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Taming Models of Prospect Theory in the Wild? Estimation of Vlcek and Hens (2011)

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Non-Technical Summary

Despite its elegant approach, intuitive appeal and formal axiomatization, Expected Utility Theory (EUT) appears to struggle in explaining observed behavior of decision makers as unraveled in countless experimental studies. These shortcomings of EUT motivated researchers to reconciliate observed violations with utility theory and to propose alternative or generalized utility functions to improve descriptive accuracy. As a prominent representative of these generalized utility theories, Daniel Kahneman and Amos Tversky contemplate the possibility that the utility function is not completely concave but contains convex (concave) elements in the domain of gains (losses). Kahnemann and Tversky argue that the typical value function is rather normally concave above a certain reference point and often convex below, tantamount to risk seeking in the domain of losses and risk avoidance for gains. This S-shaped value function with decreasing marginal value for rising magnitudes of gains and losses is governed by a curvature parameter that reflects diminishing sensitivity towards variations in the respective outcomes. Based on empirical evidence indicating losses to be perceived more painful than an equal magnitude of gains, Kahneman and Tversky additionally introduce a steepness-multiplier to capture this loss aversion feature. The final distinctive feature of Prospect Theory in comparison to EUT is the treatment of physical probabilities in a non-linear fashion, leading decision makers to upward-biased decision weights if the probability on an event is sufficiently low and vice versa.

With the intention to contribute to the stream of literature on Prospect Theory and its parameterization in finance, we select and modify the model of Martin Vlcek and Thorsten Hens due to its prominence, simplicity and intuitive appeal, and estimate the required parameters of Prospect Theory that comply with observed trading data of individual investors from a large German brokerage firm using a maximum likelihood.

We find evidence that the majority of individual investors appears to comply with Prospect Theory, however, our results deviate somehow from earlier (experimental) studies on the parameterizations of Prospect Theory. In particular, with respect to the individual investors in our dataset, we find quite low values for risk sensitivity, tantamount to a high curvature of the value function, which implies a pronounced diminishing sensitivity towards large variations in gains and losses.

Furthermore, unlike the frequently cited estimates from Kahneman and Tversky, we find only moderate loss aversion in our dataset, which we link to the specific features of the trading model used in our paper. With respect to the nonlinear treatment of physical probabilities however, our estimates are in line with estimates from earlier studies.

The reliability of our estimates should not be overemphasized as, due to several shortcomings in our dataset, the design of this study and of the econometric method used therein, these estimates are potentially biased towards an underestimation of loss aversion and overestimation of risk sensitivity, as our estimates are by parts associated with large standard errors.

TAMING MODELS OF PROSPECT THEORY IN THE WILD? AN ESTIMATION OF VLCEK AND HENS (2011)

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Abstract. Shortcomings revealed by experimental and theoretical researchers such as Allais (1953), Rabin (2000) and Rabin and Thaler (2001) that put the classical expected utility paradigm von Neumann and Morgenstern (1947) into question, led to the proposition of alternative and generalized utility functions, that intend to improve descriptive accuracy. The perhaps best known among those alternative preference theories, that has attracted much popularity among economists, is the so called prospect theory by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Its distinctive features, governed by its set of risk parameters such as risk sensitivity, loss aversion and decision weights, stimulated a series of economic and financial models that build on the previously estimated parameter values by Tversky and Kahneman (1992) to analyze and explain various empirical phenomena for which expected utility does not seem to offer a satisfying rationale. In this paper, after providing a brief overview of the relevant literature, we take a closer look at one of those papers, the trading model of Vlcek and Hens (2011) and analyze its implications on prospect theory parameters using an adopted maximum likelihood approach for a dataset of 656 individual investors from a large German discount brokerage firm. In contrast to existing literature, we find evidence that investors in our dataset are only moderately averse to large losses and display high risk sensitivity, supporting the main assumptions of prospect theory. Illustrating simulations show that, for those investors, who can be characterized by these parameter estimates, realized returns and roundtrip length statistically resembles those in our dataset.

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1. Introduction

Despite its elegant approach, intuitive appeal, and formal axiomatization, expected utility theory appears to struggle in explaining the observed behavior of decision makers (as unraveled in experimental studies such as those of Allais (1953) and others). These shortcomings in expected utility theory have motivated researchers to reconcile observed violations with utility theory and to propose alternative or generalized utility functions to improve descriptive accuracy.[1] These efforts prompted the creation of alternatives preferences such as rank-dependent utility (Karmarkar (1978), Karmarkar (1979), Quiggin (1982), Wakker (1994)), designed to capture the nonlinear treatment of physical probabilities but hold on to expected utility.[2] Although risk-averse behavior can be reflected by nonlinearities in the decision weights, as shown by Yaari (1965), these modifications of the decision weight were found to be insufficient to explain the observed asymmetric treatment of gains and losses.

Based on experimental evidence, Kahneman and Tversky (1979) contemplated the possibility that the utility function (in accordance with Markowitz (1952)) is not completely concave for gains (convex for losses) but contains convex (concave) elements in the domain of gains (losses).[3] They argued that the typical value function is rather normally concave above a certain reference point and often convex below, tantamount to risk seeking in the domain of losses (concavity) and risk avoidance for gains (concavity). This S-shaped value function with decreasing marginal value for rising magnitudes of gains and losses is governed by a curvature parameter α, which usually refers to diminishing sensitivity toward variations in the respective outcomes.[4] Based on empirical evidence indicating that losses are perceived as more painful than an equal magnitude of gains, Kahneman and Tversky additionally introduced a steepness multiplier λ, where the magnitude of λ determines a stretching or a buckling of the value function. For cases where $\lambda > 1$, the attitude

[1]For example, Friedman and Savage (1948) proposed a utility-of-income function that incorporates a convex (risk-seeking) segment surrounded by two concave (risk-averse) segments to explain the simultaneous demand for insurance and risky gambles. Markowitz (1952) found, that the Friedman–Savage utility function leads to unrealistic predictions, such as too much gambling, and concluded that the inflection point where the concave region becomes convex should be located at the level of current wealth. Furthermore, the author suggested the use of gains and losses instead of terminal wealth as arguments and proposes two additional inflection points. Although increasing marginal utility causes certain discomfort for economists, Hershey and Schoemaker (1980) argued, in accordance with Markowitz, that a concave and convex utility function on gains or losses where the specific name *value function* replaces the usual terminology *utility function* can account for all non-expected utility behavior.

[2]According to rank-dependent utility, the overall utility $U_{RD}(X_i, p_i)$ from an outcome X_i arising in state i with probability p_i depends on an utility function of the risky outcome still being formulated in terms of final wealth (Quiggin (1982)).

[3]In their attempt to eliminate flaws from the original version of prospect theory and to capture individual preferences more accurately, Tversky and Kahneman (1992) combined the ideas of Quiggin (1982) with their original prospect theory and posed what they called *cumulative prospect theory*. Put on an axiomatic basis by Wakker and Tversky (1993), this theory conflates the advantages of rank-dependent utility and concurrently eliminates some deficiencies of the original prospect theory, such as the preference for stochastically dominated lotteries and the restriction to simple binary lotteries. Kahneman and Tversky (1979), p. 275 already noted the problem of stochastic dominance and solved it by the "detection of dominance," assuming that dominated alternatives are eliminated during the editing phase (for further details, see Wakker (2010)).

[4]Risk seeking in the domain of losses has empirical support and arises from the idea that individuals dislike losses and therefore try to gamble for resurrection and are thus willing to take on more risk.

toward gains and losses is commonly labeled loss aversion.[5] The final distinctive feature of prospect theory in comparison to expected utility theory is the treatment of physical probabilities in a nonlinear fashion, leading decision makers to upward-biased decision weights if the probability of an event is sufficiently low and vice versa, captured by a function of physical probabilities $\omega(p)$.

Financial models that are built around these specific features often refer to the parameters estimated by Tversky and Kahneman (1992), although it is not sure whether these parameter estimates fit for models such that the implications drawn from them provide a good description with regard to decision making and asset pricing in financial markets. With the intention of contributing to the stream of literature on prospect theory and its parameterization in finance (and for trading models in particular), we select the model of Vlcek and Hens (2011) due to its similarity to the difference-in-utility approach of Currim and Sarin (1989) and Hey and Orme (1994), which allows us to construct an econometric trading model to estimate the required parameters α, λ, and γ that comply with observed trading data using a maximum likelihood approach derived from their model.

In detail, this paper is organized as follows: In Chapter 2, we briefly review studies that deal with prospect theory in finance, particularly those establishing a connection between the characteristics of prospect theory and investor trading behavior. Among the studies presented in this chapter, we select and analyze the model of Vlcek and Hens (2011). Since their model is constructed in a rather theoretical environment, in Chapter 3 we modify and transform their model into an econometric model that allows for the estimation of the prospect theory parameters used. Therein, we also discuss the necessary modifications, the underlying assumptions we made, and the estimation procedure that allows us to estimate the prospect theory parameters of their (modified) model using the trade data of individual investors from a large German brokerage firm. Chapter 4 presents the results of these estimations, which we relate to the results of empirical and theoretical studies and highlight their commonalities and differences. In contrast to the literature on experimental studies, where the loss aversion parameter λ was estimated to be 2.25 and where the risk sensitivity parameter α was found to be 0.88, we find only moderate loss aversion ranging near unity and a high curvature of the prospect value functional, where α ranges near 0.37. Regarding the parameters of the decision weighting function, γ, we find values near 0.72 that seem to resemble those estimated by Tversky and Kahneman (1992), who found γ to be near 0.69.

2. Prospect Theory: Fit for Finance? A Brief Literature Review

Although not seen as a definitive theory (e.g., Birnbaum et al. (1999), Starmer (2000)) but backed by various studies (Currim and Sarin (1989), Camerer and Ho (1994), Hey and Orme (1994), Fennema and Wakker (1997), Loomes et al. (2002), Wu et al. (2005)), prospect theory has gained popularity among economists. For instance, theoretical literature suspected various empirically supported phenomena to be related to prospect theory, such as matters of portfolio choice (Berkelaar et al. (2004), Gomes (2005), Jin and Zhou (2008)), as well as some aspects of asset pricing (Benartzi and Thaler (1995), Barberis and Huang (2001), Barberis et al. (2001)). Prospect theory is also seen as a driving factor for various effects affecting the trading decisions of individual investors and its consequences, such as the presence

[5] Evidence of loss aversion and initial wealth as reference points is supported by Rabin (2000) and Rabin and Thaler (2001).

of an equity premium (Benartzi and Thaler (1995)), excess stock return volatility (Barberis et al. (2001)), overinsurance (Cutler and Zeckhauser (2004)), and stock market momentum (Grinblatt and Han (2005b), Grinblatt and Han (2005a)), as well as its implications on market liquidity (Pasquariello (2008)), return forecasts (Barberis and Huang (2001)), and herding behavior in stock markets (Lin and Hu (2010)).

The perhaps most frequently cited manifestation of prospect theory in financial markets has been in the so-called *disposition effect*, the tendency of investors to hold onto assets that have lost value compared to those assets whose value rose.[6] The hypothesis that the disposition effect is engendered by differences in the values attached to potential gains and losses was initially listed by Shefrin and Statman (1985)[7] and has led subsequent studies to cite prospect theory as the main if not the only driver of the disposition effect (Weber and Camerer (1998), Odean (1998), Garvey and Murphy (2004), Jordan and Diltz (2004), Lehenkari and Perttunen (2004), Dhar and Zhu (2006), Frazzini (2006)).[8] If prospect theory triggers phenomena such as the disposition effect, this particular behavioral pattern should be observable in other environments as well. In fact, evidence of the disposition effect has been found among individual investors in the stock market (e.g., Schlarbaum et al. (1978a), Ferris et al. (1988), Odean (1998), Odean (1999)) and in the financial advice of stock brokers (Shapira and Venezia (2001)), the behavior of future trades (Heisler (1994), Frino et al. (2004), Coval and Shumway (2005), Locke and Mann (2005)), IPO trading volumes (Kaustia (2004a)), real estate markets (Genesove and Mayer (2001)), insurance contracts (e.g. Camerer and Kunreuther (1989), Schoemaker and Kunreuther (1979)) and risk behavior observed in laboratory environments for stocks (Weber and Camerer (1998), Chui (2001), Dhar and Zhu (2006), Vlcek and Wang (2007), Talpsepp et al. (2014)).

Regardless of its popularity, the theoretical connection between prospect theory and the disposition effect does not appear as clear as intuition suggests, given the estimated parameters from experimental studies such as those of Tversky and Kahneman (1992), Camerer and Ho (1994), Tversky and Fox (1995), Wu and Gonzalez (1996), Birnbaum and Chavez (1997), Fennema and van Assen (1999), Gonzalez and Wu (1999a), Abdellaoui (2000), Bleichrodt and Pinto (2000), Abdellaoui et al. (2005), Abdellaoui et al. (2007). There are various inconsistencies in the hypothesis

[6]The term *disposition effect* was initially coined by Shefrin and Statman (1985) and was brought to prominence by Ferris et al. (1988) and Odean (1998) and repeatedly found in empirical and experimental settings. Note that the disposition effect has been defined in various ways, such as the behavioral pattern where investors linger on stocks that have lately depreciated in value and are anxious to sell those whose price has risen (Shefrin (2008), p. 419), as the tendency to hold losers too long and sell winners too soon (Odean (1998), p. 1775), or in terms of probability, whereby investors are more likely to sell winners than losers (Odean (1998), p. 1779).

[7]Although Shefrin and Statman (1985) support roles for avoiding regret and seeking pride (Muermann and Volkmann (2006)), the role of emotions is not fully explored and leads to an unclear explanation, especially in the case of gains, and can even result in behavioral patterns that are inconsistent with the disposition effect (Shefrin and Statman (1985), Shefrin (2008)). O'Curry Fogel and Berry (2006) discussed the potential role of regret and pride in the context of losers, but without separating regret and disappointment.

[8]Andreassen (1988), however, suspected that investors believe that, after a stock has reached its peak, its price is more likely to decline, whereas they perceive losing stocks as having reached bottom and being likely to rise, no matter whether a lack in mean reversion is detected (e.g. Odean (1998) and Murstein (2003) reported only approximately 5% of all stocks to be mean reverting). Early empirical investigations indicated that the trading pattern of individual investors, lacking strong demand for stocks with past underperformance, is inconsistent with the widespread belief in mean-reverting stock prices (Odean (1998), Zuchel (2001), Kaustia (2004b)).

that prospect theory leads to phenomena such as the disposition effect mentioned in the literature. Exemplarily, studies such as those of Kaustia (2004b), Vlcek and Hens (2011), Kaustia (2010), and Barberis and Xiong (2009) identified a logical flaw in the argumentation of Shefrin and Statman (1985). They found that, if investors are modeled as myopic decision makers, following prospect theory in a multiperiod setting (as implicitly assumed by Shefrin and Statman (1985)), common parameter estimates cause inconsistencies in those models and fail to explain the disposition effect.[9] Other studies, such as those of Arkes et al. (2008), Meng (2010), and Ingersoll and Jin (2012), suggest a modification of the reference point to rematch empirical trading profiles with trading implications derived from prospect theory. In another rescue attempt that tries to reestablish the link between prospect theory and the disposition effect, Barberis and Xiong (2010), Barberis and Xiong (2009) and Ingersoll and Jin (2012) introduced the concept of realization utility, according to which investors only experience an increase in utility if the assets are sold (in contrast to paper gains and losses). Despite its ingenuity, however, the concept of realization utility received only weak support from empirical studies (Ben-David and Hirshleifer (2012)).

Furthermore, experimental studies have been criticized for their artificial settings, particularly regarding their hypothetical payoff structure (e.g., Kahneman and Tversky (1979), Hershey and Schoemaker (1985), McCord and DeNeufville (1986), Tversky and Kahneman (1992), Etchart-Vincent (2004), Laury and Holt (2005)), as well as the way relevant information was presented, since the participants were told the exact relevant probabilities and returns (or at least they had the chance to infer them from the setting). In financial decision making, however, investors cannot be expected to derive the relevant probabilities and returns from the underlying stochastic process (e.g., Ellsberg (1961)). Thus, the way market parameters are estimated, particularly the dynamics of market parameter estimates, could influence the magnitude of the parameters of prospect theory.

Indeed, parameter values adopted from experimental studies such as that of Tversky and Kahneman (1992), which are used to calibrate financial models, are often deemed implausible or inconsistent for reconciling the conclusions drawn from these models with evidence from financial markets.[10] Consequently, a discussion of prospect theory parameter assumptions in financial markets seems inevitable in light of empirical data to determine which assumptions regarding risk sensitivity, loss aversion, and decision weights are consistent with observed trading behavior. The paper of Vlcek and Hens (2011), in which the authors concluded that prospect theory parameters need to differ significantly from those of Tversky and Kahneman (1992) to (consistently) explain the disposition effect, is our starting point to address this question. In this context, it is worth noting that, in a related study, Vlcek and Wang (2007) investigated the relation between risk sensitivity, loss aversion, and decision weighting and the trading behavior of individuals in a controlled experimental setup and found parameter values close to those of Tversky and Kahneman (1992). Although the authors detected a decision pattern similar to the disposition effect, based on the results of a logistic regression, they concluded that

[9] However, due to subtle features of the decision weights, dynamic optimization under prospect theory can generate time-inconsistent trading strategies and thus trading patterns such as the disposition effect. Barberis (2012) demonstrated that this can be traced back to the interplay between the curvature of the value function and the inherent nonlinearities of the decision weights.

[10] The earliest reference, to the best of our knowledge, is that by Siegmann (2002), who recognized that the curvature of the S-shaped value function (given common magnitudes of loss aversion) is insufficient to trigger a phenomenon such as the disposition effect.

the parameterization of the prospect theory function does not seem to explain much of the observed trading decisions. However, their study only shows that parameters obtained from an experimental setting cannot be directly applied to explain decisions in a different (stock trading) environment, emphasizing the need to estimate the parameters from trading decisions directly.

3. An Empirical Estimation by Vlcek and Hens (2011): Stock Market, Trading Behavior, and the Elicitation Procedure

Intending to make a statement regarding prospect theory parameters in financial markets, Vlcek and Hens (2011) offered an intertemporal decision model that directly allows conclusions regarding the parameterization of prospect theory that is consistent with observed trading behavior. To start with, Vlcek and Hens (2011) captured the evolution of the stock price by a simple binomial process (Cox et al. (1979)), as has been done in the context of prospect theory (Barberis and Xiong (2009), Roger (2009)). In this setting, two possible states of the world are identified, namely, an upside state U, realized with probability $p > 0$ at time t, where the stock price follows a rise and yields an upside return $R_U > 1$, and a downside state D with probability $1 - p$, accompanied by a downside return $0 \leq R_D < 1$.

With regard to empirical data, constant amplitudes for upside and downside returns as modeled by Vlcek and Hens (2011) cannot be expected in real stock markets; therefore, this assumption needs to be relaxed. As a proposed modification, at any time t, only two outcomes, indicated by the index t and written as gross returns $R_{D,t}$ and $R_{U,t}$, are possible, where we allow p_t, $R_{D,t}$, and $R_{U,t}$ to vary in time. To keep the notation manageable, we denote the possible upside and downside returns by a common variable $R_{S,t}$, where $S \in \{U; D\}$ unless stated otherwise. Note that, regarding $R_{D,t}$ and $R_{U,t}$, positive prices require the satisfaction of the non-arbitrage condition $0 \leq R_{D,t} < 1 \leq R_{f,t} < R_{U,t}$ with $p_t R_{U,t} + (1-p_t) R_{D,t} > R_{f,t}$, where $R_{f,t}$ represents an alternative risk-free investment (notably the gross return of a bank account). Accordingly, at date t there are $t + 1$ possible states in the tree, where, for $j = 1, 2, \ldots, t + 1$, the case $j = 1$ denotes the highest node and $t + 1$ the lowest node.[11] The price of the risky stock at node j at time t is therefore

$$P_{t,j} = P_0 R_{U,t}^{t-j+1} R_{D,t}^{j-1}. \tag{3.1}$$

To identify investors whose behavior is driven by prospect theory, Vlcek and Hens (2011) followed Kahneman and Tversky (1979) and assumed that the preferences of an individual investor k are based on changes of the initially invested amount of wealth W_0 (Garvey and Murphy (2004); for other possible reference points, see Grinblatt and Keloharju (2001b), Kaustia (2010), Meng (2010)) evolving as in (3.1) and being repeatedly evaluated at any point in time $t \in \{1, \ldots, T\}$, a day between the purchase and the sale day T. In the stock market, the investor faces the choice between an investment in a risky stock bestowing a daily gross return of either $R_{U,t}$ or $R_{D,t}$ or, alternatively, an investment in a money market account from which the investor receives a daily gross return of $R_{f,t}$, both alternatives modeled as being mutually exclusive.[12] In this vein, Vlcek and Hens (2011) modeled an

[11] Time-dependent upside and downside returns and probabilities induce a path dependence that generally leads to 2^t nodes. However, we follow the usual convention of the literature of defining the gross returns $R_{U,t}$ and $R_{D,t}$ as inversely related to each another, which results in a recombining tree with $t + 1$ terminal nodes.

[12] It should be noted that modeling portfolio strategies such as those of Vlcek and Hens (2011) are in line with theoretical models on static portfolio choice under prospect theory, such as those of Schmidt and Zank (2007), Jin and Zhou (2008), Bernard and Ghossoub (2010), and He and

investor's decision to trade a stock based on differences in utilities from the stock and the risk-free asset, denoted $\Delta_t(U_k|\boldsymbol{\theta_k})$, given the prospect theory parameter set $\boldsymbol{\theta}_k = \{\alpha, \lambda, \gamma\}$, where α denotes the curvature of the prospect theory value functional and to which Vlcek and Hens (2011) referred as *risk sensitivity*, λ denotes the *loss aversion* parameter, and γ represents a decision weight parameter defined according to Tversky and Kahneman (1992).

Although trying to keep as close as possible to the approach of Vlcek and Hens (2011), we need to deviate with respect to some aspects to capture certain features of our dataset. First, modeling the disposition effect with respect to prospect theory parameters appears to be too restrictive, since investors could also exhibit the opposite disposition effect (Weber et al. (2014)). In addition, Vlcek and Hens (2011) noted that, to generate a pattern that resembles the disposition effect, not all parameter combinations allow a consistent model of purchases and sales of the risky asset.[13] A second significant deviation is the introduction of a more general formulation for intermediate gains and losses (Odean (1998)), Vlcek and Hens (2011) allowed gains and losses to be only within a short range (i.e., R_U and R_D, respectively) due to their two-period setting, which appears to be too restrictive for our dataset.

Modeling an investor's trading behavior under generalized current gains and losses, as Vlcek and Hens (2011), has important implications on the model setup, since the dynamics of gains and losses determine the cases for which a loss aversion parameter applies. The specification of intermediate gains and losses, denoted $\hat{R}_{S,t}$, where $S \in \{U; D\}$, indicating a current gain where $S = U$ such that $\hat{R}_{S,t} \geq 1$ or a current loss where $S = D$ with $1 > \hat{R}_{S,t} \geq 0$, is crucial to $\Delta_t(U_k|\boldsymbol{\theta_k})$, since $\hat{R}_{S,t}$ determines the respective prospect values and the application of the loss aversion parameter λ. Accordingly, the prospect value of the risk-free asset, denoted $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\boldsymbol{\theta}_k)$, is assumed to be $(W_0\hat{R}_{S,t}R_{f,t} - W_0)^\alpha$ if $\hat{R}_{S,t} \geq 1$ and $-\lambda(W_0\hat{R}_{S,t}R_{f,t} - W_0)^\alpha$ if $\hat{R}_{S,t} < 1$ is sufficiently low such that $\hat{R}_{S,t}R_{f,t} < 1$.

In distinction from the two-period setting of Vlcek and Hens (2011), we define risk in terms of $\hat{R}_{S,t}$ rather than $R_{D,t}$, which, in turn, requires an extended case distinction. If accumulated gains $\hat{R}_{S,t}$ are high enough such that $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq \hat{R}_{S,t}R_{D,t} \geq 1$ (Case 1), the investors' overall prospect value for the stock

Zhou (2011)), who found that investors with prospect theory preferences may find corner solutions optimal and prefer full sales of existing positions (see Gomes (2005) for a constant relative risk aversion form of prospect theory and Polkovnichenko (2005) for a model under rank-dependent utility). Note that, once multiperiod settings are considered, corner solutions are no longer necessarily optimal (e.g., Gollier (1997), Vlcek (2006), Barberis and Xiong (2009)).

[13] Vlcek and Hens (2011) distinguished between an ex post disposition effect, for which a large scale of prospect theory parameters provide acceptable results, and an ex ante disposition effect. The authors argued that, if initial purchase decisions are considered, it can be optimal for investors not to buy the stock if they are aware of the ex post behavior in the next period (see also Barberis and Xiong (2009)). With respect to evidence from empirical studies, it seems inadvisable to rely solely on prospect theory to model purchase decisions, since, following Odean (1999), Statman et al. (2006), and Glaser and Weber (2007), these decisions are driven by different factors. For example, overconfident investors may suffer from biased beliefs about the anticipated returns they expect to generate by trading stocks, even if these investors performed averagely in the past (Barber and Odean (1999), Odean (1999), Glaser and Weber (2007)) and are thus inclined to buy stocks more readily. Consequently, this opens a wide range of other possible reasons why these investors bought the stock in the first place.

at time t, denoted $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k)$, can be written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) = \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^{\alpha} + \omega(1-p_t)(W_0\hat{R}_{S,t}R_{D,t} - W_0)^{\alpha}, \quad (3.2)$$

where W_0 denotes the amount of wealth initially invested in the stock at time 0. According to Vlcek and Hens (2011), the decision weights $\omega(p_t)$ are defined as $\omega(p_t) = p_t^{\gamma}(p_t^{\gamma} + (1-p_t)^{\gamma})^{-\frac{1}{\gamma}}$ (similarly, $\omega(1-p_t)$; see Tversky and Kahneman (1992)). If the investor is endowed with a stock that has only moderately increased in value such that $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq 1 > \hat{R}_{S,t}R_{D,t} \geq 0$ (Case 2), that investor's overall prospect value can be written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) = \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^{\alpha} - \lambda\omega(1-p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t})^{\alpha}. \quad (3.3)$$

The prospect value of the risk-free asset $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\boldsymbol{\theta}_k)$ is $(W_0\hat{R}_{S,t}R_{f,t} - W_0)^{\alpha}$, as in Case 1. The prospect values derived from holding the stock until the next period can be decomposed into the prospect value stemming from a rise of the stock price by the amount $R_{U,t}$ and a second component multiplied by loss aversion λ that represents the loss of the prospect value if the downside state occurs. These expressions are multiplied by their corresponding decision weights $\omega(p_t)$ and $\omega(1-p_t)$, expressing the impact of the upside or downside event on the overall prospect value.

Note that, in the domain of accumulated losses where $0 \leq \hat{R}_{S,t} < 1$, the investor still faces Case 2: If the losses turn out to be moderate such that $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq 1 > \hat{R}_{S,t}R_{D,t} \geq 0$, the investor still holds a chance of winning back these losses and ending up with a gain by switching to the risk-free asset. Accordingly, the investor's prospect value can be written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) = \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^{\alpha} - \lambda\omega(1-p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t})^{\alpha}. \quad (3.4)$$

The prospect value from an investment in the risk-free asset is $U_k(W_t, \hat{R}_{S,t}, R_{f,t}|\boldsymbol{\theta}_k) = (W_0\hat{R}_{S,t}R_{f,t} - W_0)^{\alpha}$, as in Case 1. Although for moderate accrued losses, the investor is in a Case 2 situation, we indicate that the investor incurred a loss and define an auxiliary Case 3. If the losses $\hat{R}_{S,t}$ turn out to be more severe, where $\hat{R}_{S,t}R_{U,t} > 1 > \hat{R}_{S,t}R_{f,t} > \hat{R}_{S,t}R_{D,t} \geq 0$ (Case 4), the investor obtains an overall prospect value from the stock of the form

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) = \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^{\alpha} - \lambda\omega(1-p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t})^{\alpha}. \quad (3.5)$$

Note that, in this case, the prospect value of the risk-free asset $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\boldsymbol{\theta}_k)$ is now negative, $-\lambda(W_0 - W_0\hat{R}_{S,t}R_{f,t})^{\alpha}$. Finally, if losses are high enough such that they cannot be offset by the proceeds from the riskless assets (i.e., $1 > \hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} > \hat{R}_{S,t}R_{D,t} \geq 0$ (Case 5)), both the prospect values from the stock and the risk-free asset are negative. In particular, the prospect value of the stock is now written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) = -\lambda\omega(p_t)(W_0 - W_0\hat{R}_{S,t}R_{U,t})^{\alpha} - \lambda\omega(1-p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t})^{\alpha} \quad (3.6)$$

and the prospect of the risk-free asset, $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\boldsymbol{\theta}_k)$, takes the form $-\lambda(W_0 - W_0\hat{R}_{S,t}R_{f,t})^\alpha$. Therefore, due to the applicability of the loss aversion parameter λ, it is imperative to distinguish whether the losses can still be offset by the proceeds of the riskless investment alternative or the upside returns from the risky asset. Accordingly, in the framework of Vlcek and Hens (2011), an investor buys or holds the stock whenever $\Delta_t(U_k|\boldsymbol{\theta_k}) := U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) - U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\boldsymbol{\theta}_k) \geq 0$ and vice versa. It is important to note that the initially invested amount of wealth W_0 can be truncated from both sides of the inequality; therefore, in this form, our analysis is insensitive and robust regarding an individual investor's budget.

4. Extension toward an Econometric Decision Model

In light of empirical evidence on trading behavior in stock markets, it seems reasonable to assume that an individual investor's decisions to sell or buy stocks are not solely driven by differences in prospect values $\Delta_t(U_k|\boldsymbol{\theta_k})$ but also dependent on other, independent factors. As a logical consequence, we need to address these factors and extend the model of Vlcek and Hens (2011) by an investor-specific and additively separable stochastic component ϵ_k to introduce uncertainty in the decision process (Cramer (1986), Train (1986), Rust (1994), Train (2009)), such that we arrive at the decomposition $V_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) = U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) + \epsilon_k$. Accordingly, an investor buys or holds the risky asset whenever $\Delta_t(U_k|\boldsymbol{\theta_k}) + \epsilon_k \geq 0$, including in the case where the difference in prospect value is negative but, due to other factors, the error is large enough to counterbalance the inequality. By specifying the underlying stochastic process of the investor-specific error term ϵ_k, we can derive the likelihood function of investor k, denoted $L(\Delta_t(U_k|\boldsymbol{\theta_k}))$. As a technical remark, we assume that the risk-free return $R_{f,t}$ is in fact risk free, which, in turn, allows us to assume that the investor-specific error is zero for payoffs generated by the risk-free asset. This technical assumption avoids the necessity of evaluating the covariance matrix of errors along with $\boldsymbol{\theta}_k$. Note that the standard errors estimated for $\boldsymbol{\theta}_k$ depend on the correlation structure of the error terms but should not have an impact on the estimated parameters (for details, see Train (2009)).

A frequent approach regarding the stochastic characteristics of ϵ_k, which consequently determines the functional form of $L(\Delta_t(U_k|\boldsymbol{\theta_k}))$, is to assume that ϵ_k are normally distributed, $\epsilon_k \sim N(0, \sigma_k^2)$, where the density of the error is characterized by $\phi(\epsilon_k) = (2\pi\sigma_k^2)^{-\frac{1}{2}} e^{-\frac{1}{2}(\epsilon_k/\sigma_k)^2}$ (Hey and Orme (1994), Carbone and Hey (2000)). By assuming normally distributed errors, we implicitly assume that other factors driving the purchase and sales decisions of investors are unsystematic with respect to utility $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k)$, although other assumptions of ϵ_k are possible (e.g., Harless and Camerer (1994), Hey and Orme (1994), Loomes and Sugden (1995), Wilcox (2008), Booij et al. (2009). We refer to Harrison and Rutstrom (2008) for a discussion of the different specifications of ϵ_k). The introduction of a buy-or-hold index $I_{k,t} := I[\Delta_t(U_k|\boldsymbol{\theta_k}) + \epsilon_k \geq 0]$ allows us to derive the respective choice probabilities for $\Delta_t(U_k|\boldsymbol{\theta_k})$: Given a normal distribution of ϵ_k, the conditional choice probability of holding the stock is defined as the cumulative normal density function $\Phi\left(\Delta_t(U_k|\boldsymbol{\theta_k})/\sigma_k\right)$ and the probability of investing in the riskless asset is defined as $1 - \Phi\left(\Delta_t(U_k|\boldsymbol{\theta_k})/\sigma_k\right) = \Phi\left(-\Delta_t(U_k|\boldsymbol{\theta_k})/\sigma_k\right)$.[14] Note that the model of Vlcek and Hens (2011) represents the (extreme) cases where the probability of holding the stock converges to unity if the stock generates an infinite stream of utility (e.g., $\Delta_t(U_k|\boldsymbol{\theta_k}) \to \infty$). On the other hand, if the difference in utility is infinitively negative, such that $\Delta_t(U_k|\boldsymbol{\theta_k}) \to -\infty$, the investor's probability of holding the stock approaches zero (for a general reference, see Rust (1994)).

Given the binary choice feature of Vlcek and Hens (2011), reflected in the dichotomous variable $I_{k,t}$, combined with the assumption of the error term, the overall (logarithmized) likelihood function of an investor i is

$$\log L(\Delta_t(U_k|\boldsymbol{\theta_k})) = \sum_{t\in T} \log\left(\Phi\left(\frac{\Delta_t(U_k|\boldsymbol{\theta_k})}{\boldsymbol{\sigma}_k}\right)^{I_{k,t}} \Phi\left(-\frac{\Delta_t(U_k|\boldsymbol{\theta_k})}{\boldsymbol{\sigma}_k}\right)^{1-I_{k,t}}\right), \quad (4.1)$$

in which we omit constant terms because they add no further information about $\boldsymbol{\theta}_k$.[15] It can be shown that maximizing $\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))$ with respect to $\boldsymbol{\theta_k}$ provides asymptotically efficient and unbiased estimators $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$.[16] To obtain $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$, equation (4.1) needs to be evaluated numerically. The numerical evaluation of $\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))$ for $\boldsymbol{\theta_k}$ is performed in the `ml model` environment of the statistical software Stata v. 10.1, since it allows one to conveniently customize the likelihood function $\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))$.

5. Estimation of Prospect Theory Parameters: A Calibration of Vlcek and Hens (2011) Using Trading Data

To estimate $\boldsymbol{\theta}_k$ in the model of Vlcek and Hens (2011), it is imperative to perform the necessary analysis on a per-investor basis, since the model is formulated for an individual investor.[17] An appropriate way of conducting this analysis is to use trading data from discount brokers, as done in other studies that focus on individual investor trading behavior (e.g., Odean (1998), Barber and Odean (1999), Odean (1999), Barber and Odean (2000), Barber and Odean (2001), Kumar and Goetzmann (2008), Kumar (2009)). We emphasize that our dataset is similar to those of Odean (1999) and Barber and Odean (2000) and contains information regarding portfolio compositions and trading data for a random selection of 5,000 individual investors, where the recorded transaction in the *trade file* can be uniquely assigned to an individual investor (for details, see Weber et al. (2014)).

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[14]In detail, $p(\Delta_t(U_k|\boldsymbol{\theta_k}) > 0)$ can be derived as

$$\begin{aligned} p(\Delta_t(U_k|\boldsymbol{\theta_k}) > 0) &= \int_{-\infty}^{\infty} I[\Delta_t(U_k|\boldsymbol{\theta_k}) + \epsilon_k > 0]\phi(\epsilon_i)d\epsilon_i \\ &= \int_{-\infty}^{\frac{\Delta_t(U_k|\boldsymbol{\theta_k})}{\sigma_k}} \phi(\epsilon_k)d\epsilon_k = \Phi\left(\Delta_t(U_k|\boldsymbol{\theta_k})/\sigma_k\right). \end{aligned}$$

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[15]Similar expressions for $\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))$ are used by Hey and Orme (1994), de Palma et al. (2008), and Harrison and Rutstrom (2008).

[16]A sketch of this and further details of the maximum likelihood approach used are relegated to the appendix.

[17]Note that our procedure differs due to the high number of observations that comes along with trading data from the usual way parameter estimates are obtained in experimental studies (e.g., Harrison and Rutstrom (2008), Harrison and Rutstrom (2009), von Gaudecker et al. (2009)). In these studies, a single maximum likelihood function is evaluated across the whole sample and p-values for the estimates are obtained by Wald tests (Harrison (2008), Harrison and Rutstrom (2008)).

Regarding the setup for their model, Vlcek and Hens (2011) emphasize that empirical research provides some indication that individual investors treat other streams of income, such as dividends and other cash flows resulting from corporate actions and other stocks (Shefrin and Statman (1984), Baker and Wurgler (2004)), in different mental accounts (Thaler (1985)). Furthermore, the tendency to evaluate risky lotteries separately, known as narrow framing (Barberis and Huang (2001), Barberis et al. (2001), Barberis et al. (2001), Berkelaar et al. (2004), Gomes (2005), Barberis and Huang (2009)), is in line with the work of Vlcek and Hens (2011), complementing studies on individual investor trading decisions that examine the trading decisions for each stock separately.[18] Concerning the stochastic process of the risky asset, Vlcek and Hens (2011) do not explicitly mention other risky assets such as traded fixed income investments, mutual funds, or structured products; consequently, we discard those investments and focus exclusively on stocks.[19]

With respect to the stochastic process of the stocks and the specification of (3.1), the results derived from a large number of empirical studies need to be considered when it comes to individual investors' trading behavior and formulation of expectations about p_t, $R_{D,t}$, and $R_{U,t}$ of the stocks underlying the trades in our dataset. DeBondt (1993) mentioned investors may consider recent past returns to be representative of future developments of the stock price and may formulate their expectations based on the stock's history (Kahneman and Tversky (1973), Andreassen (1987), Andreassen (1988), Andreassen and Kraus (1990)). We consider this mental pattern and formulate the return parameters $R_{D,t}$ and $R_{U,t}$ as prospects extrapolated from the past over some period n, which we set to $n = 60$ days, the default for the investors' trading tools offered by the discount broker. Note that, given the observation of past returns to formulate $R_{U,t}$ and $R_{D,t}$, due to extrapolation bias with short horizons, investors may buy stocks whose price has recently increased, especially if they are following a myopic trading strategy, ruling out implicit expectations of mean reversion (Zuchel (2001)). In doing so, we explicitly distinguish from the representativeness bias, whereupon investors base their judgments on stereotypes and seek patterns in returns or prices (e.g., Weber and Camerer (1998), Shefrin (2008)). By formulating $R_{U,t}$ and $R_{D,t}$ based on past returns over n, we align the model of Vlcek and Hens (2011) with other studies, such as those of Grinblatt and Keloharju (2000) and Kaustia (2010), who found that Finnish investors bought past winners and sold past losers, thus revealing a trend-following trading strategy, which is inconsistent with an expectation of

[18]See, for instance, Shefrin and Statman (1985), Odean (1998), Odean (1999), Barber and Odean (2000), Barberis and Huang (2001), Grinblatt and Keloharju (2001a), Grinblatt and Keloharju (2001b), Barber and Odean (2002), Dhar and Kumar (2002), Hong and Kumar (2002), Zhu (2002), Grinblatt and Han (2005b), Lim (2006), Frazzini (2006), Barber and Odean (2008).

[19]This is common practice in empirical studies on matters of individual investor trading (e.g., Barber and Odean (2000), Barber and Odean (2001), Barber et al. (2011), Graham and Kumar (2004), Mitton and Vorkink (2007), Kumar and Goetzmann (2008)), although insights from trades in securities or portfolios characterized by asymmetric payoffs (Mitton and Vorkink (2007), Barberis and Huang (2008)) regarding the decision weight parameter γ are ignored. We expect the loss of information to be negligible, since Weber et al. (2014) reports that products with asymmetric return profiles are not widespread investments in our dataset. Note that dropping certain products is in line with studies on trading behavior in mutual funds (e.g., Grinblatt and Titmann (1989), Grinblatt and Titmann (1993), Brown and Goetzmann (1995), Carhart (1997), Daniel et al. (1997), Chan et al. (2000), Wermers (2000), Coval and Moskowitz (2001), Kosowski et al. (2006); for the resulting trade pattern, see Murstein (2003)) and in financial products with asymmetric payoffs. Baule and Tallau (2011)) indicate that other trading motives may exist that mimic trade patterns from prospect theory (for trading in mutual funds, see Ivkovic and Weisbrenner (2009), Chang et al. (2012),; for trading in structured products, see Entrop et al. (2013)), thus probably leading to systematic biases in the calibration of the parameter set.

mean-reverting stock prices. Similarly, Dhar and Kumar (2002) investigated the price trends of stocks bought by more than 62,000 households using discount brokerage data and concluded that investors prefer to buy stocks that have recently enjoyed abnormal returns.

Concerning the estimation of p_t, we follow the approach presented by Weber and Camerer (1998), in which an individual investor is assumed to update a subjective probability p_t in a Bayesian fashion by observing upticks and downticks, given the investor observes a change in the prices.[20] For the specification of $R_{U,t}$ and $R_{D,t}$, we apply a method similar to that of Barberis and Xiong (2009), which draws on the assumption that the stock price is assumed to follow a binomial process, as defined in (3.1). Given the features of such a stochastic process, we estimate the expected returns μ_t and the volatility σ_t for each stock in our dataset; consequently, the values for $R_{D,t}$ and $R_{U,t}$ can be derived from μ_t and σ_t.[21]

To break down complex trades from the trade file to obtain simple and unambiguous trading sequences, commonly referred to as *round trips* (Shapira and Venezia (2001)), we follow the methods proposed by Lacey (1945), Schlarbaum et al. (1978b), Schlarbaum et al. (1978a), and Silber (1984) and apply the *first-in-first-out (FIFO)* principle throughout our dataset if not mentioned otherwise. By applying the FIFO principle (which is the implicit accounting principle according to current tax regulations in Germany), we reflect the results from empirical studies (e.g., Lakonishok and Schmidt (1986), Grinblatt and Keloharju (2004), Barber and Odean (2004), Ivkovic et al. (2005), Horn et al. (2009)) supporting the assumption

[20] We calculate the required uptick probability as $p_t = \frac{t-j+1}{t} = \frac{n_U}{n_D+n_U}$, where n_D and n_U denote the number of down or up moves, respectively, of the respective risky asset. Note that p_t is a maximum likelihood estimator for probability p given a binomial distribution $p_t = p^{t-j+1}(1-p)^{j-1}$.

[21] Given a rolling-window estimation approach over a look-back period n, the expected returns and volatility take the form $\mu_t = (R_{U,t}p_t + R_{D,t}(1-p_t))^t$ and $\sigma_t^2 = \left((R_{U,t}^2 p_t + R_{D,t}^2(1-p_t)) - (R_{U,t}p_t + R_{D,t}(1-p_t))^2\right)^t$, respectively. If the required stock parameters are aligned with these two expressions, $R_{D,t}$ and $R_{U,t}$ have to fulfill these basic equations simultaneously. By combining both equations and solving for $R_{U,t}$ and $R_{D,t}$, we obtain explicit expressions for $R_{U,t}$ and $R_{D,t}$ respectively. In detail, the values for $R_{D,t}$ and $R_{U,t}$ can be assigned by calculations using μ_t and σ_t at time t for different formation periods with $R_{U,t} = \mu_t^{\frac{1}{t}} + \sqrt{\frac{1-p_t}{p_t}\left((\mu_t^2+\sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}}\right)}$ and $R_{D,t} = \mu_t^{\frac{1}{t}} - \sqrt{\frac{p_t}{1-p_t}\left((\mu_t^2+\sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}}\right)}$, respectively. From (3.1), we can calculate the expected value and volatility of the stock for time t in terms of returns as $\mu_t = (R_{U,t}p_t + R_{D,t}(1-p_t))^t$ and $\sigma_t^2 = \left((R_{U,t}^2 p_t + R_{D,t}^2(1-p_t)) - (R_{U,t}p_t + R_{D,t}(1-p_t))^2\right)^t$, respectively. The required values of $R_{U,t}$ and $R_{D,t}$ must fulfill these two basic equations simultaneously. Adding and subtracting μ^2 combines them and yields $\mu_t^2+\sigma_t^2-\mu_t^2 = p_t R_{U,t}^2(1-p_t)+(1-p_t)R_{D,t}^2 p_t - 2p_t(1-p_t)R_{U,t}R_{D,t}$, which allows us to use the binomial formula $\mu_t^2+\sigma_t^2-\mu_t^2 = p_t(1-p_t)[R_{U,t}-R_{D,t}]^2$ and $R_{U,t}-R_{D,t} = \sqrt{(\sigma^2+\mu^2)^{\frac{1}{t}} - (\mu^2)^{\frac{1}{t}}}$. From μ_t, it follows that $R_{U,t} = \frac{1}{p_t}\mu^{\frac{1}{t}} - \frac{1-p_t}{p_t}R_{D,t}$ and $R_{D,t} = \frac{1}{1-p_t}\mu^{\frac{1}{t}} - \frac{p_t}{1-p_t}R_{D,t}$. By combining this with $\mu_t^2+\sigma_t^2-\mu_t^2$, we can calculate the required market values for $R_{U,t}$ and $R_{D,t}$ as stated above.

that the mental accounting of individual investors follows tax regulations.[22]

The likelihood function (4.1) is evaluated for each investor in our dataset to obtain estimates for $\boldsymbol{\theta}_k$ and the standard deviation of the error term σ_k, where we transform the latter by an exponential function (Rabe-Hersketh and Everitt (2004)) to guarantee strict positivity of the estimate for the error term. The numerical search algorithm is constructed by a mixed iteration procedure where we run a Newton–Ralphson procedure for the first five steps. If no solution is obtained or the algorithm fails to converge, we switch to the Davidon–Fletcher-Powell (Fletcher (1980)) algorithm for the next five iterations to push the estimates outside of the critical section of the likelihood function and then return to the former technique. Furthermore, we follow the recommendation of Cramer (1986) and restrict the number of iterations to 30.[23]

With respect to the surface of the likelihood function $\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))$, we are concerned that local maximum problems may arise due to convex segments in the prospect value function, leading to erroneous estimates for $\boldsymbol{\theta}_k$ if the numerical search algorithm gets stuck at such a local optimum. We address this problem in two ways: First, as described above, we alter the numerical search algorithm every five steps, a procedure also recommended by Judge et al. (1985), Ruud (2000) and Gould et al. (2006). Second, we decided to apply a vector of randomly selected starting values for the numerical algorithm within the boundaries of our parameter set $\boldsymbol{\theta}_k$ (Liu and Mahmassani (2000)). Every time Stata reports successful convergence, we store the estimates and repeat this procedure using a new starting vector. This procedure is repeated 11 times and the estimate with the highest absolute value for $\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))$ is selected at the end.

6. Vlcek and Hens (2011) Put to the Test

Since estimation of a multi-parameter function such as those used by Vlcek and Hens (2011) turned out to be numerically and computationally demanding and particularly time consuming, we use a reduced dataset instead of the total of $5,000$ investors. To optimize computation times but still obtain a satisfying statistical reliability for our estimation results, we pick a subset of investors for which we performed the evaluation of the likelihood function (4.1). In detail, we randomly select from our original dataset a subsample of 659 investors, which corresponds to a targeted 95% confidence interval, implying a sampling error of 3.56% if we assume a binomial distribution with a conservative probability of 50% regarding the occurrence of prospect theory being the dominant utility model. The application

[22] Although Vlcek (2006) presented an extension of the model of Vlcek and Hens (2011), where portfolio weights vary between zero and one, we ignore the underlying portfolio positions, since these might not fully reflect risk preferences due to other factors (e.g., portfolio inertia; see Calvet et al. (2009), Bilias et al. (2010)), which potentially affects our results for risk preferences. However, we admit that trade data can also be contaminated by other factors, such as stale limit orders (in the context of the disposition effect, see Linnainmaa (2010)). It should be noted that portfolio positions can be retrieved by reconstructing residual positions, as described by Barber et al. (2007) and Barber et al. (2009) to reflect active decision making. Under this approach, however, initial portfolio positions can be pronouncedly volatile due to the inherent initial share of idiosyncratic risk erroneously indicating lower risk aversion if used for the estimation of an individual's risk aversion before ramping up enough portfolio volume for further analysis.

[23] A trial-and-error search in terms of the number of iterations and computational times showed that, among the available numerical search techniques within Stata v. 10.1, the (Berndt et al. (1974)) algorithm performed the worst, which left us with the Newton–Ralphson and Davidon–Fletcher–Powell algorithms (Fletcher (1980)), a result in line with those of Griffiths et al. (1987).

of an overlapping-window procedure in our estimation of the stocks' characteristics μ_t, σ_t, and p_t reduces the number of likelihood functions and cuts the number of investors to 656, since three investors had to be dropped because their time series spanned fewer than 60 days. The trading history of the remaining 656 individual investors covers $3,724$ distinct securities, for which we could construct single likelihood functions for each day of their trading history, summing to $17,186,660$ single likelihood functions needing to be evaluated. However, further inspections lead to the exclusion of three investors from this reduced dataset, since, for our stock parameter estimates μ_t and σ_t, the variance–covariance matrix of the investors' portfolio holdings is not positive semidefinite and is thus internally inconsistent. The final reduced dataset thus comprises $38,903$ round trips conducted between January 1999 and November 2011 in stocks, with an average of approximately 107 and a median of 65 round trips per investor.

Given this set of observations, we evaluate $2,612$ prospect theory and nuisance parameters numerically, from which we actually estimate $1,084$ parameters successfully, summing to a total number of 271 out of 653 investors. Note that these 271 investors correspond to those for which prospect theory was found to be the best-fitting utility model, given their trade history (Hackethal et al. (2015)). In detail, from these 271 investors, the trading patterns of 228 of them can be best characterized by the original version of prospect theory (Kahneman and Tversky (1979)). In a myopic forward-looking setting, as for Vlcek and Hens (2011), where cumulative prospect theory as proposed by Tversky and Kahneman (1992) coincides with the original version, another 33 investors can be added to our sample. If prospect theory yields the second rank but statistical tests show no significant differences from the first-ranked model at the 10% significance level, we can enrich our sample by another 10 investors.

Regarding this reduced dataset of 271 investors, it is inevitable to check whether the identification of prospect theory as the best-fitting utility model in the evaluation process could somehow bias our results. This could be the case if the characteristics of the original sample of 653 investors significantly differ from those of this reduced dataset. A tabulation of the market parameters $R_{U,t}, R_{D,t}, andp_t$ and the underlying parameters of the binomial process (see Table 3), as well as realized trade returns and intermediate gains and losses (see Tables 1 and 2), which serve as arguments for the likelihood $\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))$, reveals no significant differences between the original and reduced datasets. The t-tests we performed to compare the means of both datasets showed no significant differences between the market parameters μ_t, σ_t, and p_t used to derive $R_{U,t}$ and $R_{D,t}$. The population stability index with respect to realized gains and losses[24] is below 0.091, indicating only minor differences. A Kolmogoroff–Smirnoff test in the differences between the reduced dataset of 653 investors and the dataset of 271 investors hints at no significant differences between the two datasets either (p-value 0.384). With respect to the comparison of the means of trade returns and accrued returns, similar t-tests indicated no significant differences between the datasets, also at the 10% level, such that we conclude that no systematic differences are due to the reduction of our dataset.

[24]The population stability index, calculated as $(\%TradeReturn_{653} - \%TradeReturn_{271}) \ln(\%TradeReturn_{653} - \%TradeReturn_{271})$ and ranging between zero and infinity, is used to obtain an indicator of the representativeness of a dataset.

Figure 1. Descriptive Summary of Trade Returns

The top panel provides a descriptive summary of realized trade (round-trip) returns across all 653 investors. The bottom panel summarizes the results for individual investors when the likelihood function (4.1) is successfully evaluated and the investors can be classified as prospect theory investors. For both panels, trade returns $\hat{R}_{S,T}$ are taken directly from the trade records and reported as daily gross returns. The term *Obs.* denotes the number of observed round trips in the dataset; *Mean* and *Median* denote the arithmetic mean and median of returns, respectively; *Std.* denotes the standard deviation of returns; and 5*p*, 25*p*, 75*p*, and 95*p* denote the fifth, 25th, 75th, and 95th percentiles of the returns, respectively.

	Mean	Std.	Median	5p	25p	75p	95p	Obs.
Trade Returns for all 653 Investors								
Total	0.9885	0.7099	0.9922	0.1825	0.7928	1.1025	1.6885	38903
Case 1	1.4040	0.9835	1.1765	1.0328	1.0854	1.4118	2.4120	15141
Case 2	1.0642	0.4553	1.0230	1.0024	1.0103	1.0471	1.1403	3031
Case 3	0.9999	0.0001	0.9999	0.9998	0.9999	1.0000	1.0000	4
Case 4	0.9649	0.0489	0.9804	0.8824	0.9595	0.9911	0.9975	2731
Case 5	0.6531	0.2896	0.7542	0.0569	0.4581	0.8928	0.9650	17996
Trade Return for reduced dataset of 271 Investors								
Total	1.0082	0.9826	0.7486	0.1292	0.7478	1.1358	1.9181	9048
Case 1	1.4778	1.2118	0.9450	1.0311	1.0926	1.4931	2.7779	3277
Case 2	1.0796	1.0199	0.5595	1.0022	1.0083	1.0474	1.1986	519
Case 3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
Case 4	0.9667	0.9830	0.0464	0.8793	0.9596	0.9926	0.9981	515
Case 5	0.6363	0.7362	0.3022	0.0505	0.3984	0.8943	0.9711	4032

Figure 2. Descriptive Summary of Accrued Returns

The top panel provides a descriptive summary of accrued returns across all 653 investors. The bottom panel summarizes the results for investors where the likelihood function (4.1) is successfully evaluated and the investors can be classified as prospect theory investors. Accrued returns are calculated according to Vlcek and Hens (2011) to obtain $\hat{R}_{S,t}$ and are reported as daily gross returns. The term *Obs.* denotes the number of observations in days in the dataset; *Mean* and *Median* denote the arithmetic mean and median of returns, respectively; *Std.* denotes the standard deviation of returns; and 5*p*, 25*p*, 75*p*, and 95*p* denote the fifth, 25th, 75th, and 95th percentiles of the returns, respectively.

	Mean	Std.	Median	5p	25p	75p	95p	Obs.
Accrued Returns for all 653 Investors								
Total	0.8967	0.8696	0.8343	0.0915	0.4290	1.1027	1.9816	17186660
Case 1	1.5515	1.1090	1.2558	1.0345	1.1086	1.6032	2.9528	3193124
Case 2	1.0592	0.6166	1.0163	1.0019	1.0074	1.0341	1.1200	617604
Case 3	0.9999	0.0001	0.9999	0.9996	0.9998	1.0000	1.0000	1210
Case 4	0.9758	0.0380	0.9862	0.9209	0.9731	0.9934	0.9983	538436
Case 5	0.5518	0.2944	0.5880	0.0556	0.2868	0.8213	0.9525	8430401
Accrued Returns for reduced dataset of 271 Investors								
Total	0.9243	0.8365	0.8791	0.1137	0.4308	1.1250	2.1240	5497408
Case 1	1.6238	1.2928	1.1122	1.0365	1.1224	1.6965	3.2610	1862136
Case 2	1.0480	1.0140	0.5695	1.0017	1.0067	1.0298	1.1105	161075
Case 3	0.9999	0.9999	0.0001	0.9996	0.9998	1.0000	1.0000	386
Case 4	0.9787	0.9877	0.0323	0.9304	0.9765	0.9940	0.9987	152638
Case 5	0.5435	0.5677	0.2898	0.0701	0.2763	0.8092	0.9509	3321173

FIGURE 3. Descriptive Summary of Estimated Market Parameters

The left panel captures the characteristics of the market parameters in this dataset for 653 investors. The panel on the right summarizes the results for individual investors where the likelihood function (4.1) is successfully evaluated and where the investors can be classified as prospect theory investors. Daily gross returns are calculated as $R_{t+1,t} = e^{\ln(P_{t+1}/P_t)} + 1$. The values for $R_{D,t}$ and $R_{U,t}$ are derived from μ_t and σ_t of the formation period at time t by $R_{U,t} = \mu_t^{\frac{1}{t}} + \sqrt{\frac{1-p_t}{p_t}\left((\mu_t^2+\sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}}\right)}$ and $R_{D,t} = \mu_t^{\frac{1}{t}} - \sqrt{\frac{p_t}{1-p_t}\left((\mu_t^2+\sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}}\right)}$, respectively. The probabilities are estimated by $p_t = \frac{t-j+1}{t} = \frac{n_U}{n_D+n_U}$, where n_D and n_U denote the numbers of down or up moves of the respective stock. The three-month Euribor retrieved from Thompson Reuters Datastream serves as a proxy for the risk-free return $R_{f,t}$. Accrued returns are calculated according to equation (3.1) to obtain $\hat{R}_{S,t}$; however, final realized returns $\hat{R}_{S,T}$ are taken directly from the trade records. All values are reported as daily gross returns and calculated using a look-back window of $l = 60$ observations or trading days. Here, *Mean* and *Median* denote the arithmetic mean and median of returns, respectively; *Std.* denotes the standard deviation of returns; and $5p$, $25p$, $75p$, and $95p$ denote the fifth, 25th, 75th, and 95th percentiles of the returns, respectively.

Dataset of all 653 investors

	Mean	Std.	Median	5p	25p	75p	95p
Total Dataset							
μ_t	0.9995	1.0054	1.0000	0.9909	0.9976	1.0019	1.0063
σ_t	0.0344	0.0345	0.0260	0.0103	0.0169	0.0409	0.0834
p_t	0.4718	0.1646	0.4678	0.1991	0.3740	0.5622	0.7707
$R_{U,t}$	1.0363	0.0681	1.0242	1.0092	1.0158	1.0388	1.0869
$R_{D,t}$	0.9577	0.0666	0.9726	0.8858	0.9521	0.9835	0.9912
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0002	1.0003
Where Indicator $I_{k,t} = 1$							
μ_t	0.9994	1.0052	0.9999	0.9912	0.9976	1.0018	1.0058
σ_t	0.0338	0.0332	0.0255	0.0103	0.0167	0.0400	0.0831
p_t	0.4727	0.1646	0.4687	0.1990	0.3750	0.5630	0.7719
$R_{U,t}$	1.0355	0.0640	1.0238	1.0091	1.0155	1.0380	1.0865
$R_{D,t}$	0.9584	0.0632	0.9731	0.8868	0.9531	0.9838	0.9913
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0001	1.0003
Where Indicator $I_{k,t} = 0$							
μ_t	0.9996	1.0061	1.0000	0.9901	0.9975	1.0022	1.0076
σ_t	0.0361	0.0379	0.0277	0.0106	0.0177	0.0434	0.0843
p_t	0.4693	0.1644	0.4651	0.1996	0.3709	0.5596	0.7674
$R_{U,t}$	1.0386	0.0789	1.0255	1.0095	1.0165	1.0410	1.0882
$R_{D,t}$	0.9554	0.0758	0.9708	0.8830	0.9494	0.9826	0.9911
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0002	1.0004

Reduced Dataset of 271 Investors

	Mean	Std.	Median	5p	25p	75p	95p
Total Dataset							
μ_t	0.9996	1.0047	1.0000	0.9923	0.9980	1.0018	1.0054
σ_t	0.0301	0.0292	0.0230	0.0099	0.0155	0.0356	0.0709
p_t	0.4767	0.1619	0.4720	0.2073	0.3814	0.5653	0.7711
$R_{U,t}$	1.0318	0.0520	1.0220	1.0088	1.0146	1.0344	1.0753
$R_{D,t}$	0.9640	0.0550	0.9761	0.9041	0.9591	0.9851	0.9917
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0001	1.0003
Where Indicator $I_{k,t} = 1$							
μ_t	0.9996	1.0046	1.0000	0.9923	0.9980	1.0018	1.0052
σ_t	0.0299	0.0285	0.0228	0.0099	0.0154	0.0353	0.0703
p_t	0.4774	0.1622	0.4725	0.2070	0.3821	0.5659	0.7732
$R_{U,t}$	1.0316	0.0526	1.0218	1.0088	1.0145	1.0342	1.0750
$R_{D,t}$	0.9643	0.0543	0.9762	0.9051	0.9593	0.9852	0.9917
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0001	1.0003
Where Indicator $I_{k,t} = 0$							
μ_t	0.9999	1.0052	1.0002	0.9921	0.9981	1.0021	1.0068
σ_t	0.0313	0.0325	0.0240	0.0099	0.0160	0.0371	0.0736
p_t	0.4731	0.1602	0.4695	0.2090	0.3778	0.5617	0.7605
$R_{U,t}$	1.0326	0.0485	1.0228	1.0089	1.0149	1.0359	1.0770
$R_{D,t}$	0.9623	0.0587	0.9752	0.8988	0.9576	0.9847	0.9918
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0002	1.0004

7. Presentation of the Estimation Results

Figure 4. Estimated Parameters for Prospect Theory

This table summarizes the results of the evaluation of the maximum likelihood function (4.1) and the results of a one-sided t-test of the presumption regarding the parameter set $\alpha < 1$, $\lambda > 1$, and $\gamma < 1$. The term $Var.$ represents the prospect theory parameter, $Case\ Type$ denotes the round-trip category as described in the text, and $Mean$ denotes the arithmetic mean of the estimates across all investors for which the likelihood function (4.1) is successfully evaluated. The results from Wald tests performed at the investor level are not reported. Case 3 has been omitted because no Case 3 round trips are observed.

Var.	Case Type	Mean of Estimates	Standard Error	p-value $\alpha, \gamma < 1$ $\lambda > 1$	Lower 95% Confidence Interval	Upper 95% Confidence Interval	Number of Obs.
α	Total	0.3738	0.0111	0.0000	0.3520	0.3956	271
	Case 1	0.4733	0.0168	0.0000	0.4402	0.5065	129
	Case 2	0.3511	0.0631	0.0000	0.1967	0.5056	7
	Case 4	0.3307	0.0458	0.0000	0.2223	0.4391	8
	Case 5	0.2733	0.0102	0.0000	0.2531	0.2935	127
λ	Total	1.0940	0.0080	0.0000	1.0782	1.1097	271
	Case 1	1.0497	0.0129	0.0003	1.0242	1.0752	129
	Case 2	1.0716	0.0378	0.0748	0.9792	1.1640	7
	Case 4	1.0719	0.0310	0.0407	0.9986	1.1452	8
	Case 5	1.1480	0.0082	0.0000	1.1319	1.1642	127
γ	Total	0.7242	0.0084	0.0000	0.7077	0.7407	271
	Case 1	0.7376	0.0148	0.0000	0.7084	0.7669	129
	Case 2	0.7117	0.0519	0.0007	0.5846	0.8387	7
	Case 4	0.6726	0.0514	0.0002	0.5511	0.7941	8
	Case 5	0.7246	0.0092	0.0000	0.7064	0.7429	127

Table 4 reveals that our estimates for a risk sensitivity parameter α of 0.3738, on average, with a median value of α near 0.357, tend to stay below 0.88 (one-sided t-test p-value < 0.001), the frequently cited estimates of Tversky and Kahneman (1992), which reflect the high curvature of the prospect value function, confirming the usual prior of diminishing risk sensitivity.[25] The p-values derived from the one-sided t-tests, which are appropriate for testing the presumption that the prospect value function displays significant curvature,[26] indicate that we can reject the hypothesis that our estimates for α are significantly larger than one at the 1% significance level.

Part of a possible explanation for the observed low estimates of α is probably rooted in the implications of the model of Vlcek and Hens (2011), particularly regarding Case 1 and Case 5 round trips. These implications could drive our results, since a large fraction of these round trips can be found in our dataset. In detail, it can be shown that, for a Case 1 round trip to occur, α is required to be significantly lower than unity to observe a sale. To see this, assume that $\hat{R}_{S,T}$ suffices for $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq \hat{R}_{S,t}R_{D,t} \geq 1$ and the no-arbitrage condition holds. Then

[25] Furthermore, our estimates for α display a moderate dispersion of 0.183 and seem to be slightly positively skewed (skewness 0.688).

[26] We refer to Train (2009) for the use of t-tests for the evaluation of the estimated parameters of the likelihood function and Harrison (2008) for its use in the context of prospect theory.

parameter values exist for α such that a stock sale is inconsistent with prospect theory.[27] We can quickly prove the statement by contradiction. Assume that a stock sale is consistent with prospect theory for all parameter values of α:

$$\omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^\alpha + \omega(1-p_t)(W_0\hat{R}_{S,t}R_{D,t} - W_0)^\alpha \\ - (W_0\hat{R}_{S,t}R_{f,t} - W_0)^\alpha \leq 0 \;\forall\; \alpha \in \mathbb{R}. \quad (7.1)$$

On the other hand, further inspection of the term on the left-hand side (denoted $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k)$) shows that

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) \geq \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^\alpha - (W_0\hat{R}_{S,t}R_{f,t} - W_0)^\alpha. \quad (7.2)$$

By the non-arbitrage condition, we have $0 \leq R_{D,t} < 1 \leq R_{f,t} < R_{U,t}$, which implies $x := W_0\hat{R}_{S,t}R_{U,t} - W_0 > W_0\hat{R}_{S,t}R_{f,t} - W_0 =: y$. In addition, we can find $m > 0$ such that $x = (1+m)y$. With this new notation, we can rewrite the previous inequality as

$$\begin{aligned} U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) &\geq \omega(p_t)x^\alpha - y^\alpha \\ &= \omega(p_t)(1+m)^\alpha y^\alpha - y^\alpha \\ &= y^\alpha(\omega(p_t)(1+m)^\alpha - 1). \end{aligned} \quad (7.3)$$

Since $(1+r)^\alpha \to \infty$ as $\alpha \to \infty$ and $y^a > 0 \;\forall\; \alpha \in \mathbb{R}$, we can find $\bar{\alpha} \in \mathbb{R}$ such that $y^\alpha(\omega(p_t)(1+m)^\alpha - 1)$ for all $\alpha > \bar{\alpha}$. That this implies that $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) > 0$ for all $\alpha > \bar{\alpha}$, which is inconsistent with a stock sale under prospect theory. A similar proof can be conducted for Case 5 round trips.

Diminishing risk sensitivity is generally in line with evidence from experimental studies fitting variants of prospect theory with a power function, where α normally falls in the range $0.5 \leq \alpha < 1$ (the properties of diminishing sensitivity due to variations were confirmed in the areas of gains and losses by Wakker and Deneffe (1996) and Fennema and van Assen (1999) and by Fox and Tversky (1998), respectively). Exemplarily, Tversky and Kahneman (1992) estimated the parameters of prospect theory conducting a controlled lottery questioning and elicited parameter values by applying nonlinear regression, concluding α to be close to 0.88. These results are predominantly confirmed; however, some studies, such as that of Fennema and van Assen (1999), provide mixed results, where the outcomes of the estimation depend on the method applied and range from $\alpha = 0.39$ ($\alpha = 0.39$) for gains to $\alpha = 0.84$ ($\alpha = 0.34$) for losses. Although some studies have found values of α as low as 0.22 (Loomes et al. (2002)) or slightly above (Camerer and Ho (1994), Wu and Gonzalez (1996), Gonzalez and Wu (1999a)), the majority of experimental studies point toward weak sensitivity, tantamount to high values for α.[28] Table 2 summarizes the findings.

However, in contrast to the majority of experimental studies, a comparison of our findings to the results derived from theoretical and empirical studies on decision making in financial markets seems to support the direction of α falling below 0.88 (we summarize the results from a selection of theoretical studies in Table 1). For the Finnish stock market, Kaustia (2004b) and Kaustia (2010) tested implications derived from prospect theory with empirical investor data using a probit model, concluding that prospect theory could cause the disposition effect only if

[27] Note that Vlcek and Hens (2011) discussed the cases where $\alpha = 0$ and $\gamma = 1$ for $\lambda \leq 1$, concluding that, in their model, an investor under prospect theory is prone to what they called the *ex post* disposition effect once a state similar to Case 4 occurs.

[28] The results from these and other studies are discussed by Stott (2006) and Booij et al. (2010).

α is sufficiently low. For given market parameters of expected return and volatility, the author found that sales are only compatible with prospect theory if α falls substantially below 0.7 or, alternatively, loss aversion does not exceed 1.6, while an investor, who realizes a gain around 7% matches with $\lambda \leq 1.2$ and $\alpha \geq 0.7$ (Kaustia (2004b), pp. 10–11, Kaustia (2010), p. 9). Barberis and Xiong (2009) argued that, as soon as α falls below 0.88, a trading pattern similar to the disposition effect can be observed more often; for an expected value of 10% and a volatility of 30%, α needs to decline sufficiently, particularly for the case at hand, to below 0.77.[29]

Table 1. Parameter Values and the Disposition Effect

The parameter listed for the studies cited are the boundaries mentioned regarding the occurrence of the disposition effect. The market parameters are in the order *upside return, downside return, risk-free return, and probability* . Whenever missing or not reported, the values for $R_{D,t}$ and $R_{U,t}$ are derived from μ_t and σ_t if mentioned in the study, by $R_{U,t} = \mu_t^{\frac{1}{t}} + \sqrt{\frac{1-p_t}{p_t}\left((\mu_t^2+\sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}}\right)}$ and $R_{D,t} = \mu_t^{\frac{1}{t}} - \sqrt{\frac{p_t}{1-p_t}\left((\mu_t^2+\sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}}\right)}$, respectively. The studies differ in their market parameters as well as their methodology and underlying model and the definition of the disposition effect. We calculate the required market values for Kaustia (2010), Roger (2009), and Henderson (2012) according to Barberis and Xiong (2009) for one period. Li and Yang (2009) match values for λ and α to the disposition measure of Dhar and Zhu (2006) and other market parameters, such as momentum, in an earlier version of their paper. Yao and Li (2013) match their estimates to the data points provided by Odean (1998). Neither study provides direct market parameters. The parameter values of Vlcek and Hens (2011) are a selection of the parameters mentioned in the study .

	Parameter Specifications					
Theoretical Study	Boundary Values		Market Values			
Kaustia (2004b)	$\alpha \leq 0.67$	$\lambda \leq 1.5$	1.334	0.85	1.038	0.5
Vlcek and Hens (2011)	$\alpha \leq 0.88$	$\lambda \leq 5$	1.330	0.770	1.10	0.5
Barberis and Xiong (2009)	$\alpha \leq 0.77$	$\lambda \leq 2.25$	1.16	0.89	1.00	0.5
Henderson (2012)	$\alpha \leq 0.50$	$\lambda \leq 2.2$	1.180	0.84	1.00	-
Li and Yang (2009)	$\alpha \leq 0.37$	$\lambda = 1.0$	-	-	1.038	0.5
Roger (2009)	$\alpha \leq 1.00$	$\lambda \leq 2.65$	1.255	0.854	1.006	0.5
Kaustia (2010)	$\alpha \leq 0.7$	$\lambda \leq 1.6$	1.62	0.62	1.016	-
Yao and Li (2013)	$\alpha \leq 0.74$	$\lambda \leq 1.61$	-	-	-	-

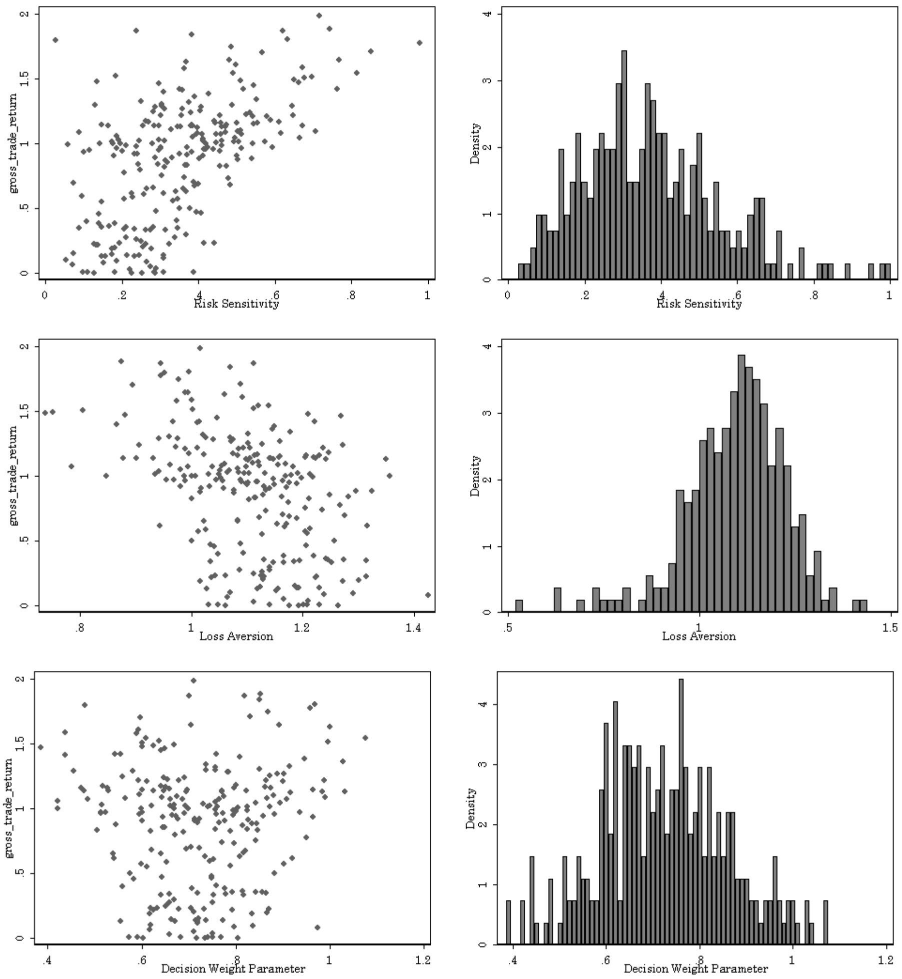
Our results for loss aversion λ indicate that loss aversion is not very prevalent in the trading behavior within our dataset.[30] According to Table 4, our estimates for loss aversion λ are characterized by a mean value of 1.094 and a median of 1.106,

[29]Barberis and Xiong (2009) argued that the investor does not gamble toward the edge of the concave region any longer and therefore decides to take smaller positions at the beginning. In the domain of losses, lower values of α lead to increased convexity and thus to increased positions in the risky asset after a loss (Barberis and Xiong (2009), p. 771). Applying a full-market model, Li and Yang (2009) highlighted that conclusions such as the inexplicability of the disposition effect through prospect value functions could be partly to blame for high expected values and the near-risk neutrality reflected in the mild concavity and convexity of the value functions for high values of α (Barberis and Xiong (2009), p. 769, Li and Yang (2009), p. 27).

[30]Recall that $\lambda > 1$ is commonly equivalent to *loss aversion* (e.g., Kahneman and Tversky (1979), Bowman et al. (1999), Neilson (2002), Koebberling and Wakker (2005)). Although Wakker and Tversky (1993) and Schmidt and Zank (2008) provide a framework for loss aversion under cumulative prospect theory, there is no agreement about what comprises loss aversion and how it can be implemented in a mathematical framework (Neilson (2002), Schmidt and Zank (2005), Koebberling and Wakker (2005), Booij et al. (2010)). Abdellaoui et al. (2007) compared several definitions that have been proposed in the literature and concluded that the definitions proposed

FIGURE 5. Distribution of Estimated Parameters

The panels on the left illustrate the dependence between gross trade returns and the risk sensitivity parameter α (upper left), the loss aversion parameter λ (middle left), and the decision weighting parameter γ (lower left). The panels on the right display the associated histograms for the risk sensitivity parameter α (upper right), the loss aversion parameter λ (middle right), and the decision weighting parameter γ (lower right).



thus varying around unity, with a tendency to be slightly above one, indicating only weak forms of loss aversion.[31] One-sided t-tests show that, across all round trips, λ is distinct from one, although for Case 2 and Case 4 round trips, due to the low number of observations, loss aversion parameter λ is statistically distinct from one at the 10% and 5% significance levels, respectively. However, for these round trips, λ is significantly smaller than the frequently cited values of 2.25 (p-value ≤ 0.001 for all Case 2 and Case 4 round trips) mentioned by Tversky and Kahneman (1992).

In light of these estimates, it is noteworthy that Vlcek and Hens (2011), concluded that, for $\lambda \leq 1$ and $\alpha = 0$, their prospect theory model favors occurrence of

by Kahneman and Tversky (1979) and Koebberling and Wakker (2005) were the most satisfactory in classifying most subjects according to their attitude toward losses.

[31]Our estimates for λ display a standard deviation of 0.132 with a negative skewness of -0.869.

the disposition effect.[32] Empirical evidence from financial studies is mixed, since our results for loss aversion seem to be confirmed by Dimmock and Kouwenberg (2010), who found λ to be lower for investors who invest in stocks, but contrasted by others, such as Hwang and Satchell (2011), who based their analysis on the asset allocation decisions of pension funds. According to them, one reason for our low values for λ could be driven by selection bias, since those investors whose $\lambda \geq 1$ tend to stay away from investing in stocks, since they prefer low proportions of stocks in their portfolio (e.g., Ang et al. (2004), Berkelaar et al. (2004), Gomes (2005), Polkovnichenko (2005), Barberis and Huang (2006), Dimmock and Kouwenberg (2010)). From a market-based view, however, Shumway (1997) investigated an equilibrium asset pricing model with prospect theory preferences, finding λ to be close to 3.11 and α near 0.758. The results were fitted to stock market returns and display a strong dependency concerning the evaluation period in question. Note that, for three-month returns, as in our case, λ was reported to be less than one, whereas for one-month returns, α is found to be 1.367, implying risk seeking in the domain of gains. Benartzi and Thaler (1995) explained the observed magnitude of equity premium through a loss aversion equal to 2.77.

Table 2. Parameter Values in the Laboratory

This table provides an overview of a representative selection of studies investigating particular parameter value characteristics. These studies differ in their methodology, reported mean or median, and presupposed functional form. Here, *CE* denotes the method is *certainty equivalent based* and *LE* indicates the *lottery equivalent method* .

	Parameter Estimates			
Elicitation Study	Method	Alpha	Gamma	
Tversky and Kahneman (1992)	*CE*	$\alpha = 0.88$	$\gamma^{+} = 0.61$	$\gamma^{-} = 0.69$
Camerer and Ho (1994)	*LE*	$\alpha = 0.37$	$\gamma^{+} = 0.56$	$\gamma^{-} = 0.56$
Tversky and Fox (1995)	*CE*	$\alpha = 0.88$	$\gamma^{+} = 0.69$	$\gamma^{-} = 0.69$
Wu and Gonzalez (1996)	*LE*	$\alpha = 0.50$	$\gamma^{+} = 0.71$	$\gamma^{-} = 0.71$
Birnbaum and Chavez (1997)	*LE*	$\alpha = 0.82$	-	-
Fennema and van Assen (1999)	*CE*	$\alpha = 0.39$	-	-
Gonzalez and Wu (1999a)	*CE*	$\alpha = 0.49$	$\gamma^{+} = 0.44$	$\gamma^{-} = 0.44$
Bleichrodt and Pinto (2000)	*CE*	$\alpha = 0.77$	$\gamma^{+} = 0.67$	$\gamma^{-} = 0.67$
Abdellaoui (2000)	*CE*	$\alpha = 0.89$	$\gamma^{+} = 0.60$	$\gamma^{-} = 0.70$
Kilka and Weber (2001)	*CE*	$\alpha = 0.88$	$\gamma^{+} = 0.49$	$\gamma^{-} = 0.42$
Etchart-Vincent (2004)	*CE*	$\alpha = 0.97$	-	$\gamma^{-} = 0.87$
Abdellaoui et al. (2005)	*CE*	$\alpha = 0.91$	$\gamma^{+} = 0.83$	$\gamma^{-} = 0.83$
Stott (2006)	*LE*	$\alpha = 0.19$	$\gamma^{+} = 0.96$	-
Abdellaoui et al. (2007)	*LE*	$\alpha = 0.73$	-	-

[32]For given risk sensitivity and market parameters, Barberis and Xiong (2009) presented evidence that the disposition effect is less likely to hold as soon as loss aversion disappears. They offered a rationale whereby individual investors take more aggressive positions in the risky asset to begin with and cut back the position to prevent their wealth dipping into losses if asset values decline. Henderson (2012), p. 20 drew a similar conclusion for loss aversion around 2.25 with the probability of selling at a gain being close to unity, although in the absence of loss aversion the probability is still high.

Our estimates for the decision weight indicate that γ takes values below one (we find a mean value for γ of 0.724 and a median of 0.719).[33] One-sided t-tests show that, for all round trips, γ is larger than 0.65, as estimated by Tversky and Kahneman (1992) with a p-value < 0.001, but significantly lower than one (p-value < 0.001).

Above, we provided an argument that particular values in some parameter values could be due to the way Vlcek and Hens (2011) constructed their model. This argumentation can also be applied here with respect to the interplay between α and γ. Assume $\hat{R}_{S,t}$ suffices for $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq \hat{R}_{S,t}R_{D,t} \geq 1$, the no-arbitrage condition holds, and, in particular, the expected stock return exceeds the risk-free return such that $\mu_t > R_{f,t}$. Then parameter combinations of α and γ exist where both are smaller than unity such that a stock sale is inconsistent with prospect theory. As in the case for α, we can prove this statement quickly by contradiction. Assume that a stock sale is consistent with prospect theory for all parameter values of α and γ:

$$\begin{aligned}\omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^\alpha + \omega(1-p_t)(W_0\hat{R}_{S,t}R_{D,t} - W_0)^\alpha \\ - (W_0\hat{R}_{S,t}R_{f,t} - W_0)^\alpha \leq 0 \;\forall\; \alpha \in \mathbb{R}. \quad (7.4)\end{aligned}$$

For simplicity, let us again denote the left-hand term by $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k)$. Using $\lim_{\gamma\to 1}\omega(p_t) \to p_t$, by the continuity of $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k)$ in α and γ, we obtain

$$\begin{aligned}\lim_{\alpha,\gamma\to 1} U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k) &= p_t(W_0\hat{R}_{S,t}R_{U,t} - W_0) + (1-p_t)(W_0\hat{R}_{S,t}R_{D,t} - W_0) \\ &\quad - (W_0\hat{R}_{S,t}R_{f,t} - W_0) \\ &= W_0\hat{R}_{S,t}(p_tR_{U,t} + (1-p_t)R_{D,t} - R_{f,t}) > 0,\end{aligned} \quad (7.5)$$

where the last inequality uses the assumption that the expected stock return $\mu_t = p_tR_{U,t} + (1-p_t)R_{D,t}$ exceeds the risk-free return $R_{f,t}$. As a result of the intermediate value theorem, by the continuity of $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k)$ in α and γ, values for $\bar{\alpha} \leq 1$ and $\bar{\gamma} \leq 1$ exist such that $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\boldsymbol{\theta}_k)$, where for $(\boldsymbol{\theta}_k = \{\bar{\alpha}, \bar{\gamma}\})$ and where $\bar{\alpha}, \bar{\gamma} \geq 0$, which is inconsistent with a stock sale under prospect theory. Similarly, for round trips that satisfy the conditions for Case 5 round trips, a similar proof can be conducted.

To illustrate the economic effects of a change in the parameter estimates with regard to round-trip length (in days) and realized returns, we supplement our previous results with a comparative static analysis of the simulated trading behavior of a prospect theory investor over a time span of $1,260$ trading days, covering roughly five years, and given the average market parameters in our dataset, taken from Table 3. In particular, the effect of a change in the respective prospect theory parameter can be obtained from the difference in round-trip length and realized returns between the simulated trade history using the original parameter estimates mentioned by Tversky and Kahneman (1992) and those of these simulated trades, where, ceteris paribus, the parameter of interest is replaced by one of our estimates. For the original values according to Tversky and Kahneman (1992), where $\alpha = 0.88$, $\lambda = -2.25$, and $\gamma = 0.65$, the simulated trade sequences are characterized by a round-trip length of 115.62 days and a realized return of 1.065, or 6.496%, on

[33] For our estimates of γ, we find a standard deviation of 0.138 with a positive skewness of 0.089.

average, per year.[34]

If α is reduced to 0.3738, the average round-trip length is reduced to 89.47 days and realized returns change to 1.0458, on average, which is statistically distinct from the average number of days and realized returns according to Welch's t-test (p-value < 0.001). Recall that parameter α governs investor prospect value sensitivity to variations in gains or losses and determines the curvature of the prospect value function. A reduction in α is equivalent to an increase in concavity (or convexity in the domain of losses) of the prospect value function and equipollent prospects near the reference point are of greater utility for the investor compared to large prospects, which are located in the flat part of the prospect value function. According to this increased concavity in the region of moderate gains, in association with outcomes that are linked to the accrued return, small deviations from the purchase price, for example, moderate returns, bestow the investor with a stronger increase in prospect value, whereas changes given large deviations do not contribute much to the investor's prospect value anymore (see also Tversky and Kahneman (1992), p. 303), which does not favor holding onto a risky asset once gains or losses fall in the proximity of the flat section of the prospect value function.

A change in the loss aversion parameter from -2.25 to -1.094 yields an average trade duration of 129.04 days and a realized return of 1.104, where the difference in the round-trip length is significant at the 1% level (p-value of Welch's t-test is 0.008) and the difference in realized returns is only significant at the 10% level (p-value of Welch's t-test is 0.059). A first inspection of the results reveals that the realized returns are decreasing in the loss aversion parameter λ. We offer the economic rationale that the accrued returns serve as a cushion against possibly unfavorable prospects, since they absorb the impact of potential losses given the bad state occurs in the subsequent period. This protection against this probable downside risk-driven cutback in prospect value increases as the realizable return rises, working in favor of holding the stock and, given a positive drift, earning a higher return. Finally, an increase of the decision weighting parameter γ from 0.65 to 0.724 results in a moderate shortening of the round-trip length to 112.04 days and a reduction of the realized return of 1.062 per year; however, both values are not significantly distinct from the case where $\gamma = 0.65$ (the p-values of Welch's t-test are 0.190 for the round-trip duration and 0.391 for the differences in realized returns).

With respect to the overall picture of prospect theory, the interdependence between α, λ, and γ has been discussed by Vlcek and Hens (2011) but is also debated in theoretical studies (e.g., Kaustia (2004b), Kaustia (2010), Polkovnichenko (2005), Dacey and Zielonka (2008), Barberis and Xiong (2009), Li and Yang (2009). In addition, from an econometric point of view, significant correlation among the estimators may point toward multicollinearity issues, which affects the quality of our estimators, since the unbiasedness of estimators only holds asymptotically (Gonzalez and Wu (1999b)). Thus, the correlation structure across our dataset is of some interest. Given our data, we find a weak but statistically significant positive

[34]The standard deviation of the round-trip length is 39.44 days (the minimum duration was one day and the maximum round-trip length in our simulation was 182 days) and the standard deviation of the realized returns is 0.2515, with a minimum of 0.959 (or -4.124%, respectively) and a maximum of 1.554, corresponding to a net return of 55.485%.

correlation between α and γ and between λ and γ, but a negative correlation between λ and α.[35] A statistically significant negative relation seems to exist between α and λ, which is not only in line with the work of Vlcek and Hens (2011), but also in accordance with other theoretical studies (Kaustia (2004b), Kaustia (2010), Dacey and Zielonka (2008), Barberis and Xiong (2009)).[36] Our simulated trade sequences also show this effect as a reduction in loss aversion λ and risk sensitivity α is non-monotonic with respect to realized returns and trade duration. One possible interpretations is that, for the high degrees of risk sensitivity we detected, α already reflects an aversion against risky outcomes. Consequently, other risk preference parameters such as the loss aversion λ are moderate, since α captures most of the effect. Regarding the reliability of the prospect theory parameters $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$), the low correlation we detect between the parameters also reflects a low level of multicollinearity, measured in terms of the off-diagonal elements of the inverse Hessian matrix $\boldsymbol{H}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$. This is also reflected in the low standard errors of our estimates $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$, since the inverse of the Hessian matrix serves as the (asymptotic) covariance matrix of the estimates (Cramer (1986)).[37]

8. Sensitivity Analysis of the Prospect Theory Parameters

Our results were derived given the reference point specification of Vlcek and Hens (2011); however, the sensitivity of our estimators toward the reference point, one of the essential ingredients of prospect theory (Kahneman and Tversky (1981), Kahneman and Tversky (2000)), could be of interest, since Kahneman and Tversky (1979) failed to specify where the reference point should be located. In the context of prospect theory and its relevance to the disposition effect, Vlcek and Hens (2011) assumed that the initial wealth serves as a fixed reference point, a view similar to that of studies on the disposition effect (e.g., Weber and Camerer (1998)). Despite the intuitive appeal of using the level of initial wealth, other studies chose a different definition of what constitutes a loss (for previous and initial stock prices, see Weber and Camerer (1998); for historical high prices, see Odean (1998), Garvey and Murphy (2004), Jordan and Diltz (2004), Lehenkari and Perttunen (2004), Gneezy (2005); for wealth times the risk-free return, see Dhar and Zhu (2006), Frazzini (2006), Barberis and Xiong (2009)). Given the possibility of

[35]The correlation between α and γ is 0.1333 (p-value 0.0282) and the correlation between λ and γ is 0.1752 (p-value 0.0038).

[36]The correlation between α and λ is significantly negative, at -0.4185 (p-value< 0.001). A notable exception is the study by Li and Yang (2009), where loss aversion λ does not seem to have an impact on the magnitude of the disposition effect. The authors did not provide specific ranges for possible parameters but discussed the effects of a decrease in risk sensitivity on the interaction between stock market momentum and the disposition effect. As long as α does not exceed a critical value, an increasing risk sensitivity increases the sales of winner stocks for given return specifications. Once the benchmark value is undershot, fewer winners are sold, albeit, for losing stocks, a similar turnaround was not detected. The authors traced this paradox back to the influence of α on momentum, which leads to an increase in the attraction of holding onto winning stock, thus counterbalancing the direct effect α has on the disposition effect.

[37]Note that the confidence interval boundaries from the per-investor estimation determines the boundaries of the confidence intervals of our t-tests. Multicollinearity is expressed in high standard errors, as inferred from the inverse Hessian matrix $\boldsymbol{H}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$, yielding wide confidence intervals due to the flat surface of the likelihood function. Since the width of the confidence intervals of the t-test cannot be smaller than the confidence interval derived from the maximum likelihood estimation, multicollinearity should be reflected in our t-tests. Regarding the validity of our parameter estimates, in a preceding simulation study, we find that, for prospect theory utility models, the estimated parameters for loss aversion and risk sensitivity and the parameter for the decision weighting function do not diverge significantly from the input parameter settings (Jakusch (2013)).

individual investors adapting their reference point to their expectations or recent gains or losses (Andreassen (1988), Arkes et al. (2008), Meng (2010), Ingersoll and Jin (2012)), prospect theory appears to be reconciled with empirical trading patterns. In contrast to Vlcek and Hens (2011), who assumed the reference point to be fixed at the initial wealth W_0, Meng (2010) suggested that the reference point is subject to a dynamic adaption process and could be equal to the expected wealth (see also Chen and Rao (2002), Arkes et al. (2008), Arkes et al. (2010)). However, recall that the decision model of Vlcek and Hens (2011) is insensitive to assumptions regarding the initial wealth W_0 invested, since W_0 can be truncated on both sides of the inequality,. Consequently, since this characteristic of their decision model remains intact for any assumptions regarding the reference point, we do not expect significant changes in our estimates if we modify equation (4.1), where we replace the initial wealth W_0 by its expected value $W_0\mu_t$ and rerun the evaluation of equation (4.1) to reestimate $\boldsymbol{\theta_k}$.

Since the definition of a round trip appears to be a crucial ingredient for our likelihood function, a sensitivity analysis of the impacts of an alternative assumption regarding the underlying accounting principle is necessary to evaluate the robustness of our parameter estimates $\boldsymbol{\theta_k}$ with respect to round-trip length. Shifting from the previously applied FIFO principle to a *last-in-first-out (LIFO)* principle significantly shortens the round-trip length, particularly since the majority (about $77,6\%$) of the purchase and sales orders constitute complex trades where the individual investors in our dataset ramp up an initial position in a stock over time until it is finally sold off. A reestimation of $\boldsymbol{\theta_k}$ for round trips under the LIFO accounting principle shows that the mean of α is now 0.375, the mean of the loss aversion parameter λ is around 1.149, and the mean of γ is near 0.974 across all round trips.[38] These parameter estimates are statistically distinct from unity (p-value < 0.001 for all $\boldsymbol{\theta_k}$) across all trades, although for some cases (particularly for Cases 2 and 4), the decision weighting parameter γ is not statistically distinct from unity (see Table 6).[39]

The results are presented in Table 6. Since the accounting principle changes, we obtain a different distribution of round trips in terms of our classifications for Case 1 to Case 5 trades. Nevertheless, the risk sensitivity α appears to be robust to changes of the accounting principle. Since the same individual investor is reestimated under the LIFO principle, a paired t-test shows that, across all round trips, the difference between both α is not significant (p-value 0.4639). Concerning loss aversion λ, however, the difference in the base case estimates appears to be substantial (p-value < 0.001). A shift in accounting principles from FIFO to LIFO also affects γ, which is now significantly increased in comparison to our results if the reference point is assumed to be equal to the initial wealth (p-value < 0.001). With regard to the correlations between the various parameters, the correlation structure does not seem to be very affected: The correlations between α and γ and between λ and γ are still positive and significantly different from zero at the 1% significance level. A significantly negative relation seems to be prevalent between α and λ, which is in line with the relevant theoretical literature (see Kaustia (2004b), Kaustia (2010),

[38] For the risk sensitivity parameter α, we find a median value of 0.352, a standard deviation of 0.193, and a skewness of 0.812. Regarding the loss aversion parameter λ, we find a median value of 1.156, a standard deviation of 0.113, and a skewness of -0.612. The parameter γ displays a median of 0.979, a standard deviation of 0.102, and a skewness of -0.118.

[39] Note that a similar argumentation applies regarding lump-sum trading costs C for purchases and sales, since proportional trading cost factors c can be truncated from $\Delta_t(U_k|\boldsymbol{\theta_k})$ if based on the respective realized gain or loss.

Figure 6. Estimated Parameters for *LIFO*

This table summarizes the result of the evaluation of the maximum likelihood function (4.1) and the results of a one-sided t-test of the presumption regarding the parameter set $\alpha < 1$, $\lambda > 1$, and $\gamma < 1$ under the LIFO principle. The term $Var.$ indicates a prospect theory parameter, *Case Type* denotes the round-trip category as described in the text, and *Mean* denotes the arithmetic mean of the estimates across all investors for which the likelihood function (4.1) is successfully evaluated. The results from Wald tests performed at the investor level are not reported. Case 3 has been omitted because no Case 3 round trips are observed.

Var.	Case Type	Mean of Estimates	Standard Error	p-value $\alpha, \gamma < 1$ $\lambda > 1$	Lower 95% Confidence Interval	Upper 95% Confidence Interval	Number of Obs.
α	Total	0.3751	0.0117	0.0000	0.3520	0.3982	271
	Case 1	0.4554	0.0190	0.0000	0.4177	0.4931	107
	Case 2	0.4436	0.0362	0.0000	0.3629	0.5243	11
	Case 4	0.3943	0.0334	0.0000	0.3227	0.4659	15
	Case 5	0.2774	0.0114	0.0000	0.2548	0.3000	138
λ	Total	1.1492	0.0068	0.0000	1.1357	1.1627	271
	Case 1	1.1327	0.0118	0.0000	1.1094	1.1561	107
	Case 2	1.1909	0.0287	0.0000	1.1269	1.2549	11
	Case 4	1.1603	0.0197	0.0000	1.1180	1.2026	15
	Case 5	1.1566	0.0089	0.0000	1.1390	1.1742	138
γ	Total	0.9752	0.0063	0.0000	0.9629	0.9876	271
	Case 1	0.9745	0.0099	0.0057	0.9549	0.9941	107
	Case 2	1.0165	0.0288	0.7106	0.9524	1.0806	11
	Case 4	0.9838	0.0202	0.2175	0.9405	1.0271	15
	Case 5	0.9668	0.0089	0.0001	0.9492	0.9844	138

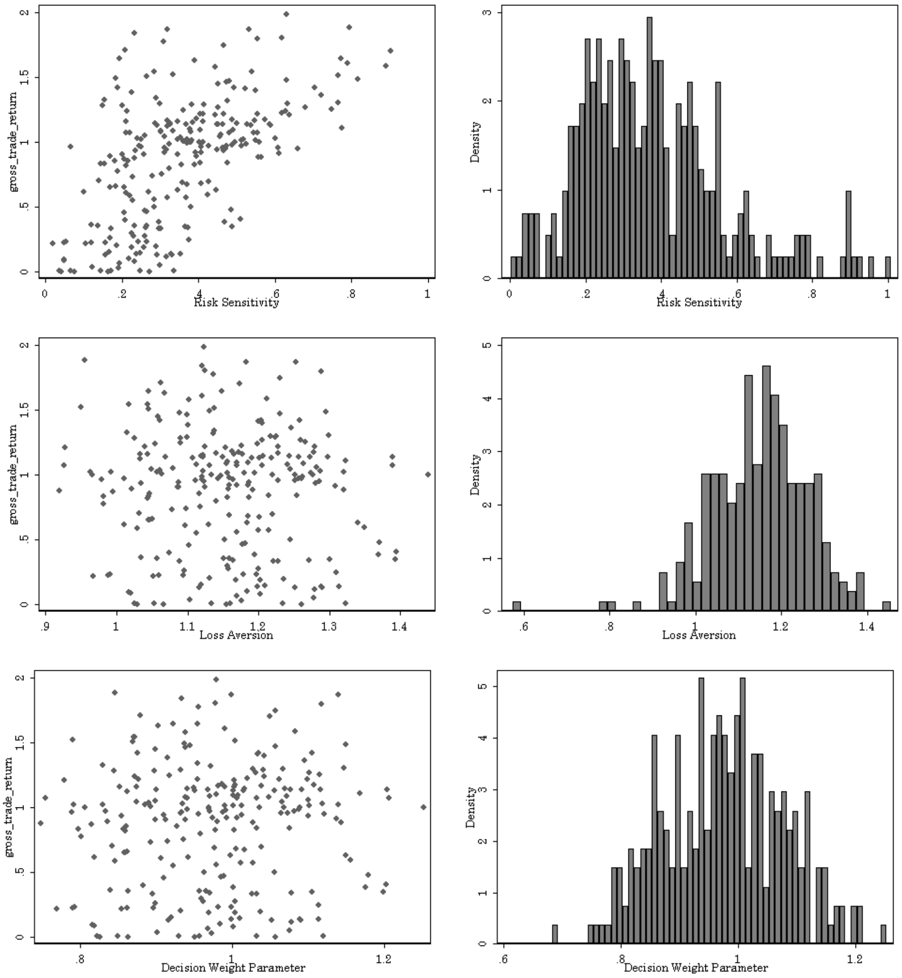
Vlcek and Hens (2011), Barberis and Xiong (2009) on trading; Polkovnichenko (2005), Dacey and Zielonka (2008), Roger (2009)).[40]

Although the maximum likelihood approach we adopt is the state of the art in experimental economics since Hey and Orme (1994) (e.g., Carbone and Hey (1994), Orme (1995), Hey (1995), Hey and Carbone (1995), Carbone and Hey (1995), Loomes and Sugden (1995), Carbone (1997), Carbone and Hey (2000), Loomes et al. (2002), Stott (2006); for an overview, see Harrison and Rutstrom (2008), de Palma et al. (2008)), several shortcomings in the evaluation of the likelihood function (4.1) can affect our prospect theory estimates (Cramer (1986), Liu and Mahmassani (2000), Rabe-Hersketh and Everitt (2004), Gould et al. (2006)) if these shortcomings are correlated with $\boldsymbol{\theta_k}$. To see whether our estimates change if we apply a different estimation method, we adopt an alternative calibration approach in which we minimize the (normalized) squared difference of the prospect values $\Delta_t(U_k|\boldsymbol{\theta_k})$. According to the Weierstrass theorem, a solution can be obtained for a continuous spectrum of $\boldsymbol{\theta_k}$ and by an auxiliary definition of border values. The objective function in our case is continuous in $\boldsymbol{\theta_k}$ and constrained such that a solution for the optimal vector of $\boldsymbol{\theta_k}$ can be found. Minimization with respect to $\boldsymbol{\theta_k}$ is performed in Stata, using the `optimize` command embedded in Stata's matrix calculation environment Mata. Standard errors at the investor level are derived

[40]The correlation between α and γ is 0.5264 (p-value < 0.001) and the correlation between λ and γ is 0.2349 (p-value < 0.001), whereas the correlation between α and λ is significantly negative, -0.2608 (p-value < 0.001).

Figure 7. Distribution of Estimated Parameters for *LIFO*

The panels on the left illustrate the dependence between gross trade returns and the risk sensitivity parameter α (upper left), the loss aversion parameter λ (middle left), and the decision weighting parameter γ (lower left). The panels on the right display the associated histogram for the risk sensitivity parameter α (upper right), the loss aversion parameter λ (middle right), and the decision weighting parameter γ (lower right).



from the inverse of the Hessian matrix of the objective function (for details on nonlinear least squares methods, see, e.g., Bard (1974), Seber and Wild (1989), Wooldridge (2010), Chap. 12). For the numerical search algorithm, we specify the Newton–Ralphson algorithm as the search method, since our pretests revealed that the Newton–Ralphson algorithm seems to converge more reliably when minimizing the squared difference to find $\boldsymbol{\theta_k}$. However, by minimizing the squared difference, the outliers achieve greater weight compared to minimization of the absolute difference according to Vlcek and Wang (2007).

A comparison between the results in Table 8 with our estimates in Table 4 shows that both parameter estimates reveal a certain similarity. According to paired t-tests, the correct test in this case, since the trade history of the same individual investor is evaluated by two different methods, we find, for the risk sensitivity parameter α under the nonlinear least squares method, that their difference is not

FIGURE 8. Estimated Parameters (Nonlinear Least Squares)

This table summarizes the results of the nonlinear least squares estimation and of a one-sided t-test of the presumption regarding the parameter set $\alpha < 1$, $\lambda > 1$, and $\gamma < 1$. The term *Var.* represent the prospect theory parameter, *Case Type* denotes the round-trip category as described in the text, and *Mean* denotes the arithmetic mean of the estimates across all investors for which the likelihood function (4.1) is successfully evaluated. The results from Wald tests performed at the investor level are not reported. Case 3 has been omitted because no Case 3 round trips are observed.

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Var.	Case Type	Mean of Estimates	Standard Error	p-value $\alpha, \gamma < 1$ $\lambda > 1$	Lower 95% Confidence Interval	Upper 95% Confidence Interval	Number of Obs.
α	Total	0.3408	0.0132	0.0000	0.3149	0.3667	271
	Case 1	0.4466	0.0205	0.0000	0.4061	0.4870	129
	Case 2	0.4077	0.0402	0.0000	0.3094	0.5060	7
	Case 4	0.2671	0.0522	0.0000	0.1436	0.3905	8
	Case 5	0.2282	0.0122	0.0000	0.2042	0.2523	127
λ	Total	1.0564	0.0077	0.0000	1.0412	1.0716	271
	Case 1	1.0283	0.0117	0.0084	1.0052	1.0514	129
	Case 2	1.1071	0.0300	0.0059	1.0338	1.1805	7
	Case 4	1.0544	0.0169	0.0073	1.0145	1.0943	8
	Case 5	1.0959	0.0096	0.0000	1.0770	1.1149	127
γ	Total	0.7169	0.0090	0.0000	0.6992	0.7346	271
	Case 1	0.7085	0.0148	0.0000	0.6792	0.7378	129
	Case 2	0.7594	0.0350	0.0002	0.6738	0.8450	7
	Case 4	0.6411	0.0499	0.0001	0.5231	0.7591	8
	Case 5	0.7269	0.0112	0.0000	0.7047	0.7491	127

significant (p-value 0.5780) and similarly for the loss aversion parameter λ (p-value 0.2180) and the decision weighting parameter γ (p-value 0.5673). The correlation structure also seems to be preserved and is similar to the maximum likelihood estimators. We still detect a positive correlation between α and γ, as well as between λ and γ, and negative correlation between α and λ.[41] We suspect the remarkable similarity between our maximum likelihood estimators and those estimators obtained from the nonlinear least squares method to be systematic. Recall that the maximum likelihood estimation was performed using normally distributed error terms ϵk. Given this structure, the nonlinear least squares method and the maximum likelihood approach converge, as shown by Seber and Wild (1989).

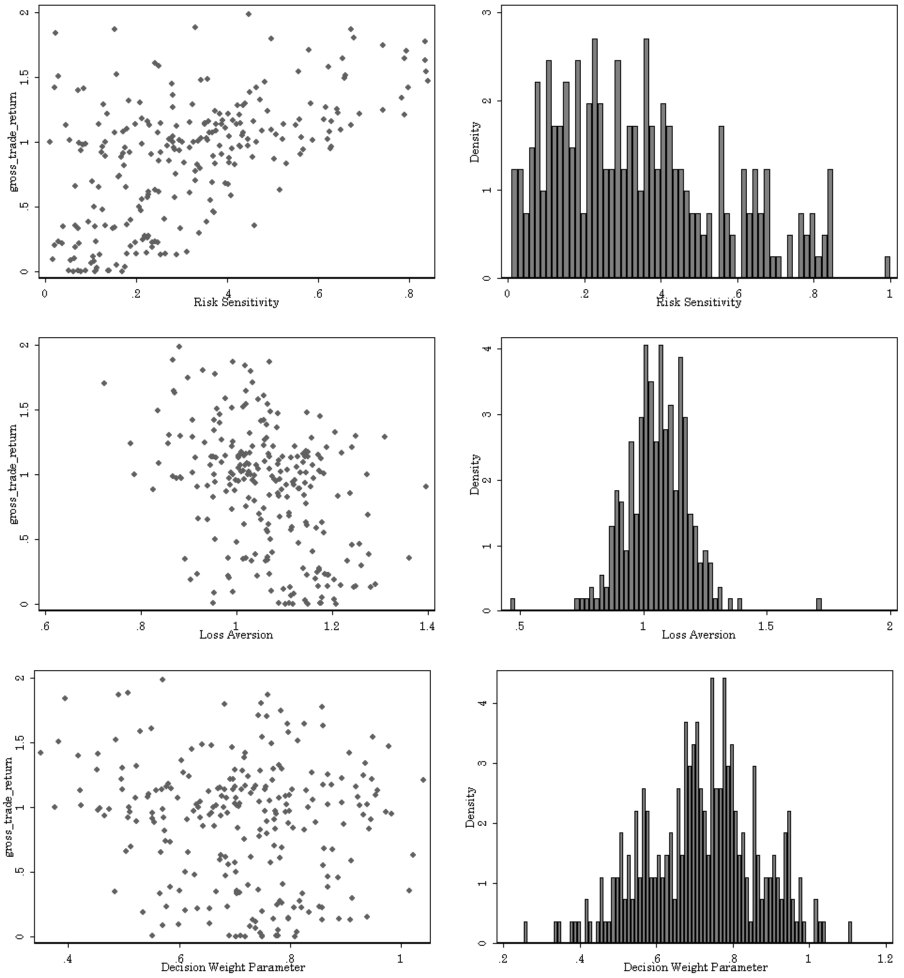
9. Discussion and Summary

Hitherto, following a brief review of the application of prospect theory in finance in general and trading models in particular, we selected the model of Vlcek and Hens (2011) due to its similarity to the difference-in-utility approach proposed by Currim and Sarin (1989). Since their model was constructed for a rather theoretical environment, we needed to extend their framework to capture the features of our dataset to address the question of which prospect theory parameters comply with

[41] In detail, the correlation between α and γ is 0.6974 (p-value < 0.001) and that between λ and γ is 0.2969 (p-value < 0.001), while the correlation between α and λ is negative, at -0.1401 (p-value 0.0211).

FIGURE 9. Distribution of Estimated Parameters (Nonlinear Least Squares)

The panels on the left illustrate the dependence between gross trade returns and the risk sensitivity parameter α (upper left), the loss aversion parameter λ (middle left), and the decision weighting parameter γ (lower left). The panels on the right display the histograms for the risk sensitivity parameter α (upper right), the loss aversion parameter λ (middle right), and the decision weighting parameter γ (lower right).



observed trading behavior. Given a dataset of trading data of individual investors from a large German discount brokerage firm, we estimated the prospect theory parameters, discussed its implications and limitations with regard to the outcomes of our estimation, and compared them to the results of related studies.

Models such as that of Vlcek and Hens (2011) illustrate the decision process as a myopic optimization problem, which implicitly results in an underestimation of the value of waiting (Henderson (2012)). If the disposition effect is modeled as a result of sequential decision making instead (Zuchel (2001)), models that apply intertemporal optimization, as those of Kyle et al. (2006) and Henderson (2012), address this feature more adequately. These models have been recently elaborated by Nielssen and Jaffray (2004), Barberis and Xiong (2009), and Ebert and Strack (2012), among others. Moreover, given the full spectrum of prospect theory, parameters can lead to more subtle explanations for the interdependence between prospect theory and trading patterns such as the disposition effect (Barberis (2012)).

Another aspect of the model of Vlcek and Hens (2011) we did not address in this paper is whether the mathematical specification of prospect theory, which the authors used in their model, is the one that provides the best fit to our data. Although Kahneman and Tversky (1979) provided mathematical reasons for the power function used in prospect theory, whether this functional form fits in finance is not without debate. According to Vlcek and Hens (2011), it appears to be difficult to reconcile prospect theory under a power function with a trading pattern such as the disposition effect. A number of recent studies in finance have challenged the idea of a power functional and its ability to capture individual investors' trading behavior. As an example, Rieger and Wang (2008) refined prospect theory for application in continuous-outcome environments, since it is common to model financial markets and assets using stochastic calculus. DeGiorgi and Hens (2006) discussed the idea of a piecewise negative exponential value function (DeGeorgi et al. (2004)) to capture trading patterns such as the disposition effect. They argued that, given the power function as used by Vlcek and Hens (2011), investors would not choose to invest in risky assets at the beginning; however, under a piecewise negative exponential value function, the optimal solution can generate a trading pattern similar to the disposition effect. DeGiorgi and Hens (2006) mentioned that, under an exponential form instead of a power form of the prospect value function, the problem of whether the asset is held ex ante can be solved, due to the fact that a negative exponential displays greater curvature at the edges of the return distribution. It should be noted that, as soon as non-negative skewness is present in the return distribution of the stock, where an increase in the stock value shifts the position into the domain of large gains and a decline makes a relatively small dent in the investor's wealth position, the mild curvature of the S-shaped prospect function given a power functional is sufficient to give rise to the disposition effect. In that case, the investor needs a larger stock position after a gain compared to the position after a loss to gamble to the edge of the respective part of his or her prospect value function and the disposition effect may hold for the given market parameters (Barberis and Xiong (2009), Li and Yang (2009)).

The difficulty Vlcek and Hens (2011) had in explaining the disposition effect is also related to the market parameters we observed, particularly the low expected returns μ_t from the risky assets (recall our results in Table 3). Kaustia (2010) noted that low expected values yield inconsistencies if the investor considers whether the asset should be held ex ante, a point that has been notes in recent literature (Kaustia (2004b), Barberis and Xiong (2009)), whereas Kyle et al. (2006) emphasize that this inconsistency does not arise with the piecewise negative exponential value function. However, Henderson (2012) demonstrated that, under S-shaped preferences, the risky stock can display low Sharpe ratios, which are equivalent to relatively poor expected returns, and still be held ex ante if the individual investor gambles on the possibility of liquidating at a small gain. This is a surprising implication, since Vlcek and Hens (2011) remarked that prospect theory cannot completely account for the disposition effect if the investor takes into account the decision to buy the stock ex ante.

Another critical assumption of the study of Vlcek and Hens (2011) that we relaxed is the assumption of stable market parameters. For our estimated parameters, according to Vlcek and Hens (2011), the ex ante disposition effect cannot be explained by prospect theory due to inconsistencies in the optimal solutions across time. However, if the drift parameter μ_t or volatility changes over time, this result could be disputable, particularly if the stock market were more favorable at the beginning of the round trip. We captured this effect with our models of the upside and downside returns; however, the implications regarding the disposition effect are left to future research.

Despite the work of DeGeorgi et al. (2004), DeGiorgi and Hens (2006), Kyle et al. (2006), and Rieger and Wang (2008), prospect theory with fixed reference points and a power functional is still the most common functional form in financial studies, backed by recent studies that deal with the best-fitting shape (Wakker (2008)). For instance, Blondel (2002) fitted linear, power, and exponential functions to experimental data and found strong evidence in favor for the power and exponential functions, concluding that these forms provided a better fit to the author's data than linear functions did. Furthermore, the author noted that power functions fit slightly better than exponential ones. Stott (2006) examined the best fit for power and exponential functions and found that quadratic and linear specifications fit the worst. Stott (2006) found (cumulative) prospect theory to be most predictive if the power value function is combined with the probability weighting function of Prelec (1998) when using a logit stochastic process. A further comparison between the power and exponential functional forms shows, in line with Blondel (2002), that power specifications fit even better to experimental data.[42] Other experimental studies (e.g., Lattimore et al. (1992), Hey and Orme (1994), Abdellaoui (2000)) assess parametric forms at the level of individual subjects. From the perspective of experimental studies, the results are most consistent with an inverse S-shaped probability weighting function (Wu and Gonzalez (1996), Wu and Gonzalez (1999), Abdellaoui (2000), Bleichrodt and Pinto (2000), Abdellaoui et al. (2005)). However, to the best of our knowledge, nobody has tried to test a piecewise negative exponential function yet.

[42]Levy and Levy (2002), however, challenged the idea of the S-shaped prospect theory value function, since their data support Markowitz's hypothesis of an inverse S-shape. They used a stochastic dominance approach to conclude that investors are not generally risk loving over losses but are more likely to exhibit risk aversion in both the gain and loss domains. In contrast, Wakker (2003) showed that Levy and Levy's mistake was to neglect the probability weighting function. Once it is incorporated into their analysis, their data support prospect theory.

Appendix A. Remarks on the Maximum Likelihood Approach

As elaborated, experimental studies maximize the overall likelihood of an investor or decision maker, given the assumption of stochastically independent error terms yielding the likelihood function for a utility model of type k, expressed as

$$\log L(\Delta_t(U_k|\boldsymbol{\theta_k})) = \sum_{t\in T}\sum_{I\in I_{k,t}} I_{k,t}\log p_{I_{k,t}}(\Delta_t(U_k|\boldsymbol{\theta_k})),$$

in which it is required that $\Delta_t(U_k|\boldsymbol{\theta_k})$ is a one-to-one relationship connecting the functional values to particular values of $\boldsymbol{\theta}_k$ and where $p_{I_{k,t}}(\Delta_t(U_k|\boldsymbol{\theta_k}))$ denotes the respective conditional probabilities. To clarify notation and provided there exists a unique solution to the maximizing problem within the possible range of $\boldsymbol{\theta}_k$, maximizing the likelihood function (A.1) for a given sample and time periods $t \in \{1,\ldots,T\}$ returns a maximum likelihood estimate $\hat{\boldsymbol{\theta}}_{k|n,t}$, depending on the sample size, of the true but unknown parameter $\hat{\boldsymbol{\theta}}_k$, briefly denoted as

$$\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}} = \arg\max_{\theta_k\in\boldsymbol{\theta_k}} \log L(\Delta_t(U_k|\boldsymbol{\theta_k})). \tag{A.1}$$

Accordingly, the obtained estimator $\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}$ is characterized by the usual standard conditions concerning the score vector $\boldsymbol{S}(\Delta_t(U_k|\boldsymbol{\theta_k}))$, which should be equal to a zero vector, and the Hessian matrix $\boldsymbol{H}(\Delta_t(U_k|\boldsymbol{\theta_k}))$, consequently being positive definite. Ignoring σ_t for a moment and following Edwards (1992), the score vector $\boldsymbol{S}(\Delta_t(U_k|\boldsymbol{\theta_k}))$ is

$$\boldsymbol{S}(\Delta_t(U_k|\boldsymbol{\theta_k})) = \sum_{I\in I_{k,t}} \boldsymbol{\delta}(U_k|\boldsymbol{\theta_k})\boldsymbol{S}(\Delta_t(\boldsymbol{\theta}_k)) \tag{A.2}$$

where we use the abbreviation $\boldsymbol{\delta_t}(U_k|\boldsymbol{\theta_k})$ to denote the square matrix of first derivatives of $\Delta_t(U_k|\boldsymbol{\theta_k})$ with respect to each of its parameters and denote the $(K_k\times 1)$ vector of outer derivatives of the likelihood function as $\boldsymbol{S}(\Delta_t(\boldsymbol{\theta}_k))$, being the product of a diagonal matrix $\boldsymbol{I}$ with elements $I_{k,t}/p_{I_{k,t}}$ and the diagonal matrix $\boldsymbol{P_I}$ containing the outer derivatives of $p_{I_{k,t}}$. Following this notation, the Hessian matrix $\boldsymbol{H}(\Delta_t(U_k|\boldsymbol{\theta_k}))$ consists of two terms, namely a matrix containing partial derivatives of the elements of $\boldsymbol{\delta}(U_k|\boldsymbol{\theta_k})$ and a matrix collecting the second derivatives of $\Delta_t(U_k|\boldsymbol{\theta_k})$ with respect to its parameters (see Edwards (1992) for details).[43]

To obtain the Information matrix $\boldsymbol{I}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$, the sign of the Hessian needs to be reversed and taken by its expectations, where we can use the fact that $E(I_{k,t}) = p_{I_{k,t}}$. Since the sum of the choice probabilities equals 1 $\sum_{I\in I_{k,t}} p_{I_{k,t}} = 1$, the last term of the Hessian vanishes if evaluated at $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ such that the last term can be greatly simplified (Fisher (1956), Edwards (1992), Theorem 7.2.2) to

$$\boldsymbol{I}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k)) = \sum_{I\in I_{k,t}} \boldsymbol{\delta}(U_k|\boldsymbol{\theta_k})\boldsymbol{I}(\Delta_t(\boldsymbol{\theta_k}))\boldsymbol{\delta}(\boldsymbol{U_k}|\boldsymbol{\theta_k})'. \tag{A.3}$$

Here, $\boldsymbol{\delta_t}(U_k|\boldsymbol{\theta_k})$ denotes the square matrix of first derivatives of $\Delta_t(U_k|\boldsymbol{\theta_k})$ with respect to each of its parameters and $\boldsymbol{I}(\Delta_t(\boldsymbol{\theta_k})) = \boldsymbol{P_I}\boldsymbol{P_I'}\boldsymbol{I}$ being the product of a diagonal matrix $\boldsymbol{I}$ with elements $I_{k,t}/p_{I_{k,t}}$ and the diagonal matrix $\boldsymbol{P_I}$ containing the outer derivatives of $p_{I_{k,t}}$. It is evident from this structure that for each $I_{k,t}$th term, the Hessian is a positive semi-definite matrix since $\boldsymbol{I}(\Delta_t(\boldsymbol{\theta_k})) = \boldsymbol{P_I}\boldsymbol{P_I'}\boldsymbol{I}$ is

[43] Note that due to the independence assumption, each element of the score vector and the Hessian matrix consist of a series of sums. This is not surprising since, according to the independence assumption across time and choice sets, the log-likelihood function inherits the regularity property in the sense that differentiation and summation are interchangeable (e.g., Cramer (1986)), which in turn carries over to the entire sample if it holds for any single observation.

symmetrical. Disregarding the possibility that $\boldsymbol{H}(\Delta_t(U_k|\boldsymbol{\theta_k}))$ is singular, the Hessian is in fact positive definite. This implies that $\boldsymbol{I}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k))$ is also a positive definite matrix over reasonable values of $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$.

We mentioned above that the usual invariance and asymptotic properties can be applied to show that maximizing the log-likelihood function for each of the K_k elements of $\boldsymbol{\theta_k}$ and nuisance parameter σ_t of the score vector returns estimators that are consistent and asymptotically efficient. Until now, we used $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ and the sample size–dependent estimate $\hat{\boldsymbol{\theta}}_{\boldsymbol{k|t,n}}$ interchangeably and implicitly assumed that the latter is asymptotically consistent with the former. Showing that $\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}$ is indeed a consistent and asymptotically efficient estimator of $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ is conceptually straightforward and based on several existing insights on parameter transformation from likelihood theory (for the classical proof see Wald (1949) and Chung (1974), Serfling (1974), Spanos (1999) and DeGroot and Schervish (2002) for more recent sources). In the case at hand, it must be shown that $\lim_{nt\to\infty} P\left(|\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})| > \nu\right) = 0$ for any arbitrarily small positive value of ν, a feature that, according to the Slutsky Theorem, carries over to the estimators $\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}$. To sketch this, we return to a series of convergence theorems, pre-supposing that certain criteria for their application are met (Gnedenko (1962)). In accordance with the usual line of argumentation, we define mean expected values of the likelihood function $\log L(\Delta_t(U_k|\boldsymbol{\theta}_k))$ and information matrix $\boldsymbol{I}(\Delta_t(U_k|\boldsymbol{\theta}_k))$ as

$$\bar{L}(\Delta_t(U_k|\boldsymbol{\theta}_k)) = \frac{1}{nt}E\left(\log L(\Delta_t(U_k|\boldsymbol{\theta_k}))\right) \text{ and } \bar{\boldsymbol{I}}(\Delta_t(U_k|\boldsymbol{\theta_k})) = \frac{1}{nt}E\left(\boldsymbol{I}(\Delta_t(U_k|\boldsymbol{\theta_k}))\right).$$

It should be remembered that $\Delta_t(U_k|\boldsymbol{\theta_k})$ are independent but not identically distributed since their density depends on the current characterization of the market parameters for the lookback period–and it can be expected that these values differ across time t and stock n. Consequently, the score vector $\boldsymbol{L}(\Delta_t(U_k|\boldsymbol{\theta_k}))$ and the Hessian $\boldsymbol{H}(\Delta_t(U_k|\boldsymbol{\theta_k}))$ are not identically distributed either–a feature that carries over to its mean values. To make this distinction clearer, we denote the respective estimates and terms with subscripts n, t. Invoking the Chebychev version of the Weak Law of Large Numbers, we know that

$$\frac{1}{nt}\log L(\Delta_t(U_k|\boldsymbol{\theta_k})) \xrightarrow{p} \bar{\boldsymbol{L}}(\Delta_t(U_k|\boldsymbol{\theta_k})) \tag{A.4}$$

whereupon the sample mean converges in probability to its expectations at any $\boldsymbol{\theta} \in \boldsymbol{\theta}_k$. According to Gnedenko (1962) and Rao (1973), this determines the characteristics of the maximands $\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}$ for (A.1) as

$$\max_{\theta_k\in\boldsymbol{\theta_k}} \frac{1}{nt}\log L(\Delta_t(U_k|\boldsymbol{\theta_k})) \xrightarrow{p} \max_{\theta_k\in\boldsymbol{\theta_k}} \bar{\boldsymbol{L}}(\Delta_t(U_k|\boldsymbol{\theta_k})). \tag{A.5}$$

We can directly make use of this result and expand the score vector of a given sample size in a Taylor series around each of the K_k true parameters to obtain the approximation

$$\begin{aligned}&\boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}})) \approx \\ &\quad\approx \boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})) + \boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))\end{aligned}$$

Since $\boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$ is a zero vector at $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$, it is possible to isolate the parts of the utility difference that contain the true estimator $\hat{\boldsymbol{\theta}}_k$ of model k by rearrangement of the former expression to obtain

$$(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})) \approx -\boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))^{-1}\boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$$

or accordingly

$$\sqrt{nt}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})) \approx \\ \approx \left(-\frac{1}{\sqrt{nt}}\boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))\right)^{-1} \frac{1}{\sqrt{nt}}\boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})). \tag{A.6}$$

If either the number of stocks traded by this particular investor n increases (i.e., the investor engages in day-trading) or we can keep track of the investor's trading history for a longer period of time, meaning that t extends considerably (i.e., the investor's security account was opened in the past and has been actively used ever since), the Chebychev Weak Law of Large Numbers implies that

$$-\frac{1}{\sqrt{nt}}\boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})) \xrightarrow{p} \bar{\boldsymbol{I}}(\Delta_t(U_k|\boldsymbol{\theta}_k)). \tag{A.7}$$

Since inverting $\boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$ can be treated as a function of the Hessian matrix, we know by the Slutsky Theorem (Cramer (1946), Theil (1971), Serfling (1974)) that the results from above also hold for

$$\left(-\frac{1}{\sqrt{nt}}\boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))\right)^{-1} \xrightarrow{p} \left(\bar{\boldsymbol{I}}(\Delta_t(U_k|\boldsymbol{\theta}_k))\right)^{-1}.$$

The classical proof would continue from here, but we need to remember that, as pointed out earlier, the mean values are not identically distributed. To account for this heterogeneity, we introduce parameter σ_t such that we need to add an intermediate step and use the Liapounov Central Limit Theorem for non-identically distributed variables to argue that their distribution also converges asymptotically to a normal distribution (see Gnedenko (1962)). Keeping in mind that $\boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$ equals zero if evaluated at $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$, its variance is

$$E\left(\frac{1}{\sqrt{nt}}\boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))\boldsymbol{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))^T\frac{1}{\sqrt{nt}}\right) = \frac{1}{\sqrt{nt}}\boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})),$$

of which we already know that $\frac{1}{\sqrt{nt}}\boldsymbol{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})) = \bar{\boldsymbol{I}}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}))$. By combining this with equation (A.6) and invoking the Chebychev Weak Law of Large Numbers once more, we obtain in the limit

$$\sqrt{nt}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})) \xrightarrow{L} N\left(0, \sum_{I\in I_{k,t}} \boldsymbol{\delta}(U_k|\boldsymbol{\theta}_{\boldsymbol{k}})'\boldsymbol{I}^{-1}(\Delta_t(\boldsymbol{\theta}_{\boldsymbol{k}}))\boldsymbol{\delta}(U_k|\boldsymbol{\theta}_{\boldsymbol{k}})\right) \tag{A.8}$$

as claimed (Cramer (1946)). These are important results for our likelihood approach, since, due to its limiting distribution being normal, it allows us to use simple t-tests to evaluate the statistical significance of each of our maximum likelihood estimators, although the likelihood function is highly nonlinear due to $\Delta(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})$. By making use of the Chebychev Inequality, we complete the final step and establish a connection to the probability statement as claimed in the text. In principle, the statement posits that the probability of a positive difference is below a certain bound, defined in terms of variance

$$P\left(|\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k}})| > \nu\right) \leq \frac{\boldsymbol{H}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}}))}{\nu^2 nt} \tag{A.9}$$

where, according to Rao (1945) and Cramer (1946), the lower bound of the variance of $\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t})$ is defined by the inverse of the information matrix

$$\boldsymbol{H}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{\boldsymbol{k|n,t}})) \geq \boldsymbol{I}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t}))^{-1} \tag{A.10}$$

as n or t goes to infinity as shown in (A.7), the right-hand side approaches zero. To complete the statement, according to the Slutsky Theorem, this carries over to the estimates for $\boldsymbol{\hat{\theta}_{k|n,t}}$. Concerning these estimates, Lehmann (1983) shows, furthermore, that, under certain regularity conditions, the estimator $\boldsymbol{\hat{\theta}}_{k|n,t}$ leads to the best possible inference in terms of being efficient if measured according to the Cramér-Rao Lower Bound.

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