,t}}{N_{i,t}}R_{f,t}.$$

The interpretation of Equation (45) is that the return on a firm's equity can be written as a weighted average of the levered return on type-K capital and the (unlevered) return on type-H capital. Thanks to the simplifying assumption of perfect substitutability between type-K and type-H capital, firms' asset collateralizability can be summarized by a single state variable, $\omega_{i,t}$. The expected return on equity depends in its loadings on type-H and type-K capital. The firm-level state variable $\omega_{i,t}$ is a sufficient statistic summarizing all information relevant for expected returns. In our model, firms with high productivity and low net worth have a higher demand for external financing. They optimally choose to acquire more type-K capital because it can be used to increase leverage and acquire more capital.

From equation (45), we see that high asset collateralizability has two offsetting effects. First, it allows the firm to borrow more debt and raise leverage, which tends to increase the expected return on equity. Note that whenever the collateral constraint (14) is binding, $B_{i,t} = \zeta q_{K,t}K_{i,t+1}$. The levered return on type-K can therefore be written as

$$\bar{R}^{Lev}_{K,t+1} = \frac{1}{1-\zeta}(\bar{R}_{K,t+1} - R_{f,t}) + R_{f,t}.\qquad(46)$$

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As long as $E_t \left[\bar{R}_{K,t+1} \right] > R_{f,t}$, higher asset collateralizability tends to increase the expected return on firm equity due to the leverage effect captured by the coefficient $\frac{1}{1-\zeta}$ in the above equation.

Second, the collateralizable asset itself has a lower expected return because of the Lagrangian multiplier component of asset price. Equation (45) implies that in the cross section, high asset collateralizability is associated with lower expected returns if the collateralizability effect dominates the leverage effect. The relative riskiness of type-K versus type-H capital thus depends on the relative contributions of the collateral and the offsetting leverage effect. As shown in Table 4, our model produces a sizable negative average return spread of -7.5% between levered collateralizable capital and unlevered non-collateralizable capital. Therefore, in our calibration, quantitatively, the leverage effect is relatively small, and the collateralizability premium is negative.

We now turn to the implications of our model on the cross-section of collateralizability-sorted portfolios. We simulate firms from the model, measure the collateralizability of firm assets, and conduct the same collateralizability-based portfolio-sorting procedure as in the data. In Table 5, we report the average returns of the sorted portfolios along with several other characteristics from the data and those from the simulated model.

As in the data, firms with high asset collateralizability have a significantly lower average return than those with low asset collateralizability in our model. Quantitatively, our model produces a sizable collateralizability spread of around 4%, accounting for more than 50% of the spread in the data.

Table 5 also reports several other characteristics of the collateralizability-sorted portfolios that are informative about the economic mechanism we emphasize in our model. First, not surprisingly, the collateralizability measure is monotonically increasing for collateralizability-sorted portfolios. In fact, asset collateralizability in our model is similar in magnitude to that in the data.

Second, as in the data, leverage is increasing in asset collateralizability. This implication of our model is consistent with the data and the broader corporate finance literature that emphasizes the importance of collateral in firms' capital structure decisions (e.g., Rampini

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Electronic copy available at: https://ssrn.com/abstract=3474975

and Viswanathan (2013)). The dispersion in leverage in our model is somewhat higher than in the data. This finding is not surprising, as in our model, asset collateralizability is the only factor determining leverage, whereas in the data there are many other determinants of the capital structure. Note that despite the higher leverage, high collateralizability portfolios have a lower average return. As we explained earlier, this is because in our model, $E\left[\bar{R}_{K,t+1}^{Lev}\right] < E\left[\bar{R}_{H,t+1}\right]$. This feature of our model also helps to reconcile the mixed evidence on the relationship between leverage and expected stock returns, as discussed in Gomes and Schmid (2008).

Third, as in the data, high collateralizability firms also tend to have higher asset growth rates and higher return on equity (ROE). In our model, other things being equal, firms that experienced a history of positive productivity shocks have a higher financial need and optimally chose to obtain higher asset collateralizability. In the model, a history of higher productivity shocks is also associated with higher asset growth rates and higher ROE. As we show in Table 5, this feature of our model is also consistent with the pattern in the data. As a result, our model also provides an explanation for the empirical fact that firms with high asset growth have lower average returns, as documented in the literature, for example, Cooper, Gulen, and Schill (2008).

6.5 Testable implications of the Lagrangian multiplier effect

In Appendix B, we provide additional empirical evidence on the relation between collateralizability and the cross section of stock returns. In particular, we consider two alternative proxies for financial shocks: (ΔEM), the change in the general cost of external finance (debt and equity), as suggested by Eisfeldt and Muir (2016), and $(\Delta\sigma_{CS})$, the log change in the cross-sectional dispersion of firm-level cash flow growth, in a spirit similar to Elenev, Landvoigt, and Van Nieuwerburgh (2018).

First and most importantly, in Section B.1, we show that the cash flows of high asset collateralizability firms exhibit less negative sensitivity to financial shocks than those of firms with low collateralizability. Second, In Section B.2, we conduct a standard Fama and MacBeth (1973) two-pass regression and show that the proxies for financial shocks are

significantly negatively priced.

Taken together, high collateralizability firms are less negatively exposed to shocks with a negative market price of risk, so that ceteris paribus their expected returns are lower. This finding strongly corroborates the model mechanism that collateralizable assets provide insurance against aggregate shocks through the Lagrangian multiplier effect in asset prices.

7 Conclusion

In this paper, we present a general equilibrium asset pricing model with heterogeneous firms and collateral constraints. Our model predicts that collateralizable assets provide insurance against aggregate shocks and should therefore earn a lower expected return. They relax the collateral constraint, which is more binding in recessions.

We develop an empirical collateralizability measure for a firm's assets and provide empirical evidence consistent with the predictions of our model. In particular, we find in the data that the difference in average equity returns between firms with a low and a high degree of asset collateralizability amounts to almost 8% per year. When we calibrate our model to the dynamics of macroeconomic quantities, we show that this credit market friction is a quantitatively important determinant for the cross-section of asset returns.

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Table 1: Capital Structure Regressions

This table reports the results for the regression

$$\frac{B_{i,t}}{AT_{i,t}} = \zeta_S StructShare_{l(i),t} + \zeta_E EquipShare_{l(i),t} + \gamma X_{i,t} + \varepsilon_{i,t},$$

where, for a given firm i, $l(i)$ denotes the industry l that firm i belongs to in year t. The sample starts in 1978 and ends at 2016, at annual frequency. *StructShare* and *EquipShare* are the respective shares of structure and equipment capital in a given industry, computed according to Table D.11 in the appendix. We assume all firms within the same industry have the same structure and equipment shares. $X_{i,t}$ represents a vector of controls typically used in capital structure regressions, including size, book-to-market ratio, profitability, marginal tax rate, earnings volatility, and bond ratings. $B_{i,t}$ is total debt, defined as the sum of long-term and short-term financial debt ($DLTT + DLC$). Additionally, to capture non-financial debt, we adjust debt by adding the capitalized value of operating leases to the financial debt, following Li, Whited, and Wu (2016). The column labeled "Full" corresponds to the regression performed on all firms. The columns labeled "Non-Dividend," "SA cons.," and "WW cons." show the results for the samples of firms classified as constrained based on thir not having paid dividends, their SA index (Hadlock and Pierce (2010)) being above the median, or their WW index (Whited and Wu (2006)) being above the median in year $t-1$, respectively. The column labeled "All Cons." refers to the regression for the sample of firms that are classified as constrained with respect to all three measures. All right-hand side variables, except *Struct Share* and *Equip Share*, are demeaned. Standard errors are clustered at the firm-year level.

	(1) Full	(2) Non-Dividend	(3) SA cons.	(4) WW cons.	(5) All cons.
Struct Share	0.624***	0.714***	0.648***	0.677***	0.686***
	(33.96)	(23.62)	(12.41)	(18.79)	(10.65)
Equip Share	0.412***	0.501***	0.406***	0.455***	0.436***
	(27.56)	(19.85)	(13.01)	(18.58)	(11.96)
Observations	63,691	31,461	21,808	27,122	15,944
R^2	0.644	0.620	0.553	0.599	0.572

t-statistics in parentheses
***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$

Table 2: Portfolios Sorted on Collateralizability

This table reports average value-weighted excess returns for portfolios sorted on collateralizability. The sample period is from July 1979 to December 2016. At the end of June of each year t, we sort the constrained firms into quintiles based on their collateralizability measures available at the end of year $t-1$, where quintile 1 (quintile 5) contains the firms with the lowest (highest) share of collateralizable assets. We hold the portfolios for one year, from July of year t until June of year $t+1$. A firm is classified as financially constrained in year t if its WW (Whited and Wu (2006)) index is greater than the corresponding cross-sectional median in year-end $t-1$, and the same for the SA index (Hadlock and Pierce (2010)). "Non-Dividend" means that a firm has not paid any dividends in year $t-1$. The t-statistics are computed based on Newey-West adjusted standard errors. For each portfolio, as well as for the long-short portfolio denoted by "1-5", the table reports its average excess return $E[R] - R_f$ (in annualized percentage term), the associated t-statistic, its return volatility σ (in annualized percentage terms), the Sharpe ratio (SR), the market beta β_{Mkt}, and the associated t-statistic. We annualize returns by multiplying by 12.

	1	2	3	4	5	1-5
	Financially constrained firms - WW index					
$E[R] - R_f$	13.27	10.76	10.90	7.74	5.41	7.86
$t_{E[R]-R_f}$	2.75	2.36	2.54	1.94	1.29	2.53
σ	29.26	26.26	24.31	23.72	22.87	18.05
SR	0.45	0.41	0.45	0.33	0.24	0.44
β_{Mkt}	1.27	1.24	1.19	1.22	1.18	0.09
t_β	16.68	19.50	20.80	23.47	21.45	1.25
	Financially constrained firms - SA index					
$E[R] - R_f(\%)$	12.03	11.97	11.39	6.57	4.52	7.51
$t_{E[R]-R_f}$	2.38	2.48	2.70	1.53	1.02	2.48
$\sigma(\%)$	30.23	27.22	24.42	24.81	23.99	17.67
SR	0.40	0.44	0.47	0.26	0.19	0.42
β_{Mkt}	1.34	1.25	1.16	1.28	1.25	0.08
t_β	16.47	17.07	19.39	23.39	23.92	1.16
	Financially constrained firms - Non-Dividend					
$E[R] - R_f(\%)$	16.12	10.12	7.50	8.45	8.14	7.98
$t_{E[R]-R_f}$	3.72	2.20	1.72	1.88	2.01	2.85
$\sigma(\%)$	27.13	26.05	25.56	25.67	24.17	16.73
SR	0.59	0.39	0.29	0.33	0.34	0.48
β_{Mkt}	1.34	1.35	1.37	1.41	1.32	0.02
t_β	20.67	21.41	26.96	27.65	25.65	0.22

Table 3: Calibrated Parameter Values

This table reports the parameter values used for our monthly calibrations.

Parameter	Symbol	Value
Relative risk aversion	γ	20
IES	ψ	2.3
Capital share in production	α	0.33
Span of control parameter	ν	0.85
Mean productivity growth rate	$e^{a_{ss}}$	0.067
Time discount rate	β	0.996
Share of type-K investment	ϕ	0.543
Capital depreciation rate	δ	0.11/12
Average death rate of entrepreneurs	$\bar{\lambda}$	0.01
Collateralizability parameter	ζ	0.513
Transfer to entering entrepreneurs	χ	0.994
Persistence of TFP shocks	ρ_A	0.999
Vol. of TFP shock	σ_A	0.009
Persistence of financial shocks	ρ_x	0.961
Vol. of financial shocks	σ_x	0.035
Corr. between TFP and financial shocks	$corr(\varepsilon_A, \varepsilon_x) = -0.8$	-1
Capital adj. cost parameter	τ	25
Mean idio. productivity growth	μ_ϵ	0.0083
Vol. of idio. productivity growth	σ_ϵ	0.072

Table 4: Aggregate Moments: Model and Data

This table presents annualized moments from the model simulations and the data. The moments for the model are obtained from repetitions of small samples. We simulate the model at the monthly frequency based on the calibration in Table 3 and then time-aggregate the monthly observations to annual frequency. Data refer to the U.S. and span the period 1930-2016, unless otherwise stated. Numbers in parentheses are Newey-West adjusted standard errors. Inside our model, $\frac{B}{K+H}$ captures the book leverage ratio. The market return R^M corresponds to the aggregate return on entrepreneurs' net worth at the aggregate level. $\bar{R}_H$ denotes the return on type-H capital. $\bar{R}_K^{Lev}$ is the levered return on type-K capital. Volatility, correlations, and first-order autocorrelation are denoted as $\sigma(\cdot)$, $corr(\cdot, \cdot)$ and $AC1(\cdot)$, respectively. As the empirical counterparts, we report the lease-adjusted leverage ratio. We use physical assets (PPEGT), which consist of both structure and equipment capital, to proxy for type-K capital. Intangible capital is used to proxy for type-H. The construction of firm-level intangible capital is detailed in Appendix D.3.

	Data	Benchmark
$\sigma(\Delta y)$	3.05 (0.60)	3.50
$\sigma(\Delta c)$	2.53 (0.56)	2.92
$\sigma(\Delta i)$	10.30 (2.36)	9.75
$corr(\Delta c, \Delta i)$	0.40 (0.28)	0.51
$corr(\Delta c, \Delta y)$	0.82(0.07)	0.93
$AC1(\Delta y)$	0.49 (0.15)	0.51
$AC1(\frac{B_t}{K_t+H_t})$	0.86 (0.33)	0.81
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.33
$E[K/(H+K)]$	0.53(0.01)	0.54
$E[I_t/Y_t]$	0.17(0.01)	0.17
$E[R^M - R^f]$	5.71 (2.25)	5.71
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85
$E[R^f]$	1.10 (0.16)	0.90
$\sigma(R^f)$	0.97 (0.31)	0.89
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50

Table 5: Firm Characteristics and Expected Returns: Data and Model

This table compares the moments in the empirical data (Panel A) and the model simulated data (Panel B) at the portfolio level. Panel A reports the statistics computed from the sample of financially constrained firms in the empirical data, from July 1979 to December 2016. In each year t, a firm is classified as financially constrained if its WW index is higher than the cross-sectional median in year $t-1$. We sort the constrained firms into quintiles at the end of June of each year t, based on the collateralizability measure at the end of year $t-1$. The portfolios are held for a year and then rebalanced every year in July. We perform model simulation at the monthly frequency and then perform the same portfolio sorts as in the data. The table shows the median of firm characteristics using the value from the year end, such as the collateralizability measure, book leverage (lease adjusted), and growth rate of physical capital. We also report the value-weighted excess returns, $E[R^e](\%)$ (annualized by multiplying by 12, in percentage terms), for quintile portfolios sorted on collateralizability. In Panel A, collateralizability is constructed as in Appendix D.2 Type-K asset growth is defined as the growth rate of physical capital (PPEGT). Book leverage is adjusted for leased capital following Li, Whited, and Wu (2016). Return on equity (ROE) is defined as operating income (OIBDP) over book equity, where book equity is defined following Fama and French (1992). For the model moments in Panel B, collateralizability is computed as $\frac{\zeta K}{K+H}$, book leverage is $\frac{B}{K+H}$, and type-K asset growth is $\frac{\Delta K}{K}$. ROE in the model is defined as the firm's profit over the book equity ratio, where book equity is total assets minus total debt.

Panel A: Data

	1	2	3	4	5	1-5
Collateralizability	0.08	0.17	0.26	0.38	0.62	
Book leverage	0.10	0.16	0.23	0.34	0.46	
Type-K asset growth	0.08	0.09	0.10	0.11	0.13	
ROE	0.06	0.16	0.20	0.23	0.22	
$E[R^e]$ (%)	13.27	10.76	10.90	7.74	5.41	7.86

Panel B: Model

	1	2	3	4	5	1-5
Collateralizability	0.19	0.31	0.35	0.40	0.51	
Book leverage	0.19	0.31	0.35	0.40	0.51	
Type-K asset growth	-0.06	0.05	0.12	0.17	0.20	
ROE	0.06	0.08	0.08	0.09	0.11	
$E[R^e]$ (%)	7.96	6.60	5.96	5.24	3.82	4.14

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Internet Appendix for "The Collateralizability Premium"

A Proof of propositions

A.1 Proof of Proposition 1

It is convenient to derive the optimality conditions for firms' profit maximization using the dynamic programming formulation. Define $V_t(N_t)$ as firms' value function at time t. We have, for $t = 0, 1$,

$$V_t(N_t) = \max\{D_t + E[M_{t+1}N_{t+1}]\} \tag{A1}$$

$$D_t + q_t K_{t+1} = N_t + B_t \tag{A2}$$

$$N_{t+1} = A_{t+1}K_{t+1}(1-\delta)q_{t+1}K_{t+1} - r_t B_t \tag{A3}$$

$$B_t \leq \zeta q_t K_{t+1} \tag{A4}$$

$$D_t \geq 0. \tag{A5}$$

We first derive a set of optimality conditions that characterize the equilibrium. Taking first order conditions of (9) w.r.t. K_{t+1} and B_t, we have:

$$(1 + \phi_t)q_t = E[M_{t+1}\mu_{t+1}\{A_{t+1} + p_{t+1}\}] + \zeta\eta_t,$$

$$\bar{\mu}_t = E[M_{t+1}\mu_{t+1}r_t] + \frac{\eta_t}{q_t}.$$

The envelope condition implies $\mu_t = \bar{\mu}_t$, which we can use to simplify the above equations to write:

$$q_t = E\left[M_{t+1}\frac{\mu_{t+1}}{\mu_t}\{A_{t+1} + p_{t+1}\}\right] + \zeta\frac{\eta_t}{\mu_t}, \tag{A6}$$

$$1 = E\left[M_{t+1}\frac{\mu_{t+1}}{\mu_t}r_t\right] + \frac{\eta_t}{\mu_t}\frac{1}{q_t}. \tag{A7}$$

Also, note that whenever the collateral constraint is binding, equations (A2) and (A4) can be combined to write:

$$(1 - \zeta)q_t[(1-\delta)K_t + I_t] = N_t.$$

Using the capital producer's optimality condition, and the functional form of the adjustment cost,

we have $q_t = 1 + \tau (i_t - \delta)$. The above equation can be written as:

$$n_t = (1 - \zeta) [1 + \tau (i_t - \delta)] [1 + i_t - \delta]. \tag{A8}$$

Note that equation (A8) implicitly define i as a function of n, which we denote as $i(n)$. Given the definition of $i(n)$, we can write Tobin's q as $q_t = 1 + \tau [i(n_t) - \delta]$, and normalized consumption as $c_t = c(A_t, n_t)$, where

$$c(A, n) \equiv A - i(n) - \frac{1}{2}\tau [i(n) - \delta]^2. \tag{A9}$$

Using the above results, we can solve for the prices and quantities in period 1. In period 2, all of firms' cash flow are paid back to household as consumption goods. Therefore $\mu_2 = 1$. In addition, capital is valueless at the end of period 2 and $q_2 = 0$. Therefore equations (A6) can be written as $\mu_1 q_1 = E[M_2 A_2] + \zeta \eta_1$, and equation (A7) can be written as $\mu_1 = E[M_2] r_1 + \frac{\eta_1}{q_1}$. Under the assumption of log preference, $M_2 = \frac{C_1}{C_2} = \frac{A_1 K_1 - H(I_1, K_1)}{A_2 K_2} = \frac{c(A_1, n_1)}{A_2[(1-\delta)+i(n_1)]}$, and therefore, $M_2 A_2 = \frac{c(A_1, n_1)}{(1-\delta)+i(n_1)}$. Also, the household's intertemporal Euler equation implies $E[M_2 r_1] = 1$. Equations (A6) and (A7) can be further simplified as:

$$q_1 = \frac{1}{\mu_1} \frac{c(A_1, n_1)}{(1 - \delta) + i(n_1)} + \zeta \frac{\eta_1}{\mu_1}, \tag{A10}$$

$$\mu_1 = 1 + \frac{\eta_1}{q_1}. \tag{A11}$$

Combining equations (A10) and (A11), and using the fact that $q_1 = 1 + \tau [i(n_1) - \delta]$, we can determine η_1 and μ_1 as functions of (n_1, A_1):

$$\eta_1(A_1, n_1) = \frac{1}{1 - \zeta} \left\{ \frac{A_1 - i(n_1) - \frac{1}{2}\tau [i(n_1) - \delta]^2}{1 - \delta + i(n_1)} - [1 + \tau(i(n_1) - \delta)] \right\}, \tag{A12}$$

and

$$\mu_1(A, n) = 1 + \frac{\eta_1(A, n_1)}{1 + \tau(i(n_1) - \delta)}. \tag{A13}$$

Equation (10) then follows directly from (A10), (A12), and (A13).

To derive the law of motion of n_1, note that the binding collateral constraint in period 0 implies $B_0 = \zeta q_0 K_1$. Equation $N_1 = A_1 K_1 + p_1 K_1 - r_0 B_0$ therefore implies

$$n_1 = A_1 - (1 - \delta) q_1 - r_0 \zeta q_0. \tag{A14}$$

Using the households' consumption Euler equation, we express the interest r_0 as a function of consumption:

$$r_0 = \frac{1}{\beta E \left[\frac{C_0}{C_1}\right]} = \frac{1}{\beta c (A_0, n_0)} \frac{1}{E \left[\frac{1}{c(A_1, n_1)}\right]}. \tag{A15}$$

Equation (11) then follows from (A14) and (A15) by noting $q_t = 1 + \tau [i (n_t) - \delta]$.

A.2 Proof of Proposition 2

We prove Proposition 2 in two steps. First, we construct an equilibrium and show that under the assumptions of parameter values, the collateral constraint (A4) for both period 0 and period 1 binds. Second, we explicitly solve for the expression of the Lagrangian multipliers η_1 and μ_1 to verify the counter-cyclicality of $\frac{\eta_1}{\mu_1}$, i.e., inequality (12).

Proposed equilibrium prices and quantities Note that under the assumption of $\beta = \tau = \delta = 1$, the $i(n)$ function in (A8) and $c(A, n)$ function in (A9) take simple forms:

$$i(n) = \sqrt{\frac{n}{1 - \zeta}}, \quad c(A, n) = A - \frac{1}{2} - \frac{1}{2} \frac{n}{1 - \zeta}. \tag{A16}$$

We propose the following equilibrium prices and quantities and verify that they indeed satisfy the above listed equilibrium conditions:[21]

$$c_t = c(A_t, n_t); \quad i_t = i(n_t); \quad q_t = 1 + \tau [i(n_t) - \delta], \quad t = 0, 1 \tag{A17}$$

$$\eta_1 = \eta_1(A, n), \mu_1 = \mu_1(A, n), \tag{A18}$$

where

$$\eta_1(A, n) = \frac{1}{\sqrt{1 - \zeta} \sqrt{n}} \left\{ A - \frac{1}{2} - \frac{3}{2} \frac{n}{1 - \zeta} \right\}, \tag{A19}$$

$$\mu_1(A, n) = 1 + \frac{1}{n} \left\{ A - \frac{1}{2} - \frac{3}{2} \frac{n}{1 - \zeta} \right\}. \tag{A20}$$

and the first period net worth is given by:

$$n_1 = n(A_1 | n_0) = A_1 - x(n_0), \tag{A21}$$

---

[21]Because all optimization problems are convexity programming problems, the first order conditions are both necessary and sufficient.

where $x(n_0)$ is given by equation (A23) below.

It is straight forward to show that the proposed prices and quantities satisfy the first order conditions (A6) and (A7). Below we verify that under our assumptions, the constructed Lagrangian multipliers are strictly positive, and therefore, the proposed allocation is indeed an equilibrium in which the collateral constraints are binding in both periods.

Verifying equilibrium conditions   We verify that the collateral constraint must be binding under the proposed prices and quantities through a sequence of lemmas.

Lemma 1. *(Law of motion of net worth)*

The law of motion of net worth can be written as

$$n(A|n_0) = A - x(n_0).$$

Given $A_1 > \frac{1-\zeta}{1-2\zeta}$, $x(n_0)$ is strictly increasing with $x(0) = 0$ and $\lim\limits_{n_0 \to 2(1-\zeta)^2\left(A_0 - \frac{1}{2}\right)} x(n_0) = \infty$.

Proof. Because $\beta = 1$, $\delta = 1$, we can write equation (11) as $n(A|n_0) = A - \zeta \frac{i^2(n_0)}{c(A_0,n_0)} \frac{1}{E\left[\frac{1}{c(A,n(A|n_0))}\right]}$. Using the definition of $i(n)$ and $c(A,n)$, $n(A|n_0) = A - x$, where x is implicitly defined as $x = \frac{\zeta}{1-\zeta} \frac{n_0}{A_0 - \frac{1}{2} - \frac{n_0}{2(1-\zeta)}} \frac{1}{E\left[\frac{1}{c(A,n(A|n_0))}\right]}$. Note that by the definition of $c(A,n)$ (equation (A16)), with $n_1 = A_1 - x$, we have

$$
\begin{aligned}
c(A_1, n_1) &= A_1 - \frac{1}{2} - \frac{1}{2}\frac{A_1 - x}{1 - \zeta} \\
&= \frac{x}{2(1-\zeta)} + \left[1 - \frac{1}{2(1-\zeta)}\right]A_1 - \frac{1}{2}.
\end{aligned}
\tag{A22}
$$

Therefore, $x(n_0)$ as a function of n_0 is defined by the solution to the following equation:

$$
E\left[\frac{x}{\frac{x}{2(1-\zeta)} + \left[\frac{1-2\zeta}{2(1-\zeta)}A_1 - \frac{1}{2}\right]}\right] = \frac{\zeta}{1-\zeta}\frac{n_0}{A_0 - \frac{1}{2} - \frac{n_0}{2(1-\zeta)}}.
\tag{A23}
$$

Under the condition that $A_1 > \frac{1-\zeta}{1-2\zeta}$, the left-hand side is an increasing function of x, and as x increases from 0 to ∞, $E\left[\frac{x}{\frac{x}{2(1-\zeta)} + \left[\frac{1-2\zeta}{2(1-\zeta)}A_1 - \frac{1}{2}\right]}\right]$ increases from 0 to $2(1-\zeta)$. In addition, the right-hand side of equation (A23) is a strictly increasing function of n_0, and as n_0 increases from 0 to $2(1-\zeta)^2\left(A_0 - \frac{1}{2}\right)$, the right-hand side increases from 0 to $2(1-\zeta)$. As a result, equation

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([A23](#)) defines $x(n_0)$ as a strictly increasing function that maps $n_0 \in \left(0, 2(1-\zeta)^2\left(A_0 - \frac{1}{2}\right)\right)$ to $x \in (0, \infty)$. $\qquad\square$

The next lemma provide conditions under which the collateral constraint must be binding in period 1.

Lemma 2. *(Binding constraint for period 1)*

Assume

$$x(n_0) > \frac{2}{3}A_1, \tag{A24}$$

then the collateral constraint in period 1 is binding for all realizations of A_1, that is, $\eta_1(A_1, n_1) > 0$.

Proof. By equation ([A19](#)), the borrowing constraint binds, that is, $\eta_1(A_1, n_1) > 0$ if and only if

$$A_1 - \frac{1}{2} > \frac{3}{2}\frac{n_1}{1-\zeta}. \tag{A25}$$

Using $n_1 = A_1 - x(n_0)$, the above condition can be written as

$$\frac{3}{2(1-\zeta)}x(n_0) > \left(\frac{3}{2(1-\zeta)} - 1\right)A_1 + \frac{1}{2}. \tag{A26}$$

Note that under condition $A_1 > \frac{1-\zeta}{1-2\zeta}$, $\frac{1-2\zeta}{2(1-\zeta)}A_1 > \frac{1}{2}$. Therefore, a sufficient condition for ([A25](#)) is

$$\frac{3}{2(1-\zeta)}x(n_0) > \left(\frac{3}{2(1-\zeta)} - 1 + \frac{1-2\zeta}{2(1-\zeta)}\right)A_1 = \frac{2}{2(1-\zeta)}A_1,$$

which is equivalent to ([A19](#)). $\qquad\square$

Our next lemma provides conditions under which the collateral constraint is binding in period 0.

Lemma 3. *Suppose*

$$n_0 < \frac{1}{2}(1-\zeta)\left(A_0 - \frac{1}{2}\right) \tag{A27}$$

and

$$x(n_0) < \frac{1}{2+\zeta}\left[(1+2\zeta)A_1 + \frac{1}{2}(1-\zeta)\right], \tag{A28}$$

then the collateral constraint in period 0 must be binding, that is, $\eta_0 > 0$.

Proof. Combining equations (A6) and (A7), $\eta_0 > 0$ if and only if

$$E\left[M_1 \mu_1 \left(A_1 + (1-\delta) q_1\right)\right] > E\left[M_1 \mu_1\right] r_0 q_0.$$

Using the fact that $M_1 = \frac{c(A_0, n_0)}{c(A_1, n_1) i(n_0)}$ and $r_0 = \frac{1}{E[M_1]}$, the above condition can be written as:

$$\frac{E\left[\frac{\mu_1 A_1}{c(A_1, n_1)}\right] E\left[\frac{1}{c(A_1, n_1)}\right]}{E\left[\frac{\mu_1}{c(A_1, n_1)}\right]} \left(A_0 - \frac{1}{2} - \frac{1}{2(1-\zeta)} n_0\right) > \frac{n_0}{1-\zeta}. \tag{A29}$$

We first show that

$$\frac{E\left[\frac{\mu_1 A_1}{c(A_1, n_1)}\right] E\left[\frac{1}{c(A_1, n_1)}\right]}{E\left[\frac{\mu_1}{c(A_1, n_1)}\right]} > \frac{2(2+\zeta)}{2\zeta + 3}. \tag{A30}$$

To see this, using (A22), we have

$$
\begin{aligned}
\frac{c(A_1, n_1)}{A_1} &= \frac{\frac{x}{2(1-\zeta)} + \left[1 - \frac{1}{2(1-\zeta)}\right] A_1 - \frac{1}{2}}{A_1} \\
&< \frac{\frac{x}{2(1-\zeta)} + \left[1 - \frac{1}{2(1-\zeta)}\right] A_1}{A_1}
\end{aligned}
$$

Under assumption (A28), $x(n_0) < \frac{1+2\zeta}{2+\zeta} A_1$ and therefore,

$$\frac{c(A_1, n_1)}{A_1} < \frac{1}{2(1-\zeta)} \frac{1+2\zeta}{2+\zeta} + \left[1 - \frac{1}{2(1-\zeta)}\right] = \frac{2\zeta + 3}{2(2+\zeta)}. \tag{A31}$$

As a result,

$$
\begin{aligned}
E\left[\frac{\mu_1 A_1}{c(A_1, n_1)}\right] E\left[\frac{1}{c(A_1, n_1)}\right] &= E\left[\frac{\mu_1 A_1}{c(A_1, n_1)}\right] E\left[\frac{A_1}{c(A_1, n_1)} \frac{1}{A_1}\right] \\
&> \frac{2(2+\zeta)}{2\zeta + 3} E\left[\frac{\mu_1 A_1}{c(A_1, n_1)}\right] E\left[\frac{1}{A_1}\right] \\
&> \frac{2(2+\zeta)}{2\zeta + 3} E\left[\frac{\mu_1}{c(A_1, n_1)}\right],
\end{aligned}
$$

where the first inequality uses (A31) and the second inequality above uses the fact that $\frac{\mu_1 A_1}{c(A_1, n_1)}$ is increasing in A_1 and therefore negatively correlated with $\frac{1}{A_1}$. This establishes (A30).

Given (A30), a sufficient condition for (A29) is

$$\frac{2\zeta + 3}{2(2+\zeta)} \left(A_0 - \frac{1}{2} - \frac{1}{2(1-\zeta)} n_0\right) > \frac{n_0}{1-\zeta}. \tag{A32}$$

To see the above inequality holds, note that assumption (A27) implies

$$A_0 - \frac{1}{2} > \frac{2}{1-\zeta} n_0.$$

Therefore,

$$A_0 - \frac{1}{2} - \frac{1}{2(1-\zeta)} n_0 > \frac{3}{2(1-\zeta)} n_0 > \frac{2(2+\zeta)}{2\zeta+3} n_0,$$

which proves (A31). □

To summarize the above results, we define n^* and $\hat{n}$ as follows. We denote x^{-1} to be the inverse function of $x(n_0)$ defined in (A23), which is a strictly increasing function due to our previous discussion. We denote $A_{\min}$ to be the lowest possible realization of A_1 and $A_{\max}$ to be the highest possible realization of A_1. We set

$$n^* = \min\left\{ x^{-1}\left(\frac{2}{3} A_{\max}\right), \ \frac{1}{2}(1-\zeta)\left(A_0 - \frac{1}{2}\right)\right\}; \tag{A33}$$

$$\hat{n} = x^{-1}\left(\frac{1}{2+\zeta}\left[(1+2\zeta) A_{\min} + \frac{1}{2}(1-\zeta)\right]\right). \tag{A34}$$

By the above lemmas, if $n_0 \in (n^*, \hat{n})$, the collateral constraints in both periods are binding.

Monotonicity of the Lagrangian multiplier In this section, we prove that under the conditions outlined in the previous section, (12) holds by establishing that $\frac{\eta(n_1,A_1)}{\mu(n_1,A_1)}$ is a strictly decreasing function of A_1.

Lemma 4. *(Monotonicity of the Lagrangian multiplier)*

Under the assumption of $n_0 \in (n^*, \hat{n})$,

$$\frac{d}{dA_1}\left[\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)}\right] < 0,$$

that is, the Lagrangian multiplier component of asset price (10) is counter-cyclical.

Proof. Using equations (A19) and (A20), we have

$$\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)} = \sqrt{\frac{n_1}{1-\zeta}} \frac{A_1 - \frac{1}{2} - \frac{3}{2}\frac{n_1}{1-\zeta}}{A_1 - \frac{1}{2} - \left(\frac{3}{2}\frac{1}{1-\zeta} - 1\right) n_1}.$$

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Using the law of motion of net worth, $n_1 = A_1 - x$, we have:

$$\frac{\eta\left(n_1, A_1\right)}{\mu\left(n_1, A_1\right)} = \sqrt{\frac{A_1 - x}{1 - \zeta} \frac{\frac{3}{2(1-\zeta)}x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 1\right)A_1}{\left(\frac{3}{2(1-\zeta)} - 1\right)x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 2\right)A_1}}.$$

Therefore,

$$\frac{d}{dA_1}\ln\left[\frac{\eta\left(n_1, A_1\right)}{\mu\left(n_1, A_1\right)}\right] = \frac{1}{2}\frac{1}{A_1 - x} - \frac{\frac{3}{2(1-\zeta)} - 1}{\frac{3}{2(1-\zeta)}x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 1\right)A_1} + \frac{\frac{3}{2(1-\zeta)} - 2}{\left(\frac{3}{2(1-\zeta)} - 1\right)x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 2\right)A_1}$$

To save notation, we denote $a = \frac{3}{2(1-\zeta)} - 1$. We have:

$$\begin{aligned}\frac{d}{dA_1}\ln\left[\frac{\eta\left(n_1, A_1\right)}{\mu\left(n_1, A_1\right)}\right] &= \frac{1}{2}\frac{1}{n_1} - \frac{a}{x - \frac{1}{2} - an_1} + \frac{a - 1}{x - \frac{1}{2} - (a - 1)n_1} \\ &= \frac{\left(x - \frac{1}{2}\right)\left[x - \frac{1}{2} - 2an_1\right] - n_1\left[x - \frac{1}{2} - a(a - 1)n_1\right]}{2n_1\left[x - \frac{1}{2} - an_1\right]\left[x - \frac{1}{2} - (a - 1)n_1\right]}.\end{aligned}$$

It is straightforward to show that condition (A25) implies $x - \frac{1}{2} - (a - 1)n_1 > x - \frac{1}{2} - an_1 > 0$. We only need to show that the denominator is negative. Since $\zeta < \frac{1}{2}$, $(a - 1) < 1$ and

$$\begin{aligned}\left(x - \frac{1}{2}\right)\left[x - \frac{1}{2} - 2an_1\right] - n_1\left[x - \frac{1}{2} - a(a - 1)n_1\right] &< \left(x - \frac{1}{2}\right)\left[x - \frac{1}{2} - 2an_1\right] - n_1\left[x - \frac{1}{2} - 2an_1\right] \\ &= \left(x - \frac{1}{2} - n_1\right)\left[x - \frac{1}{2} - 2an_1\right].\end{aligned}$$

Also, $\zeta > \frac{1}{4}$ implies $a > 1$. Therefore, $x - \frac{1}{2} - an_1 > 0$ implies $x - \frac{1}{2} > n_1$. It remains to show $x - \frac{1}{2} - 2an_1 < 0$. Using the definition $a = \frac{3}{2(1-\zeta)} - 1$, under assumption (A28),

$$\begin{aligned}x - \frac{1}{2} - 2an_1 &= x - \frac{1}{2} - 2a(A_1 - x) \\ &= \left(\frac{3}{(1-\zeta)} - 1\right)x - \frac{1}{2} - \left(\frac{3}{(1-\zeta)} - 2\right)A_1 \\ &< 0,\end{aligned}$$

which completes the proof. $\qquad\square$

A.3 Proof of Proposition 3

We prove Proposition 3 in two steps: first, given prices, the quantities satisfy the household's and the entrepreneurs' optimality conditions; second, the quantities satisfy the market clearing conditions.

To verify the optimality conditions, note that the optimization problems of households and firms are all standard convex programming problems; therefore, we only need to verify first order conditions. Equation (30) is the household's first-order condition. Equation (36) is a normalized version of resource constraint (24). Both of them are satisfied as listed in Proposition 3.

To verify that the entrepreneur i's allocations $\{N_{i,t}, B_{i,t}, K_{i,t}, H_{i,t}, L_{i,t}\}$ as constructed in Proposition 3 satisfy the first order conditions for the optimization problem (17), note that the first order condition with respect to $B_{i,t}$ implies

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}\right]R_t^f + \frac{\eta_t^i}{q_{K,t}}. \tag{A35}$$

Similarly, the first order condition for $K_{i,t+1}$ is

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}^i \frac{\frac{\partial}{\partial K_{i,t+1}}\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta)q_{K,t+1}}{q_{K,t}}\right] + \zeta\frac{\eta_t^i}{q_{K,t}}. \tag{A36}$$

Finally, optimality with respect to the choice of type-H capital implies

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}^i \frac{\frac{\partial}{\partial H_{i,t+1}}\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta)q_{H,t+1}}{q_{H,t}}\right]. \tag{A37}$$

Next, the law of motion of the endogenous state variable n can be constructed from equation (18):[22]

$$
\begin{aligned}
n' &= (1-\lambda)\left[\alpha\nu A' + \phi(1-\delta)q_K\left(A',n'\right) + (1-\phi)(1-\delta)q_H\left(A',n'\right) - \zeta\phi q_K\left(A,n\right)R_f\left(A,n\right)\right]\\
&\quad + \lambda\chi\frac{n}{\Gamma\left(A,n\right)}. \tag{A38}
\end{aligned}
$$

With the law of motion of the state variables, we can construct the normalized utility of the

---

[22]We make use of the property that the ratio of K over H is always equal to $\phi/(1-\phi)$, as implied by the law of motion of the capital stock in (25).

household as the fixed point of

$$u\left(A,n\right)=\left\{(1-\beta)c\left(A,n\right)^{1-\frac{1}{\psi}}+\beta\Gamma\left(A,n\right)^{1-\frac{1}{\psi}}\left(E[u\left(A',n'\right)^{1-\gamma}]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The stochastic discount factors must be consistent with household utility maximization:

$$M' = \beta\left[\frac{c\left(A',n'\right)\Gamma\left(A,n\right)}{c\left(A,n\right)}\right]^{-\frac{1}{\psi}}\left[\frac{u\left(A',n'\right)}{E\left[u\left(A',n'\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} \tag{A39}$$

$$\tilde{M}' = M'[(1-\lambda)\,\mu\left(A',n'\right)+\lambda]. \tag{A40}$$

In our setup, thanks to the assumptions that the idiosyncratic shock $z_{i,t+1}$ is observed before the decisions on $K_{i,t+1}$ and $H_{i,t+1}$ are made, we can construct an equilibrium in which μ_t^i and η_t^i are equalized across all the firms because $\frac{\partial}{\partial H_{i,t+1}}\pi\left(\bar{A}_{t+1},z_{i,t+1},K_{i,t+1},H_{i,t+1}\right)=\frac{\partial}{\partial K_{i,t+1}}\pi\left(\bar{A}_{t+1},z_{i,t+1},K_{i,t+1},H_{i,t+1}\right)$ are the same for all i.

Our next step is to verify the market clearing conditions. Given the initial conditions (initial net worth N_0, $\frac{K_1}{H_1}=\frac{\phi}{1-\phi}$, $N_{i,0}=z_{i,1}N_0$) and the net worth injection rule for the new entrant firms ($N_{t+1}^{entrant}=\chi N_t$ for all t), we establish the market clearing conditions through the following lemma. For simplicity, we assume the collateral constraint to be binding. The case in which this constraint is not binding can be dealt with in a similar way.

Lemma 5. *The optimal allocations* $\{N_{i,t},B_{i,t},K_{i,t+1},H_{i,t+1}\}$ *constructed as in Proposition 3 satisfy the market clearing conditions, i.e.,*

$$K_{t+1}=\int K_{i,t+1}\,di,\quad H_{t+1}=\int H_{i,t+1}\,di,\quad N_t=\int N_{i,t}\,di \tag{A41}$$

for all $t\geq 0$.

First, in each period t, given prices and $N_{i,t}$, the individual entrepreneur i's capital decisions $\{K_{i,t+1},H_{i,t+1}\}$ must satisfy the condition

$$N_{i,t}=(1-\zeta)\,q_{K,t}K_{i,t+1}+q_{H,t}H_{i,t+1} \tag{A42}$$

and the optimal decision rule (27). Equation (A42) is obtained by combining the entrepreneur's budget constraint (13) with a binding collateral constraint (14).

Next, we show by induction, that, given the initial conditions, market clearing conditions (A41) hold for all $t \geq 0$. In period 0, we start from the initial conditions. First, $N_{i,0} = z_{i,1}N_0$, where $z_{i,1}$ is chosen from the stationary distribution of z. Then, given $z_{i,1}$ for each firm i, we use equations (A42) and (27) to solve for $K_{i,1}$ and $H_{i,1}$. Clearly, $K_{i,1} = z_{i,1}K_1$ and $H_{i,1} = z_{i,1}H_1$. Therefore, the market clearing conditions (A41) hold for $t = 0$, i.e.,

$$\int K_{i,1}\, di = K_1, \quad \int H_{i,1}\, di = H_1, \quad \int N_{i,0}\, di = N_0. \tag{A43}$$

To complete the induction argument, we need to show that if market clearing holds for $t+1$, it must hold for $t+2$ for all t, which is the following claim:

Claim 1. *Suppose $\int K_{i,t+1}\, di = K_{t+1}$, $\int H_{i,t+1}\, di = H_{t+1}$, $\int N_{i,t}\, di = N_t$, and $N_{t+1}^{entrant} = \chi N_t$, then*

$$\int K_{i,t+2}di = K_{t+2} \quad \int H_{i,t+2}di = H_{t+2} \quad \int N_{i,t+1}di = N_{t+1} \tag{A44}$$

for all $t \geq 0$.

1. Using the law of motion for the net worth of existing firms, one can show that the total net worth of all surviving firms can be rewritten as follows:

$$(1-\lambda)\int N_{i,t+1}di$$
$$= (1-\lambda)\int \left[A_{t+1}\left(K_{i,t+1} + H_{i,t+1}\right) + (1-\delta)q_{K,t+1}K_{i,t+1} + (1-\delta)q_{H,t+1}H_{i,t+1} - R_{f,t}B_{i,t}\right]di,$$
$$= (1-\lambda)\left[A_{t+1}\left(K_{t+1} + H_{t+1}\right) + (1-\delta)q_{K,t}K_{t+1} + (1-\delta)q_{H,t}H_{t+1} - R_{f,t}B_t\right],$$

   since by assumption $\int K_{i,t+1}\, di = K_{t+1}$, $\int H_{i,t+1}\, di = H_{t+1}$, and $\int B_{i,t}\, di = B_t = \zeta q_{K,t}K_{t+1}$. Using the assignment rule for the net worth of new entrants, $N_{t+1}^{entrant} = \chi N_t$, we can show that the total net worth at the end of period $t+1$ across survivors and new entrants together satisfies $\int N_{i,t+1}\, di = N_{t+1}$, where aggregate net worth N_{t+1} is given by equation (18).

2. At the end of period $t+1$, we have a pool of firms consisting of old ones with net worth given by (16) and new entrants. All of them will observe $z_{i,t+2}$ (for the new entrants $z_{i,t+2} = \bar{z}$) and produce at the beginning of the period $t+1$.

   We compute the capital holdings for period $t+2$ for each firm i using (A42) and (27). At this point, the capital holdings and the net worth of all existing firms will not be proportional to $z_{i,t+2}$ due to heterogeneity in the shocks. However, we know that $\int N_{i,t+1}\, di = N_{t+1}$, and

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$\int z_{i,t+2}\, di = 1$. Integrating (A42) and (27) across all i yields the two equations

$$(1-\zeta)\, q_{K,t+1} \int K_{i,t+2}\, di + q_{H,t+1} \int H_{i,t+2}\, di \;=\; N_{t+1} \qquad (A45)$$

$$\int K_{i,t+2}\, di + \int H_{i,t+2}\, di \;=\; K_{t+2} + H_{t+2}, \qquad (A46)$$

where we have used $\int N_{i,t+1}\, di = N_{t+1}$ and $\int z_{i,t+2}\, di = 1$. Given that the constraints of all entrepreneurs are binding, the budget constraint (A42) also holds at the aggregate level, i.e.,

$$N_{t+1} = (1-\zeta)\, q_{K,t+1} K_{t+2} + q_{H,t+1} H_{t+2}.$$

Together with the above system, this implies $\int K_{i,t+2}\, di = K_{t+2}$ and $\int H_{i,t+2}\, di = H_{t+2}$. Therefore, the claim is proved.

In summary, we have proved that the equilibrium prices and quantities constructed in Proposition 3 satisfy the household's and entrepreneur's optimality conditions, and that the quantities satisfy market clearing conditions.

Finally, we provide a recursive relationship that can be used to solve for $\theta\,(A, n)$ given the equilibrium constructed in Proposition 3. The recursion (17) implies

$$
\begin{aligned}
\mu_t N_{i,t} + \theta_t z_{i,t+1}\,(K_t + H_t) =\, & E_t M_{t+1} \left[(1-\lambda)\left(\mu_{t+1} N_{i,t+1} + \theta_{t+1}\,(K_{t+1} + H_{t+1})\, z_{i,t+2}\right) + \lambda N_{i,t+1} \right] \\
=\, & E_t M_{t+1}\left[\left\{ (1-\lambda)\,\mu_{t+1} + \lambda \right\} N_{i,t+1} \right] + (1-\lambda)\, z_{i,t+1} E_t \left[M_{t+1} \theta_{t+1}\,(K_{t+1} + H_{t+1}) \right].
\end{aligned}
$$
$$(A47)$$

Below, we first focus on simplifying the term $E_t M_{t+1}\left[\left\{ (1-\lambda)\,\mu_{t+1} + \lambda \right\} N_{i,t+1} \right]$. Note that a binding collateral constraint together with the entrepreneur's budget constraint (13) implies

$$(1-\zeta)\, q_{K,t} K_{i,t+1} + q_{H,t} H_{i,t+1} = N_{i,t}. \qquad (A48)$$

Equation (A48) together with the optimality condition (27) determine $K_{i,t+1}$ and $H_{i,t+1}$ as functions of $N_{i,t}$ and $z_{i,t+1}$:

$$K_{i,t+1} = \frac{q_{H,t} z_{i,t+1}\,(K_{t+1} + H_{t+1}) - N_{i,t}}{q_{H,t} - (1-\zeta)\, q_{K,t}}; \quad H_{i,t+1} = \frac{N_{i,t} - (1-\zeta)\, q_{K,t} z_{i,t+1}\,(K_{t+1} + H_{t+1})}{q_{H,t} - (1-\zeta)\, q_{K,t}}. \quad (A49)$$

Using Equation (A49) and the law of motion of net worth (16), we can represent $N_{i,t+1}$ as a linear

function of $N_{i,t}$ and $z_{i,t+1}$:

$$
\begin{aligned}
N_{i,t+1} &= z_{i,t+1}\alpha A_{t+1}\left(K_{t+1}+H_{t+1}\right)+\left(1-\delta\right)q_{K,t+1}\frac{q_{H,t}z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)-N_{i,t}}{q_{H,t}-\left(1-\zeta\right)q_{K,t}} \\
&\quad +\left(1-\delta\right)q_{H,t+1}\frac{N_{i,t}-\left(1-\zeta\right)q_{K,t}z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)}{q_{H,t}-\left(1-\zeta\right)q_{K,t}}-R_{f,t}\zeta q_{K,t}\frac{q_{H,t}z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)-N_{i,t}}{q_{H,t}-\left(1-\zeta\right)q_{K,t}}.
\end{aligned}
$$

Because we are only interested in the coefficients on $z_{i,t+1}$, collecting the terms that involves $z_{i,t+1}$ on both sides of (A47), we have:

$$
\theta_t z_{i,t+1}\left(K_t+H_t\right)=z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)\times Term,
$$

where

$$
Term = E_t\left[\tilde{M}_{t+1}\left\{\begin{array}{c}\alpha A_{t+1}+\left(1-\delta\right)q_{K,t+1}\frac{q_{H,t}}{q_{H,t}-\left(1-\zeta\right)q_{K,t}}\\-\left(1-\delta\right)q_{H,t+1}\frac{\left(1-\zeta\right)q_{K,t}}{q_{H,t}-\left(1-\zeta\right)q_{K,t}}-R_{f,t}\zeta q_{K,t}\frac{q_{H,t}}{q_{H,t}-\left(1-\zeta\right)q_{K,t}}\end{array}\right\}\right]+\left(1-\lambda\right)E_t\left[M_{t+1}\theta_{t+1}\right].
$$

We can simplify the first term using the first order conditions (31)-(33) to get

$$
E_t\left[\tilde{M}_{t+1}\left\{\alpha\left(1-\nu\right)A_{t+1}\right\}\right].
$$

Therefore, we have the following recursive relationship for $\theta\left(A,n\right)$:

$$
\theta\left(A,n\right)=\left[1-\delta+i\left(A,n\right)\right]\left\{\alpha\left(1-\nu\right)E\left[M'\left\{\lambda+\left(1-\lambda\right)\mu\left(A',n'\right)\right\}A'\right]+\left(1-\lambda\right)E\left[M'\theta\left(A',n'\right)\right]\right\}. \tag{A50}
$$

The term $\alpha\left(1-\nu\right)A'$ is the profit for the firm due to decreasing return to scale. Clearly, $\theta\left(A,n\right)$ has the interpretation of the present value of profit. In the case of constant returns to scale, $\theta\left(A,n\right)=0$.

B   Empirical Analysis

In this section, we provide empirical evidence on the relation between collateralizability and the cross-section of stock returns. First and most importantly, we show that high asset collateralizability firms have lower cash flow betas with respect two alternative proxies for financial shocks. Second, we conduct standard Fama and MacBeth (1973) two-pass regression and show the proxies of financial shocks are significantly negatively priced. High collateralizability firms are less negatively exposed to these shocks. These two pieces of evidence taken together strongly corroborate the model mechanism

that collateralizable assets provide an insurance against aggregate shocks. We then perform other standard multi-factor asset prcing tests, and investigate the joint link between collateralizability and other firm characteristics on one hand and future stock returns on the other using multivariate regressions.

B.1 Cash flow risks of collateralizability-sorted portfolios

Our theory suggests that the collateralizability premium comes from the countercyclicality of the marginal value of collateralizable capital. In our model, firms, rather than households, directly trade physical assets directly, because they are more efficient than households in deploying these assets. Since firms are constrained, type-K and type-H capital, whose prices contain a Lagrangian multiplier component, can have different prices and expected returns even though they generate identical cash flows from the firm's perspective (measured in net worth units). The counter-cyclical nature of the Lagrangian multiplier provides a hedge against aggregate shocks and makes the price of collateralizable capital less sensitive to aggregate shocks and less cyclical. However, it is important to note, in our model, households are not constrained and free to trade the firms' equity and debt, so that differences in expected returns on the firms' equity must be due to differences in the cash flows accruing to equity holders (measured in consumption units). Put differently, the Lagrangian multiplier component of asset prices affects the risk exposure of cash flows to the equity holders, i.e., to households. We measure the cash flow to equity holders and show empirically at the portfolio level that the equity cash flows of firms with high asset collateralizability exhibit a lower, i.e., less negative, sensitivity respect to financial shocks, consistent with the model simulation.

We consider two alternative proxies for financial shocks: the change in the general cost of external finance (debt and equity) as suggested by Eisfeldt and Muir (2016) (ΔEM), and the log change in the cross-sectional dispersion of firm-level cash flow growth ($\Delta \sigma_{CS}$), similar in spirit to Elenev et al. (2018).

When we measure the cash flow accruing to equity holders at the portfolio level, we follow Belo et al. (2017a) and first aggregate cash flow (represented by EBIT) across the firms in a given portfolio and then normalize this sum by the total lagged sales (SALE) of that portfolio. We then compute the sensitivity, i.e., the beta, of the portfolio cash flow growth with respect to the two proxies of financial shocks. The results are reported in Table B.1.

We make several observations. First and importantly, one can see from Panel A, the cash flow

betas with respective the equity finance cost shock (ΔEM)) display a monotonically increasing pattern from low to high collateralizability portfolios, and cash flow beta of low collateralizability portfolio is statistically significantly more negative that that of high collateralizability portfolio. In particular, the high collateralizability quintiles 4 and 5 exhibit insignificantly negative betas. This again highlights the main economic mechanism of our model that collateralizable assets provide an insurance against aggregate shocks. We also find this increasing pattern of cash flow betas across collateralizabilty portfolios with respect to $\Delta \sigma_{CS}$, although the cash flow beta difference is less significant.

Finally, to precisely connect the empirical evidence to our model, we run the same test based on data from a simulation of our model. As we show in Panel B, our model produces the same increasing pattern of cash flow betas with respective to the financial shock ϵ_x across collateralizability portfolios. Furthermore, the cash flow of the high asset collateralizability portfolio 5 even exhibits a positive sensitivity to financial shocks. This strongly confirms our key model mechanism and is consistent with the data.

B.2   Collateralizability spreads and financial shocks

In this section, we provide empirical evidence for the link between the collateralizability spread and financial shocks consistent with our model interpretation.

Empirically, we consider a two-factor asset pricing model with the market (Mkt) and one of the two financial shock proxies (ΔEM or $\Delta \sigma_{CS}$) as factors. Following the standard approach developed by Fama and MacBeth (1973), we first estimate the exposures (betas) of excess returns of five collateralizability-sorted portfolios with respect to the market and the financial shock factor using the whole sample. Next, we run period-by-period cross-sectional regressions of realized portfolio returns on betas to estimate the market prices of risks, which are calculated as the average slopes from the period-by-period cross-sectional regressions.

We also conduct the Fama and MacBeth (1973) two-pass regression based on data from a simulation of our model. The only difference is that, rather than running a two-factor model, we run a one-factor regression with just the financial shock ϵ_x, since our model, by design, features a one-factor structure due to the perfect correlation between TFP and financial shocks.

The results are presented in Table B.2, respectively, where Panel A and Panel B present the exposures of the five portfolios to factors, while the estimated market prices of risk are shown in

Table B.1: Cash Flow Exposure to the Financial Shock

This table shows the sensitivity of cash flows of collateralizability-sorted portfolios to the financial shock. Panel A and B report exposure coefficients from empirical data and model simulated data, respectively. The portfolio-level normalized cash flow is constructed by aggregating cash flow (EBIT) within each quintile portfolio, and then dividing it by the lagged aggregate sales (SALE) of the same portfolio. In Panel A (data), we report the regression coefficients from regressing portfolio-level normalized cash flow on two alternative empirical proxies of the financial shocks: ΔEM and $\Delta \sigma_{CS}$. ΔEM is the first difference of average external finance cost from Eisfeldt and Muir (2016). $\Delta \sigma_{CS}$ is the log change of the cross-section standard deviation of firm-level cash flow growth. In the model (Panel B), we construct the portfolio-level normalized cash flow in the same way as in the data from model simulations. The financial shock series ε_x in the model is the innovation to the liquidation probability λ. Every year, we winsorize firm-level variables within each quintile at the top and bottom 1%, respectively. All shocks are normalized to have zero mean and unit standard deviation. All regressions are conducted at the annual frequency. The t-statistics (in parentheses) are adjusted following Newey and West (1987). All regression coefficients are multiplied by 100.

Panel A: Data

Financial Shocks	1	2	3	4	5	1-5
ΔEM	-1.94	-1.95	-1.41	-0.34	-0.14	-2.13
	(-1.83)	(-1.36)	(-1.81)	(-0.67)	(-0.63)	(-2.08)
$\Delta \sigma_{CS}$	-0.40	0.08	0.28	0.31	0.17	-0.55
	(-0.88)	(-0.30)	(1.51)	(1.47)	(1.12)	(-1.10)

Panel B: Model

Financial Shocks	1	2	3	4	5	1-5
ε_x	-0.46	0.27	0.66	1.47	2.91	-3.37
	(-0.44)	(0.32)	(0.83)	(1.36)	(2.25)	(-1.85)

Panel C.

We make several observations. First, the betas with respect to ΔEM display a monotonically increasing pattern from low to high collateralizability portfolios. In particular, the high collateralizability quintile exhibits a (marginally) significantly less negative beta than the low collateralizability quintile. This pattern is confirmed in an even stronger fashion when we use $\Delta \sigma_{CS}$ as a proxy for financial shocks. The betas with respect to this factor also display a monotonically increasing pattern from low to high collateralizability portfolios. It is worth noting that, with respect to this proxy of financial shocks, the difference in return beta between portfolio 1 and 5 is statistically significant. When we run the test using simulated data from the model, the return betas of collateralizability-sorted portfolios display a pattern consistent with the data.

Furthermore, in the second stage cross-sectional regressions, we use five collateralizability-sorted portfolios as the test assets, we compare the two-factor model ($Mkt + \Delta EM$ or $Mkt + \Delta \sigma_{CS}$) with the standard CAPM with only the market factor. We observe that the CAPM fails. When we add the financial shock factor, the estimated market price of risk for this new factor is negative and significant. The average pricing error (i.e., the intercept) becomes smaller and even statistically insignificant. The second stage empirical results are again confirmed by the model simulation.

B.3 Asset pricing tests

We now perform a number of standard asset pricing tests to show that the collateralizability premium cannot be explained by standard risk factors, as represented by the Carhart (1997) four factor model, the Fama and French (2015) five factor model, or the organizational capital factor proposed by Eisfeldt and Papanikolaou (2013). We also investigate the incremental predictive power of current asset collateralizability for future stock returns at the firm-level.

First, we investigate to what extent the variation in the returns of the collateralizability-sorted portfolios can be explained by standard risk factors suggested by Carhart (1997) and Fama and French (2015). In particular, we run monthly time-series regressions of the (annualized) excess returns of each portfolio on a constant and the risk factors included in the above models. Table B.3 reports the intercepts (i.e., alphas) and exposures (i.e., betas). The intercepts can be interpreted as pricing errors (abnormal returns), which remain unexplained by the given set of factors.

We make two key observations. First, the pricing errors of the collateralizablity-sorted portfolios with respect to the given sets of factors are large and statistically significant. The estimated alphas

Table B.2: Betas and Price of Risks of the Financial Shock

This table presents the risk price estimates for the financial shock. The factors considered in the empirical data are the market return and one of the two alternative empirical proxies for the financial shock, that is, external finance shock (ΔEM) and cross-sectional dispersion shock of firm-level cash flow growth ($\Delta\sigma_{CS}$). We construct the external finance shock by taking the first difference of the average costs of external finance from Eisfeldt and Muir (2016). To calculate the cross-sectional dispersion shock of firm-level cash flow growth, we first calculate the cross-sectional dispersion of cash flow growth of across all firms each year, and then we compute its log change. Panels A and B present the first-stage estimates of factor exposures of collateralizability-sorted portfolios in the data and in the model, respectively. Panel C reports the risk prices (λ_{Fin}) of the financial shock estimated from the second-stage regressions. The risk prices reported in Panel C are the mean slopes of period-by-period cross-sectional regressions of portfolio excess returns on risk exposures (betas). All shocks are normalized to have zero mean and unit standard deviation. The regressions are conducted at the annual frequency. R^2 is calculated as the mean across R^2 of the period-by-period regressions. The mean absolute pricing errors (MAE) across the test assets in Panel C are expressed in percentage terms. The t-statistics (in parentheses) are adjusted following Newey and West (1987).

Panel A: Portfolio Factor Exposures - Data

	1	2	3	4	5	1-5
ΔEM	-0.142	-0.128	-0.113	-0.091	-0.084	-0.058
	(-2.580)	(-2.804)	(-2.135)	(-2.126)	(-1.723)	(-1.598)
$\Delta\sigma_{CS}$	-0.029	-0.007	-0.003	0.007	0.016	-0.045
	(-0.990)	(-0.251)	(-0.113)	(0.260)	(0.592)	(-2.074)

Panel B: Portfolio Factor Exposures - Model

	1	2	3	4	5	1-5
ε_x	-0.050	-0.042	-0.038	-0.033	-0.023	-0.027
	(-15.904)	(-9.908)	(-11.005)	(-9.182)	(-6.687)	(-14.961)

Panel C: Price of Risks

	Data					Model
	CAPM	ΔEM	Mkt+ΔEM	$\Delta\sigma_{CS}$	Mkt+$\Delta\sigma_{CS}$	ε_x
λ_{Mkt}	0.70		-0.005		0.072	
(t)	(2.227)		(-0.011)		(0.338)	
λ_{Fin}		-1.230	-1.094	-1.435	-1.272	-1.344
(t)		(-2.478)	(-1.729)	(-2.442)	(-1.876)	(-6.830)
Intercept	-0.441	-0.045	0.103	0.091	0.009	0.005
(t)	(-1.888)	(-0.777)	(0.214)	(2.386)	(0.041)	(1.035)
MAE	4.612	3.947	3.266	4.033	3.149	0.515
R^2	0.387	0.445	0.624	0.451	0.633	0.676

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of the low-minus-high portfolio are 9.34% for the Carhart (1997) model and 5.80% for the Fama and French (2015) five-factor model, respectively, with associated t-statistics of around 3.5 and 2.1.

Second, in order to distinguish our collateralizability measure from organizational capital, we also control for this factor constructed by Eisfeldt and Papanikolaou (2013),[23] together with the three Fama-French factors.

The results are shown in Panel C of Table B.3. The pricing error of the low-minus-high portfolio is still significant in the presence of the organizational capital factor (OMK) and amounts almost 9% per year with a t-statistic of greater than 2.6. In particular, the five portfolios sorted on collateralizability are not strongly exposed to this factor, indicated by economically small and statistically insignificant coefficients, except for Quintile 5.

Taken together, the cross-sectional return spread across collateralizability sorted portfolios cannot be explained by either the Carhart (1997) four-factor model, the Fama and French (2015) five-factor model, or the organizational capital factor proposed by Eisfeldt and Papanikolaou (2013).

Second, we extend the previous analysis to the investigation of the link between collateralizability and future stock returns using firm-level multivariate regressions that include firm's collateralizability and other controls as return predictors. In particular, we perform standard firm-level cross-sectional regressions (Fama and MacBeth (1973)) to predict future stock returns:

$$R_{i,t+1} = \alpha_i + \beta \cdot Collateralizability_{i,t} + \gamma \cdot Controls_{i,t} + \varepsilon_{i,t+1},$$

where $R_{i,t+1}$ is stock i's cumulative (raw) return over the respective next year, i.e., from July of year t to June of each year $t+1$. The control variables include current collateralizability, size, book-to-market (BM), profitability (ROA), and book leverage. To avoid using future information, all the balance sheet variables are based on the values available before the end of year t. Table B.4 reports the results. The regressions exhibit a significantly negative slope coefficient for collateralizability across all specifications, which supports our theory, since a higher current degree of collateralizability implies lower overall risk exposure, so that expected future returns should indeed be smaller with higher collateralizability.

In our empirical measure, only structure and equipment capital contribute to firms' collateralizability, but not intangible capital. Therefore, by construction, our collateralizability measure weakly negatively correlates with measures of intangible capital. In order to empirically distinguish

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[23]We would like to thank Dimitris Papanikolaou for sharing this time series of the organizational factor.

our theoretical channel from the ones focusing on organizational capital (Eisfeldt and Papanikolaou (2013)) and R&D capital (Chan et al. (2001), Croce et al. (2017)), we also control for OG/AT, the ratio of organizational capital to total assets, and XRD/AT, the ratio of R&D expenses to total assets, as suggested in the literature. The results in Table B.4 show that the negative slope coefficients for collateralizability remain significant, although they become smaller in magnitude, after controlling for these two firm characteristics. Instead of using the ratio of R&D expenditure to total assets, we also used the ratio of R&D capital to total assets as a control. The results remain very similar.

B.4 Additional empirical evidence

In this section, we provide additional empirical evidence regarding the collateralizability premium. First, we demonstrate the robustness of our findings by forming collateralizability portfolios within industries to make sure that our baseline result is not driven by industry-specific effects, and by performing a rolling-window estimation of the collateralizability parameters. Second, we present correlations between collateralizability and firm characteristics. Finally, we perform double sorts with respect to collateralizability and financial leverage.

B.4.1 Alternative portfolio sorts

 To implement the first robustness check, we consider the Fama-French industry classification with 17 sectors. We sort firms into collateralizability quintiles according to their collateralizability score within their respective industry. Portfolio 1 will thus contain all firms which are in the lowest quintile relative to their industry peers, and so on for portfolios 2 to 5. By doing so, we essentially control for industry fixed effects. Table B.5 reports the results of this exercise, and one can see that the results are very close to the findings of our benchmark analysis presented in Table 2.

In our benchmark analysis, we estimate the collateralizability coefficients for structure and equipment capital, ζ_S and ζ_E, using the whole sample. One might argue that this introduces a look-ahead bias, since the estimation is based on data not observable at the time when decisions are made. To see whether a potential look-ahead bias indeed has an effect on our results, we now perform the portfolio sort in year t exclusively on information up to $t-1$. In more detail, we use estimates denoted by $\widehat{\zeta}_{S,t-1}$ and $\widehat{\zeta}_{E,t-1}$ derived from expanding window regressions using data available up to the end of year $t-1$. The first window consists of data for the period from 1975 to

Table B.3: Asset Pricing Tests of Collateralizability-sorted Portfolios

This table shows the coefficients of regressions of excess returns of collateralizability-sorted portfolios on the factors from the Carhart (1997) four-factor model (Panel A), the Fama and French (2015) five-factor model (Panel B), and a model featuring the Fama-Franch three-factor model augmented by the organizational capital factor from Eisfeldt and Papanikolaou (2013) (Panel C). The t-statistics are computed based on Newey and West (1987) adjusted standard errors. The analysis is performed for financially constrained firms. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. The sample period is from July 1979 to December 2016, with the exception of Panel C, where the sample ends in December 2008 due to the length of the organizational capital factor. We annualize returns by multiplying by 12.

Panel A: Carhart Four-Factor Model

	1	2	3	4	5	1-5
α	5.43	2.94	2.04	-1.87	-3.91	9.34
(t)	2.80	1.76	1.35	-1.45	-2.44	3.47
β_{MKT}	1.07	1.07	1.07	1.12	1.10	-0.03
(t)	25.25	27.91	32.58	35.68	26.93	-0.48
β_{HML}	-0.62	-0.49	-0.21	-0.12	0.01	-0.63
(t)	-9.72	-8.60	-3.87	-2.31	0.16	-6.03
β_{SMB}	1.34	1.11	1.06	0.97	0.84	0.50
(t)	15.66	15.77	22.71	15.28	8.72	3.27
β_{MOM}	-0.04	-0.06	-0.05	-0.02	-0.07	0.03
(t)	-0.73	-1.74	-1.27	-0.56	-1.31	0.33
R^2	0.85	0.87	0.88	0.90	0.84	0.27

Panel B: Fama-French Five-Factor Model

	1	2	3	4	5	1-5
α	13.02	12.45	12.87	9.22	7.22	5.80
(t)	2.84	2.75	3.07	2.16	1.67	2.06
β_{MKT}	0.49	0.07	0.08	0.20	0.07	0.42
(t)	0.75	0.13	0.15	0.37	0.12	1.01
β_{SMB}	2.03	1.24	1.17	1.28	1.38	0.65
(t)	2.00	1.55	1.43	1.62	1.79	1.08
β_{HML}	-3.84	-4.34	-3.67	-3.15	-2.49	-1.35
(t)	-2.55	-3.21	-2.99	-2.65	-1.92	-1.12
β_{RMW}	-2.77	-3.12	-2.32	-1.90	-1.34	-1.43
(t)	-1.47	-2.11	-1.48	-1.33	-0.97	-1.11
β_{CMA}	2.10	1.00	1.74	0.92	0.94	1.17
(t)	0.83	0.46	1.02	0.53	0.62	0.58
R^2	0.09	0.10	0.08	0.07	0.06	0.04

Panel C: Control for Organizational Capital Factor

	1	2	3	4	5	1-5
α	5.06	3.42	1.56	-0.95	-3.90	8.96
(t)	2.30	1.67	0.85	-0.61	-1.74	2.66
β_{MKT}	1.10	1.07	1.10	1.11	1.08	0.03
(t)	19.82	21.75	24.43	30.89	23.11	0.35
β_{HML}	-0.56	-0.50	-0.19	-0.14	-0.00	-0.55
(t)	-7.23	-7.46	-2.71	-1.85	-0.04	-3.68
β_{SMB}	1.40	1.12	1.05	0.97	0.81	0.59
(t)	14.95	16.91	18.30	14.51	6.09	3.07
β_{OMK}	-0.02	0.01	0.01	-0.04	-0.14	0.13
(t)	-0.31	0.23	0.14	-0.99	-2.33	1.29
R^2	0.86	0.87	0.88	0.89	0.83	0.29

Table B.4: Fama Macbeth Regressions

This table reports the results for Fama-MacBeth regressions of annual cumulative firm-level excess stock returns on lagged firm characteristics. The coefficients reported in the table are the time-series averages of the slope coefficients from year-by-year cross-sectional regressions. The reported R^2 is the time-series average of the cross-sectional R^2. The columns labeled "SA," "WW" and "Non-Dividend" refer to the financially constrained samples. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. ROA is Compustat item IB divided by book assets. OG/AT is organizational capital over total book assets, XRD/AT is R&D expenditure over total book assets. The t-statistics (in parentheses) are adjusted following Newey and West (1987). The sample period is from 1979 to 2016.

	(1) SA	(2) WW	(3) No-Dividend	(4) SA	(5) WW	(6) No-Dividend	(7) SA	(8) WW	(9) No-Dividend
Collateralizability	-0.114***	-0.118***	-0.089***	-0.083**	-0.103***	-0.079***	-0.057**	-0.056**	-0.042**
	(-3.782)	(-4.040)	(-3.523)	(-2.388)	(-3.119)	(-3.309)	(-2.600)	(-2.371)	(-2.444)
log(ME)	-0.147***	-0.142***	-0.073***	-0.143***	-0.138***	-0.070***	-0.154***	-0.150***	-0.078***
	(-4.536)	(-5.329)	(-3.786)	(-4.458)	(-5.332)	(-3.685)	(-4.668)	(-5.480)	(-3.889)
BM	0.044**	0.032**	0.053***	0.051***	0.035**	0.057***	0.051***	0.041***	0.059***
	(2.505)	(2.164)	(4.582)	(2.731)	(2.243)	(4.440)	(3.029)	(2.860)	(5.003)
Lagged return	0.012	0.003	0.018	0.010	0.002	0.016	0.012	0.004	0.019
	(0.756)	(0.194)	(1.014)	(0.649)	(0.105)	(0.910)	(0.758)	(0.261)	(1.005)
ROA	0.068	0.053	0.089	0.082	0.067	0.102	0.178***	0.186***	0.208***
	(0.994)	(0.906)	(1.438)	(1.143)	(1.094)	(1.564)	(2.769)	(3.146)	(3.700)
Book Leverage	-0.090	-0.031	-0.013	-0.070	-0.018	0.001	-0.044	0.020	0.042
	(-1.639)	(-0.640)	(-0.236)	(-1.280)	(-0.369)	(0.015)	(-0.894)	(0.470)	(0.791)
OG/AT				0.109***	0.065**	0.072**			
				(3.873)	(2.206)	(2.296)			
XRD/AT							0.642***	0.774***	0.584***
							(2.795)	(3.053)	(2.833)
Constant	-3.574***	-3.634***	-3.783***	-3.655***	-3.685***	-3.839***	-3.620***	-3.678***	-3.833***
	(-4.275)	(-4.212)	(-4.435)	(-4.323)	(-4.229)	(-4.472)	(-4.319)	(-4.266)	(-4.503)
Observations	32864	37090	43078	32864	37090	43078	32864	37090	43078
R^2	0.087	0.085	0.065	0.091	0.088	0.069	0.093	0.091	0.072

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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Table B.5: Portfolios Sorted on Collateralizability within Industries

This table reports annualized average monthly value-weighted excess returns $(E[R] - R^f)$ for collateralizability-sorted portfolios, and their alphas with respect to different factor models. The sample period is from July 1979 to December 2016. $\alpha^{FF3+MOM}$ and α^{FF5} are the alphas with respect to the Carhart (1997) four-factor model and the Fama and French (2015) five-factor model, respectively. At the end of June each year t, we consider each of the 17 Fama-French industries and sort the constrained firms in a given industry into quintiles based on their collateralizability scores at the end of year $t-1$. We hold the portfolios for a year, from July of year t until the June of year $t+1$. Portfolios are rebalanced in July every year. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. The WW and the SA index are constructed according to Whited and Wu (2006) and Hadlock and Pierce (2010), respectively. Additionally, we consider a subsample where the firms are classified as constrained by all three measures jointly. We annualize returns by multiplying by 12. The t-statistics are estimated following Newey and West (1987).

	1	2	3	4	5	1-5
	Financially constrained firms - All measures					
$E[R] - R_f(\%)$	13.14	10.46	11.67	7.97	6.86	6.28
(t)	2.63	2.22	2.41	1.79	1.46	2.60
$\alpha^{FF3+MOM}$	4.59	2.06	2.64	-1.31	-2.74	7.33
(t)	2.48	0.98	1.35	-0.68	-1.47	3.21
α^{FF5}	13.87	10.94	13.01	10.97	7.04	6.83
(t)	2.75	2.31	2.52	2.36	1.38	2.85
	Financially constrained firms - WW index					
$E[R] - R_f(\%)$	12.53	11.77	9.83	8.36	6.02	6.51
(t)	2.68	2.71	2.22	1.99	1.41	3.07
$\alpha^{FF3+MOM}$	4.23	2.83	1.62	-0.76	-3.05	7.28
(t)	2.51	1.78	0.97	-0.50	-2.37	3.57
α^{FF5}	14.25	12.26	11.76	10.28	6.37	7.88
(t)	2.97	2.75	2.56	2.35	1.33	3.58
	Financially constrained firms, SA index					
$E[R] - R_f(\%)$	11.28	11.51	8.08	8.32	6.02	5.26
(t)	2.35	2.41	1.77	1.94	1.35	2.54
$\alpha^{FF3+MOM}$	3.58	4.65	-0.62	-0.49	-2.50	6.08
(t)	1.99	2.25	-0.41	-0.33	-1.50	2.99
α^{FF5}	12.20	13.36	10.07	9.54	7.37	4.83
(t)	2.60	2.57	2.08	2.09	1.58	2.22
	Financially constrained firms, Non-Dividend					
$E[R] - R_f(\%)$	14.99	12.98	6.99	7.92	9.69	5.30
(t)	3.50	2.83	1.70	1.98	2.12	2.27
$\alpha^{FF3+MOM}$	7.39	5.23	0.39	-0.12	2.24	5.15
(t)	3.90	2.59	0.23	-0.08	1.22	2.18
α^{FF5}	14.90	14.95	8.16	8.88	11.45	3.44
(t)	3.59	3.59	2.16	2.16	2.61	1.47

1980.[24].

Table B.6 presents the results in a fashion analogous to Table 2. For all three measures for financial constraints, the collateralizability spread is positive, large, and significant. This shows that our baseline results do not suffer from a look-ahead bias with respect to the estimation of the collateralizability coefficients.

In order to capture the fact that structure capital is more collateralizable than equipment capital (Rampini and Viswanathan (2010), Campello and Giambona (2013)), we employ a constrained version of the leverage regression in Table 1 by estimating the equation

$$\frac{B_{i,t}}{AT_{i,t}} = (\zeta_E + e^\Delta)StructShare_{l,t} + \zeta_E EquipShare_{l,t} + \gamma X_{i,t} + \varepsilon_{i,t},$$

i.e., we impose the restriction $\zeta_S = \zeta_E + e^\Delta > \zeta_E$. Then we perform a maximum likelihood estimation of above equation to obtain the time series of the estimates of ζ_E and Δ. In our sample, the estimated e^Δ across expanding windows is of mean 0.15 with standard error of 0.02.

As a further note, one advantage of our approach to sort stocks into portfolios does not rely on absolute precision in the estimation of ζ_E and ζ_S (which could potentially be subject to various sources of biases, e.g., due to endogeneity of capital structure choices, measurement errors in capital etc.). The outcome of the portfolio sort only depends on the ranking of the collateralizability measure for a given firm, not on its exact magnitude. In our empirical construction of the collateralizability measure, we consider three types of capital according to BEA, structure, equipment, and intellectual capital. As long as $\zeta_S > \zeta_E$ and intellectual capital does not contribute to collateralizability, the rank of a firm with respect to asset collateralizability will depend only on the composition of its capital, not on the numerical values of the estimated ζ-coefficients.

B.4.2   Collateralizability and additional firm characteristics

As indicated by the results in Table 5, our model can quantitatively replicate the patterns of leverage, asset growth and the investment rate. In Table B.7, we now present additional characteristics of the firms in our collateralizability-sorted portfolios.

Cash flow and size are relatively flat across the five portfolios, low collateralizability firms

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[24]The regressor, marginal tax rate, is only available after 1980, therefore we drop this regressor. All other regressors are available from 1975 onwards. The results are similar if we start our sample in 1980 with marginal tax rate.

Table B.6: Portfolios Sorting based on Expanding Window Estimated Collateralizability

This table reports average value-weighted monthly excess returns (in percent and annualized) for portfolios sorted on collateralizability. The sample period is from July 1981 to December 2016. At the end of June of each year t, we sort the constrained firms into five quintiles based on their collateralizability measures (estimated using expanding window) at the end of year $t-1$, where quintile 1 (quintile 5) contains the firms with the lowest (highest) share of collateralizable assets. We hold the portfolios for a year, from July of year t until the June of year $t+1$. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. The WW and SA indices are constructed according to Whited and Wu (2006) and Hadlock and Pierce (2010), respectively. Standard errors are estimated using Newey-West estimator. The table reports average excess returns $E[R] - R_f$, as well as the associated t-statistics, and Sharpe ratios (SR). We annualize returns by multiplying by 12.

	1	2	3	4	5	1-5
	Financially constrained firms - WW index					
$E[R] - R_f(\%)$	11.76	10.68	9.80	7.18	5.20	6.55
(t)	2.33	2.31	2.24	1.71	1.30	2.18
SR	0.41	0.41	0.40	0.30	0.24	0.38
	Financially constrained firms - SA index					
$E[R] - R_f(\%)$	9.61	10.74	9.36	7.82	3.64	5.97
(t)	1.84	2.21	2.11	1.74	0.88	2.07
SR	0.32	0.40	0.38	0.31	0.16	0.35
	Financially constrained firms - Non-Dividend					
$E[R] - R_f(\%)$	14.32	9.18	6.93	7.13	6.75	7.57
(t)	3.11	2.07	1.59	1.59	1.63	2.83
SR	0.54	0.36	0.28	0.28	0.29	0.49

on average hold more cash. Although cash is not modeled in our paper, this empirical finding is still consistent with our model intuition. Firms with less collaterizable assets hold more cash to compensate for the fact that they can hardly obtain collateralized loans, and even less so in recessions. The probability of debt issuance is increasing with asset collateralizability, while the probability of equity financing shows the opposite tendency. This reflects the substitution effect between the two types of external financing. Additionally, firms with more collateralizable assets on average have more short-term and long-term debt.

In Table B.8, we report the correlations of the collateralizability measure with other firm characteristics which have been shown in the past literature to predict the cross-section of stock returns, including the book-to-market ratio (BM) , the $R\&D$-to-asset ratio (XRD/AT), the organizational capital-to-asset ratio (OG/AT), (log) size (log(ME)), the investment rate, i.e., the ratio of investment to capital (I/K), and the return on assets (ROA). Notably, the collateralizability measure and these firm characteristics are only weakly correlated, with the correlation coefficients ranging between -33% to 16%.

B.4.3 Double sorting on collateralizability and leverage

As discussed in the main text, firms with higher asset collateralizablility have higher debt capacity and thus tend to have higher financial leverage. When a firm is highly levered, its equity is more exposed to aggregate risks. The effects of collateralizability and leverage can thus offset each other in determining the overall riskiness of the firm and consequently its expected equity return.

In order to disentangle these two effects, we conduct an independent double sort on collateralizability and financial leverage. The average returns for the resulting portfolios are reported in Table B.9. First, within each quintile sorted on book leverage, the collateralizability spread is always significantly positive. Second, the average returns of the high-minus-low leverage portfolios within each collateralizability quintile are not statistically significant.

C   Sensitivity analysis

In this section, we discuss the sensitivity of our quantitative results to several important parameters. To save space, we only discuss the moments which are sensitive to the respective each parameter. The results are reported in Table C.10.

Table B.7: Firm Characteristics

This table reports the median of firm characteristics across portfolios of firms sorted on collateralizability. The sample starts in 1979 and ends in 2016. Collateralizability is defined as in Section D.2. Book leverage is lease adjusted following Li, Whited, and Wu (2016). BM is the book-to-market ratio. $\frac{I}{K+H}$ is the sum of physical investments (CAPX), R&D and organizational capital investments over the sum of PPEGT and intangible capital. More details on the definition of R&D and organizational capital investments can be found Appendix D.3. $\log(ME)$ is the nature log of the market capitalization. Cash flow is defined as OIBDP to total asset ratio. Gross profitability is defined as revenue minus cost of goods denominated by total assets. ROE is the return on equity, which is the OIBDP divided by book equity. Asset growth is the growth rate of total assets. Type-K asset growth is the growth rate of PPEGT. Age is defined as the years a firm being recorded in COMPUSTAT. WW and SA index are following Whited and Wu (2006) and Hadlock and Pierce (2010), respectively. Dividend is calculated as the mean of the dividend dummy within each portfolio, which represents the probability of a firm paying dividend of that portfolio. Cash/AT is defined as cash and cash equivalents over total asset ratio. The probability of equity (debt) issuance is defined as the mean of a dummy variable within that quintile, which takes value of one if the flow to equity (debt) is negative. Flow to equity is defined as purchases of common stock plus dividends less sale of common stock. Flow to debt is defined as debt reduction plus changes in current debt plus interest paid, less debt issuance. Probability of external financing is defined as the mean of a dummy variable, which takes value of one when the sum of flow to debt and equity are negative.

	1	2	3	4	5
Collateralizability	0.081	0.168	0.260	0.377	0.619
Book leverage	0.104	0.163	0.228	0.343	0.460
BM	0.441	0.576	0.611	0.673	0.670
$\frac{I}{K+H}$	0.174	0.169	0.162	0.165	0.191
$\log(ME)$	3.822	3.988	4.000	4.153	4.178
Cash flow	0.037	0.094	0.110	0.113	0.098
Gross profitability	0.478	0.423	0.375	0.339	0.276
ROE	0.060	0.164	0.204	0.231	0.223
Asset growth	0.003	0.048	0.068	0.079	0.116
Type-K asset growth	0.075	0.092	0.100	0.108	0.129
Age	7.000	9.000	9.000	8.000	8.000
WW	-0.159	-0.183	-0.189	-0.194	-0.191
SA	-2.284	-2.506	-2.540	-2.576	-2.580
Prob(Dividend)	0.136	0.146	0.178	0.172	0.162
Cash/AT	0.246	0.142	0.114	0.087	0.104
Prob(Equity issuance)	0.665	0.594	0.523	0.501	0.496
Prob(Debt issuance)	0.097	0.118	0.114	0.122	0.143
Prob(External finance)	0.240	0.215	0.191	0.190	0.208
Short-term debt/AT	0.007	0.011	0.012	0.015	0.017
Long-term debt/AT	0.006	0.011	0.014	0.019	0.012

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Table B.8: Correlations Among Firm Characteristics

This table reports the correlation between collateralizability and other firm characteristics. The sample period is from 1978 to 2016, it focuses on constrained firms identified using Whited and Wu (2006) index. Log(ME) is the log of market capitalization deflated by CPI. BM is the book-to-market ratio. XRD/AT is R&D expenditure over total book assets. OG/AT is organizational capital over total book assets. I/K is the investment rate, it is calculated as the Compustat item CAPX divided by PPENT. ROA is Compustat item IB divided by book assets.

Variables	Collateralizability	BM	XRD/AT	OG/AT	log(ME)	I/K	ROA
Collateralizability	1.000						
BM	0.105	1.000					
XRD/AT	-0.333	-0.180	1.000				
OG/AT	-0.233	-0.065	0.117	1.000			
log(ME)	-0.013	-0.159	-0.006	-0.207	1.000		
I/K	0.011	-0.021	0.008	-0.001	0.004	1.000	
ROA	0.161	-0.041	-0.456	-0.154	0.126	-0.015	1.000

Table B.9: Independent Double Sort on Collateralizability and Leverage

This table reports annualized average value-weighted monthly excess returns for portfolios double-sorted independently on collateralizability and leverage. The sample starts in July 1979 and ends in December 2016. At the end of June in each year t, we independently sort financially constrained firms into quintiles based on collateralizability (horizontal direction) and into quintiles based on book financial leverage (vertical direction), then we compute the value-weighted returns of each portfolio. The book financial leverage is defined as financial debt over total asset ratio. A firm is considered financially constrained in year t, if its WW index (Whited and Wu (2006)) is above the respective median at the end of year $t-1$. The t-statistics are estimated following Newey and West (1987). All returns are annualized by multiplying with 12.

	L Col	2	3	4	H Col	L-H	t-stat
L Lev	11.96	7.58	10.51	10.14	5.48	6.48	1.81
2	13.84	11.38	11.19	5.31	5.98	7.85	1.96
3	13.07	14.16	11.05	9.70	4.50	8.57	2.06
4	15.48	10.10	11.73	5.39	5.04	10.43	2.51
H Lev	16.94	10.82	10.74	8.39	7.25	9.69	2.09
H-L	4.98	3.24	0.23	-1.75	1.76		
t-stat	1.17	0.81	0.06	-0.55	0.60		

Collateralizability parameter (ζ) The parameter ζ determines the collateralizability of type-K capital. We vary this parameter by $\pm 10\%$ around the benchmark value of 0.513 from Table 3 and make the following observations.

First, since we assume the collateral constraint is binding, higher collateralizability mechanically increases the average leverage ratio. Second, higher collateralizability leads to a lower risk premium for type-K capital, but to a higher risk premium for type-H capital, which overall implies a higher collateralizability premium. This is consistent with our model mechanism. Note that the price of type-K capital contains not only the present value of future cash flows, but also the present value of Lagrangian multipliers. According to equation (10), an increase in ζ makes the second component more important, which in turn makes the hedging channel more important and type-K capital less risky. On the other hand, a higher leverage ratio makes the entrepreneur's net worth more volatile, and therefore increases the risk premium of type-H capital.

Type-K and type-H capital ratio (ϕ) We vary this parameter by $\pm 10\%$. A higher ϕ implies a larger proportion of collateralizable assets in the economy, and as a result, it mechanically increases the leverage ratio and the overall asset collateralizability. A higher leverage ratio in turn leads to a more volatile entrepreneur's net worth, and therefore, increases the risk premia for both types of capital. On the other hand, higher ϕ implies more type-K capital, which can be used to hedge against aggregate risk. Therefore, higher ϕ may also reduce the overall riskiness of the aggregate economy and lower down the risk premium. As shown in Panel B, hedging effect dominates, thus the overall risk premium is lower and return spread is also lower.

Shock correlation $(\rho_{A,x})$ As explained in Section 6.1, we assume a negative correlation between the aggregate productivity shock and the financial shock in order for the model to generate a positive correlation between consumption and investment growth, consistent with the data. For parsimony, we had imposed a perfectly negative correlation in our benchmark calibration. We vary this parameter and consider the cases $\rho_{A,x} = -0.8$ and -0.9.

In terms of results, the correlation between consumption and investment growth becomes less positive, confirming our model intuition presented in Section 6.1. Furthermore, varying this correlation parameter does not qualitatively change the collateralizability spread and has limited effects on various risk premia as well.

Table C.10: Sensitivity Analysis

The table shows the results of sensitivity analyses, where key parameters of the model are varied around the values from the benchmark calibration shown in Table 3. A star superscript denotes the parameter value from the benchmark calibration.

Panel A: the role of collateralizability parameter ζ

	Data	Benchmark	$0.9\zeta^*$	$1.1\zeta^*$
$\sigma(\Delta y)$	3.05 (0.60)	3.50	3.52	3.47
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.33	0.29	0.37
$E[R^M - R^f]$	5.71 (2.25)	5.71	5.22	6.21
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85	7.73	7.97
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50	-6.74	-8.32

Panel B: the role of capital composition ϕ: $\pm 10\%$

	Data	Benchmark	$0.9\phi^*$	$1.1\phi^*$
$\sigma(\Delta y)$	3.05 (0.60)	3.50	3.53	3.46
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.33	0.30	0.36
$E[R^M - R^f]$	5.71 (2.25)	5.71	5.83	5.58
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85	7.68	8.05
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50	-7.59	-7.38

Panel C: the role of shock correlations

	Data	Benchmark	$\rho_{A,x} = -0.8$	$\rho_{A,x} = -0.9$
$\sigma(\Delta y)$	3.05 (0.60)	3.50	3.26	3.32
$corr(\Delta c, \Delta i)$	0.40 (0.28)	0.51	0.44	0.45
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.33	0.32	0.33
$E[R^M - R^f]$	5.71 (2.25)	5.71	4.73	4.98
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85	7.33	7.48
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50	-6.86	-7.10

Table C.10: Sensitivity Analysis (Continued)

Panel D: the role of persistence of financial shock ρ_x

	Data	Benchmark	80% Half life	120% Half life
$\sigma(\Delta y)$	3.05 (0.60)	3.50	3.39	3.60
$AC1(\Delta y)$	0.49 (0.15)	0.51	0.48	0.54
$AC1(\frac{B_t}{K_t+H_t})$	0.86 (0.33)	0.81	0.81	0.82
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.33	0.33	0.33
$E[R^M - R^f]$	5.71 (2.25)	5.71	5.47	6.02
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85	7.79	7.92
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50	-6.85	-8.08

Panel E: the role of persistence of productivity shock ρ_A

	Data	Benchmark	80% Half life	120% Half life
$\sigma(\Delta y)$	3.05 (0.60)	3.50	3.49	3.50
$AC1(\Delta y)$	0.49 (0.15)	0.51	0.51	0.51
$AC1(\frac{B_t}{K_t+H_t})$	0.86 (0.33)	0.81	0.80	0.82
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.33	0.33	0.33
$E[R^M - R^f]$	5.71 (2.25)	5.71	4.93	6.30
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85	7.87	7.73
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50	-7.10	-7.80

Panel F: the role of volatility of financial shock σ_x

	Data	Benchmark	$0.9\sigma_x^*$	$1.1\sigma_x^*$
$\sigma(\Delta y)$	3.05 (0.60)	3.50	3.42	3.58
$\sigma(\Delta i)$	10.30 (2.36)	9.75	9.08	10.42
$E[R^M - R^f]$	5.71 (2.25)	5.71	6.23	5.16
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85	7.88	7.82
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50	-6.56	-8.48

Panel G: the role of volatility of productivity shock σ_A

	Data	Benchmark	$0.9\sigma_A^*$	$1.1\sigma_A^*$
$\sigma(\Delta y)$	3.05 (0.60)	3.50	3.23	3.77
$\sigma(\Delta i)$	10.30 (2.36)	9.75	9.37	10.12
$E[R^M - R^f]$	5.71 (2.25)	5.71	4.10	7.51
$\sigma(R^M - R^f)$	20.89 (2.21)	7.85	7.06	8.64
$E[\bar{R}_K^{Lev} - \bar{R}_H]$		-7.50	-6.95	-8.04

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Persistence parameters of exogenous shocks (ρ_x and ρ_A) We vary persistence parameters of exogenous shocks (ρ_x and ρ_A) one at a time. The parameter variations we consider change the half-life of a shock to x or a by $\pm 20\%$.

First, an increase in ρ_x has opposite effects on the risk premia of type-K and type-H capital. On the one hand, a more persistent financial shock makes type-K capital an even better hedging device, which reduces the equilibrium risk premium. On the other hand, entrepreneurs' net worth becomes more volatile, and as a result, the risk premium of type-H capital increases. Put together, this leads to a higher risk premium for the aggregate market and to a larger collateralizability spread. Second, an increase in ρ_A generates a stronger long-run risk channel in cash flows, and as a result, we observe higher risk premia for both type-K and type-H capital. The effects of lower ρ_x and ρ_A are exactly opposite to those generated by higher values for these parameters.

Shock volatilities (σ_x and σ_A) We vary the shock volatilities σ_x and σ_A, one at a time, by $\pm 10\%$. We observe that the effect caused by increasing the two shock volatilities are very similar. A higher σ_x or σ_A leads to an increase in both the market risk premium and the collateralizability spread, which is intuitively clear, since the economy in general becomes riskier.

D Data and measurement

We now provide details on the data sources, the construction of our empirical collateralizability measure, and on the measurement of intangible capital.

D.1 Data sources

Our major sources of data are (1) firm level balance sheet data from the CRSP/Compustat Merged Fundamentals Annual Files, (2) monthly stock returns from CRSP, and (3) industry level non-residential capital stock data from the BEA table.[25] We adopt the standard screening process for the CRSP/Compustat Merged Database. We exclude utilities and financial firms (SIC codes between 4900 and 4999 and between 6000 and 6999, respectively). Additionally, we only keep common stocks that are traded on NYSE, AMEX and NASDAQ. The accounting treatment of R&D expenses was standardized in 1975, and we allow three years for firms to adjust to the new

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[25]The BEA table is from "private fixed asset by industry", Table 3.1ESI.

accounting rules, so that our sample starts in 1978. Following Campello and Giambona (2013), we exclude firm-year observations for which the value of total assets or sales is less than $1 million. We focus on the impact of asset collateralizability on debt capacity of firms, therefore we drop small firms, which do not have much debt in the first place. In practice, we drop firm-year observations with market capitalization below $8 million, which roughly corresponds to the bottom 5% of firms. All firm characteristics are winsorized at the 1% level. The potential delisting bias of stock returns is corrected following Shumway (1997) and Shumway and Warther (1999).

In order to obtain a long sample with broader coverage,[26] we use the narrowly defined industry level non-residential fixed asset (structure, equipment and intellectual) from the BEA tables to back out industry level structure and equipment capital shares.

In Table D.11, we provide the definitions of the variables used in our empirical analyses.

D.2   Measurement of collateralizability

This section provides details on the construction of the firm specific collateralizability measure, complementing the description of the methodology provided in Section 2.

We first construct proxies for the share of the two types of capital, denoted by *StructShare* and *EquipShare*. Then we run the leverage regression (2), which allows us to later calculate the firm-specific collateralizability score.

The BEA classification features 63 industries. We match the BEA data to Compustat firm level data using NAICS codes, assuming that, for a given year, firms in the same industry have the same structure and equipment capital shares. We construct measures of structure and equipment shares for industry l in year t as

$$StructShare_{l,t} = \frac{\text{Structure}_{l,t}^{BEA}}{\text{Fixed Asset}_{l,t}^{BEA}} \frac{\text{Fixed Asset}_{l,t}^{\text{Compustat}}}{\text{PPEGT}_{l,t}^{\text{Compustat}} + \text{Intangible}_{l,t}^{\text{Compustat}}}$$

and

$$EquipShare_{l,t} = \frac{\text{Equipment}_{l,t}^{BEA}}{\text{Fixed Asset}_{l,t}^{BEA}} \frac{\text{Fixed Asset}_{l,t}^{\text{Compustat}}}{\text{PPEGT}_{l,t}^{\text{Compustat}} + \text{Intangible}_{l,t}^{\text{Compustat}}},$$

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[26]COMPUSTAT shows the components of physical capital (PPEGT) only for the period from 1969 to 1997. However, even for the years between 1969 and 1997, only 40% of the observations have non-missing entries for the components of PPEGT, which are buildings (PPENB), machinery and equipment (PPENME), land and improvements (PPENLI).

where $AT_{l,t}$ are total assets in industry l in year t, i.e., the sum of assets across all firms in our sample belonging to industry l in year t. The first component on the right hand side refers to the structure (equipment) share from BEA data, which is given as the ratio of structure (equipment) to fixed assets at the industry level. The second component refers to the industry level fixed asset to total asset ratio in Compustat. We use PPEGT in Compustat as the equivalent for fixed assets in the BEA data. By doing so, we map the BEA industry level measure of structure (equipment) to fixed asset ratio to corresponding measures in the Compustat, at the industry level. Since we distinguish assets by their collateralizability, we normalize fixed assets by the total value of physical and intangible capital.

We interpret the weighted sum, $\zeta_S StructShare_{l,t} + \zeta_E EquipShare_{l,t}$, as the contribution of structure and equipment capital to financial leverage. The product of this sum and the book value of assets, $(\zeta_S StructShare_{l,t} + \zeta_E EquipShare_{l,t}) \cdot AT_{i,t}$, then represents the total collateralizable capital of firm i in year t.[27] Given this, the collateralizability score for firm i in year t is computed as

$$\zeta_{i,t} = \frac{(\zeta_S \cdot StructShare_{l,t} + \zeta_E \cdot EquipShare_{l,t}) \cdot AT_{i,t}}{PPEGT_{i,t} + Intangible_{i,t}}, \tag{D51}$$

where $PPEGT_{i,t}$ and $Intangible_{i,t}$ are the physical capital and intangible capital of firm i in year t, respectively. The importance of taking intangible capital into account has been emphasized in the recent literature, e.g., by Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017). The asset-specific collateralizablity parameters ζ_S and ζ_E we adopt in our empirical analyses are the ones shown in the last column of Table 1, where firms are classified as constrained based jointly on all three measures (SA index, WW index, and non-dividend paying).

In the above collateralizability measure, we implicitly assume the collateralizability parameter for intangible capital to be equal to zero. We do this based on empirical evidence that intangible capital can hardly be used as collateral, since only 3% of the total value of loans to companies are actually collateralized by intangibles like patents or brands (Falato et al. (2013)). Our results remain qualitatively very similar when we exclude intangible capital from the denominator of the collateralizability measure in (D51) and only exploit the differences in collateralizability between structure and equipment capital.

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[27]Alternatively, we also used the market value of assets to compute total collateralizable capital. The empirical collteralizability spread based on this sorting measure is even stronger than that obtained in our benchmark analysis.

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D.3 Measuring intangible capital

In this section, we provide details regarding the construction of firm-specific intangible capital. The total amount of intangible capital of a firm is given by the sum of externally acquired and internally created intangible capital, where the latter consists of R&D capital and organizational capital.

Externally acquired intangible capital is given by item *INTAN* in Compustat. Firms typically capitalize this type of asset on the balance sheet as part of intangible assets. For the average firm in our sample, *INTAN* amounts to about 19% of total intangible capital with a median of 3%, consistent with Peters and Taylor (2017). We set externally acquired intangibel capital to zero, whenever the entry for INTAN is missing.

Concerning internally created intangible capital, R&D capital does not appear on the firm's balance sheet, but it can be estimated by accumulating past expenditures. Following Falato et al. (2013) and Peters and Taylor (2017), we capitalize past R&D expenditures (Compustat item *XRD*) using the so-called perpetual inventory method, i.e.,[28]

$$RD_{t+1} = (1 - \delta_{RD})RD_t + XRD_t,$$

where δ_{RD} is the depreciation rate of R&D capital. Following Peters and Taylor (2017), we set the depreciation rates for different industries following Li and Hall (2016). For unclassified industries, the depreciation rate is set to 15%.[29]

Finally, we also need the initial value RD_0. We use the first non-missing R&D expenditure, XRD_1, as the first R&D investment, and specify RD_0 as

$$RD_0 = \frac{XRD_1}{g_{RD} + \delta_{RD}}, \tag{D52}$$

where g_{RD} is the average annual growth rate of firm level R&D expenditure. In our sample, g_{RD} is around 29%.

Following Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017), our organizational capital is constructed by accumulating a fraction of Compustat item $XSGA$, "Selling, General and Administrative Expense", which indirectly reflects the reputation or human capital of a firm. However, as documented by Peters and Taylor (2017), $XSGA$ also includes R&D expenses XRD, unless

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[28]This method is also used by the BEA R&D satellite account.

[29]Our results are not sensitive to the choice of depreciation rates.

they are included in the cost of goods sold (Compustat item $COGS$). Additionally, $XSGA$ sometimes also incorporates the in-process R&D expense (Compustat item $RDIP$). Hence, following Peters and Taylor (2017), we subtract XRD and $RDIP$ from $XSGA$.[30] Additionally, also following Peters and Taylor (2017), we add the filter that when XRD exceeds $XSGA$, but is less than $COGS$, or when $XSGA$ is missing, we keep XSGA with no further adjustment. Afterwards, we replace missing XSGA with zero. As in Hulten and Hao (2008), Eisfeldt and Papanikolaou (2014), and Peters and Taylor (2017), we count only 30% of SGA expenses as investment in organizational capital, the rest is treated as operating costs.

Using a procedure analogous to the one described above for internally created R&D capital, organizational capital is constructed as

$$OG_{t+1} = (1 - \delta_{OG})OG_t + SGA_t,$$

where $SGA_t = 0.3(XSGA_t - XRD_t - RDIP_t)$ and the depreciation rate δ_{OG} is set to 20%, consistent with Falato, Kadyrzhanova, and Sim (2013) and Peters and Taylor (2017). Again analogous to the case of R&D capital we set the initial level of organizational capital OG_0 according to

$$OG_0 = \frac{SGA_1}{g_{OG} + \delta_{OG}}.$$

The average annual growth rate of firm level $XSGA$, g_{OG}, is 18.9% in our sample.

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[30] $RDIP$ (in-process R&D expense) is coded as negative in Compustat. Subtracting RDIP from XSGA means RDIP is added to XSGA. As discussed in Peters and Taylor (2017), XSGA does not include this component, so we add this component back to XSGA, then subtract the total amount of R&D expenditures.

Table D.11: Definition of Variables

Variables	Definition	Sources
Structure share	We first construct the structure shares from BEA industry capital stock data, defined as structure capital over total fixed asset ratio. Then we rescale the structure shares by the corresponding industry average of physical asset (PPEGT) to book asset ratio (AT).	BEA + Compustat
Equipment share	We first construct the equipment shares from BEA industry capital stock data, defined as equipment capital over total fixed asset ratio. Then we rescale the equipment shares by the corresponding industry average of physical asset (PPEGT) to book asset ratio (AT).	BEA + Compustat
Intangible capital	Intangible capital is defined following Peters and Taylor (2017). We capitalize R&D and SGA expenditures using the perpetual inventory method.	Compustat
Collateralizability	Collateralizable capital divided by PPEGT + Intangible. Collateralizable capital and intangible capital are defined in Section D.2	BEA + Compustat
BE	Book value of equity, computed as the book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) as the book value of preferred stock.	Compustat
ME	Market value of equity is copmputed as price per share times the number of shares outstanding. The share price is taken from CRSP, the number of shares outstandings from Compustat or CRSP, depending on availability.	CRSP+Compustat
BM	Book to market value of equity ratio.	Compustat
Tangibility	Physical capital (PPEGT) to the sum of physical (PPEGT) and intangible capital ratio.	Compustat
Book size	Natural log of the sum of PPEGT and intangible capital.	Compustat
Gross profitability	Compustat item REVT minus COGS divided by AT.	Compustat
OG/AT	Organizational capital divided by total assets (AT).	Compustat
XRD/AT	R&D expenditure to book asset ratio.	Compustat
Book leverage	Lease adjusted book leverage is defined as lease adjusted debt over total asset ratio (AT). The lease adjusted debt is the financial debt (DLTT+DLC) plus the net present value of capital lease as in Li, Whited, and Wu (2016).	Compustat
Dividend dummy	Dummy variable equal to 1, if the firm's dividend payment (DVT, DVC or DVP) over the year was positive.	Compustat
Sales growth volatility	Rolling window standard deviation of past 4 year's sales growth.	Compustat
Rating dummy	Dummy variable equal to 1, if the firm has either a bond rating or a commercial paper rating, and 0 otherwise.	Compustat
Marginal tax rate	Following Graham (2000).	John Graham's website
WW index	Following Whited and Wu (2006).	Compustat
SA index	Following Hadlock and Pierce (2010).	Compustat
Return on asset	Income before extraordinary items (IB) divided by total assets (AT).	Compustat

Table D.11: Definition of Variables (Continued)

Variables	Definition	Sources
Cash	Compustat item CHE.	Compustat
Equity issuance	The negative of flow to debt. Compustat item -(PRSTKC+DV-SSTK).	Compustat
Debt issuance	The negative of flow to debt. Compustat item -(DLTR+DLCCH+XINT-DLTIS).	Compustat
External finance	The sum of equity and debt issuance.	Compustat
Short-term debt	Compustat item DLC.	Compustat
Long-term debt	Compustat item DLTR.	Compustat
Return on equity	Operating income before depreciation (OIBDP) divided by book equity.	Compustat
Financial leverage	Total financial debt (DLTT + DLC) over total book asset (AT) ratio.	Compustat
Cash flow	Compustat item EBITDA divided by total assets	Compustat
Asset growth	Growth rate of total assets	Compustat
Type-K asset growth	Growth rate of PPEGT	Compustat
Age	The current year minus the year where a firm has the first non-missing observation.	Compustat

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