House of Finance
SAFE
Working Paper Series

Deyan Radev

Assessing Systemic Fragility – A Probabilistic Perspective

SAFE Working Paper No. 70

Non-Technical Summary

Default risk and the risk of default of sovereigns and banking institutions more specifically, has proved to be of major concern for regulators during the subprime and the current sovereign debt crises. As the European Central Bank sets the focus of systemic risk management on the *macrofinancial* level, one could conveniently observe the financial system as a portfolio of its constituents. Within such a framework the role of a supervisor is twofold: First, to monitor the stability of the portfolio of banks conditional on their individual financial soundness, as well as the soundness of their interlinkages. Second, to analyze and assess the dynamics of the interconnectedness, especially in volatile periods, characterized by extreme events. The financial stability literature outlines several conditions for a successful systemic risk model: *consistency, flexibility, forward-looking focus, correspondence with empirical data, suitability for the need of financial regulators.*

This paper outlines a comprehensive procedure for *joint* default risk assessment and introduces a series of systemic risk measures that should enhance the regulators' toolkit for analyzing financial system's distress. The procedure we propose can be divided in three steps and covers 10 euro area (EA) sovereigns and 15 EA large and complex banking institutions (LCBGs). The reason why we investigate euro area sovereign default parallel to banking default is straightforward. At the wake of the sovereign debt crisis EA banks had large exposures to EA government debt, hence a possible negative shock from sovereigns to the banking sector might cause a collapse of the whole EA financial system.

Our results show that banking systemic fragility has increased substantially since the outbreak of the subprime crisis in mid-2007. Several events seem to affect this dynamics: the Bear Stearns and Lehman Brothers problems, as well as the Greek fiscal issues and the subsequent attempts by EA authorities to defuse the sovereign debt crisis. The latter crisis clearly affects investors' perceptions about banking default risk, since the indicator for the expected fragility of the banking system more than doubles at the end of 2011, compared to its level after Lehman Brothers' collapse. Considering sovereign fragility, before September 2008, investors seem to have ignored joint sovereign default risk. These sentiments have changed since, especially after November 2009, when Greece announced its budget difficulties.

This paper is part of a broader research agenda with the main purpose to shed more light on the EA financial system distress risk during the recent crises. We believe that our study will help policy makers and regulators to get a more comprehensive perspective of the EA systemic risk in the current turbulent times.

Assessing Systemic Fragility - A Probabilistic Perspective*

Deyan Radev†

October 1, 2014

Abstract

We outline a procedure for consistent estimation of marginal and joint default risk in the euro area financial system. We interpret the latter risk as the intrinsic financial system fragility and derive several systemic fragility indicators for euro area banks and sovereigns, based on CDS prices. Our analysis documents that although the fragility of the euro area banking system had started to deteriorate before Lehman Brothers' file for bankruptcy, investors did not expect the crisis to affect euro area sovereigns' solvency until September 2008. Since then, and especially after November 2009, joint sovereign default risk has outpaced the rise of systemic risk within the banking system.

Keywords: Banking Stability, Financial Distress, Tail Risk, Contagion

JEL-Classification: C16, C61, G01, G21.

*This project was partially conducted while I was a guest researcher at the Financial Stability Surveillance Division of the European Central Bank. I would like to thank Ivan Alves, Carsten Detken, Charles Goodhart, Silviu Oprica, Maria Rodriguez-Moreno, Bernd Schwaab and Constantin Weiser for the helpful discussions and recommendations. This paper also benefited from the suggestions and comments of the remaining ECB FSS staff members and the participants of the 1st Workshop in Financial Economics at the Johannes Gutenberg University Mainz. All remaining errors are my own. I gratefully acknowledge research support from the Research Center SAFE, funded by the State of Hessen initiative for research LOEWE.

†Research Center SAFE, House of Finance, Goethe University Frankfurt, Grüneburgplatz 1, 60323 Frankfurt am Main, email: radev@safe.uni-frankfurt.de.

Contents

1 Introduction

Default risk and the risk of default of sovereigns and banking institutions more specifically, has proved to be of major concern for regulators during the subprime and the current sovereign debt crises. As the European Central Bank sets the focus of systemic risk management on the *macrofinancial* level (ECB, 2009), one could conveniently observe the financial system as a portfolio of its constituents, as stressed by Lehar (2005). According to Gremlich and Oet (2011), within such a framework the role of a supervisor is twofold. First, to monitor the stability of the portfolio of banks conditional on their individual financial soundness, as well as the soundness of their interlinkages. Second, to analyze and assess the dynamics of the interconnectedness, especially in volatile periods, characterized by extreme events. The authors outline several conditions for a successful systemic risk model: *consistency*, *flexibility*, *forward-looking focus*, *correspondence with empirical data*, *suitability for the need of financial regulators*.

This paper outlines a comprehensive procedure for *joint* default risk assessment and introduces a series of systemic risk measures that should enhance the regulators' toolkit for analyzing financial system's distress. The procedure we propose can be divided in three steps. First, we use CDS spreads to extract the perceived individual default risk of 10 euro area (EA) sovereigns and 15 EA large and complex banking institutions (LCBGs). The second step consists of estimating the multivariate probability densities of the EA banking and sovereign systems. Among the various procedures discussed in the literature, the method that we consider especially appropriate for our purposes is the Consistent Information Multivariate Density Optimization (CIMDO) procedure (Segoviano, 2006; Goodhart and Segoviano, 2009). This methodology has solid conceptual underpinnings, allowing us to focus on the market beliefs of the performance of an institution or a sovereign, while avoiding a direct examination of their capital structure. After recovering the system's multivariate probability density, we proceed to our final stage - deriving a series of systemic risk indicators that analyze the fragility of the financial system to default events.

It is important to note that by using CDS-derived default probabilities, we are concentrating on market default risk *perceptions* and not directly on *actual* default frequencies. Market perceptions about individual and joint default risk are of interest to investors and regulators, because default risk premia comprise a large part of bond yields. Since nowadays banks and sovereigns rely heavily on international financial markets to finance their liquidity needs, they should pay closer attention to market default expectations. This is especially important for regulators in their analysis of the effectiveness of policy measures.

The reason why we investigate euro area sovereign default parallel to banking default is straightforward. At the wake of the sovereign debt crisis EA banks had large exposures to EA government debt (see e.g. ECB, 2010; EBA, 2011; IMF, 2011), hence a possible negative shock from sovereigns to the banking sector might cause a collapse of the whole EA financial system.

Our results show that banking systemic fragility has increased substantially since the outbreak of the subprime crisis in mid-2007. Several events seem to affect this dynamics: the Bear Stearns and Lehman Brothers problems, as well as the Greek fiscal issues and the subsequent attempts by EA authorities to defuse the sovereign debt crisis. The latter crisis clearly affects investors' perceptions about banking default risk, since the indicator for the expected fragility of the banking system more than doubles at the end of 2011, compared to its level after Lehman Brothers' collapse. Considering sovereign fragility, before September 2008, investors seem to have ignored joint sovereign default risk. These sentiments have changed since, especially after November 2009, when Greece announced its budget difficulties.

This paper is part of a broader research agenda with the main purpose to shed more light on the EA financial system distress vulnerability during the recent crises. In Gorea and Radev (2014), we investigate the determinants of perceived *bivariate* joint default risk of euro area countries and find that investors seem to discriminate between the EA "core" (Austria, Belgium, France, Germany and Netherlands) and "periphery" (Greece, Ireland, Italy, Portugal and Spain) already during the Post-Lehman global recession. In Radev (2013), we introduce a new *multivariate* measure for default risk contributions, the change in conditional joint default probability ($\Delta CoJPoD$), and use it to analyze the susceptibility of the EA banking system to sovereign default. We find evidence for "too-big-to-save", riskiness-of-business and asset quality considerations in the investors' assessment of the EA banking system vulnerability to sovereign risk. The current work complements our previous studies and extends the regulatory toolbox of probability measures introduced by Lehar (2005), Avesani et al. (2006), Goodhart and Segoviano (2009), Giglio (2011) and Zhang et al. (2012). We believe that our study would help policy makers and regulators to get a more comprehensive perspective of the EA systemic risk vulnerabilities in the current turbulent times.

The paper is organized as follows. Section 2 presents our methodology for deriving marginal and joint probabilities of default. In section 3, we introduce our probability measures and provide guidelines for their calculation. Section 4 describes briefly our dataset, while section 5 presents our empirical results. Section 6 concludes.

2 Methodology

The first step of our procedure involves estimating expected probabilities of default (PoD) of sovereigns and banks. To this end, we use 1- to 5-year government and bank CDS premia and apply a procedure called CDS bootstrapping, based on Hull and White (2000). This method for PoD estimation is used by Gorea and

Radev (2014) and Radev (2013)[1] and the interested reader can consult with these sources for more information.

As a second step, we transfer univariate to multivariate probabilities of default, using the CIMDO approach of Segoviano (2006).[2] Let the financial system is represented by a portfolio of n entities (sovereigns or banks): X_1, X_2 to X_n, with their log-assets being x_1, x_2, to x_n. The CIMDO approach then minimizes the following Lagrangian:

$$\begin{aligned}
L(p,q) = & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\ln\left[\frac{p(x_1,x_2,...,x_n)}{q(x_1,x_2,...,x_n)}\right]dx_1\cdots dx_{n-1}dx_n \\
& + \lambda_1\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\mathbf{I}_{[\overline{\mathbf{x}}_1,\infty)}dx_1\cdots dx_{n-1}dx_n - PoD_t^1\right] \\
& + \lambda_2\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\mathbf{I}_{[\overline{\mathbf{x}}_2,\infty)}dx_1\cdots dx_{n-1}dx_n - PoD_t^2\right] \\
& + \cdots \\
& + \lambda_n\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\mathbf{I}_{[\overline{\mathbf{x}}_n,\infty)}dx_1\cdots dx_{n-1}dx_n - PoD_t^n\right] \\
& + \mu\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)dx_1\cdots dx_{n-1}dx_n - 1\right]
\end{aligned} \tag{1}$$

The first integral in equation 1 represents the cross-entropy probability difference, introduced in Kullback (1959). The idea of this approach is to minimize the distance between a *prior* distribution guess $q(x_1,x_2,...,x_n)\in\mathbb{R}^n$ and a *posterior* distribution $p(x_1,x_2,...,x_n)\in\mathbb{R}^n$ that reflects empirical market data on individual probabilities of default. The market information is included in the model by means of consistency

[1]Radev (2013) shows that this method achieves a more accurate mapping from CDS spreads to probabilities of default than a widely used simplified formula.

[2]The current section outlines the multivariate case of the CIMDO approach, following Radev (2013). For lower dimensionality problems, see Segoviano (2006) and Gorea and Radev (2014).

constraints (the lines from the second to the last but one in equation 1). In those constraints, PoD_t^1, PoD_t^2 to PoD_t^n stand for the expected probabilities of default of the respective entities, derived from CDS prices. With $\mathbf{I}_{[\overline{\mathbf{x}}_1,\infty)}$, $\mathbf{I}_{[\overline{\mathbf{x}}_2,\infty)}$ to $\mathbf{I}_{[\overline{\mathbf{x}}_n,\infty)}$ we denote a set of indicator functions that take the value of one if the respective entities' default thresholds $\overline{\mathbf{x}}_1$, $\overline{\mathbf{x}}_2$ to $\overline{\mathbf{x}}_n$ are crossed and zero in the opposite case. The empirical default thresholds are analogous to the ones in the classic structural model (Merton, 1974) and are calculating by inverting a standard normal cumulative distribution at the borrower's average estimated PoD for the sample period. To constrain the posterior distribution to the $[0, 1]^n$ region, we need to impose the additivity rule in the last line of equation 1. The coefficients μ, λ_1, λ_2 to λ_n are the Lagrange multipliers of the described constraints. The optimal CIMDO (or posterior) distribution is then represented in the following way:[3]

$$p^*(x_1, x_2, ..., x_n) = q(x_1, x_2, ..., x_n)exp\left\{-\left[1 + \mu + \sum_{i=1}^{n} \lambda_i \mathbf{I}_{[\overline{\mathbf{x}}_i,\infty)}\right]\right\} \tag{2}$$

In the next section, we explain how the optimal CIMDO distribution from equation 2 can be used to derive a series of unconditional and conditional probability measures.

3 Probability Measures

Once we have specified the default regions of the *prior* distribution and recovered the optimal Lagrange multipliers, we can completely identify the *posterior* distribution. Moreover, by coupling the different prior default regions with their respective Lagrange multiplier combinations, we can fairly easy construct practically unlimited default probability measures. The current section aims at presenting the huge variability of the possible CIMDO-derived measures.

[3]See the appendix in Radev (2013) for a complete solution of the multivariate minimum cross-entropy problem.

3.1 Systemic Fragility Measure

Our first measure is the CIMDO-derived probability of at least two entities (sovereigns or banks) defaulting jointly. As this is an unconditional measure, it represents the systemic default *potential*, hence the general vulnerability to systemic events. In other words, it is an indicator of how *fragile* the system is to default of its constituents. Therefore, we call this indicator the Systemic Fragility Measure (SFM). In essence, it is a CIMDO-derived alternative to a similar measure by Avesani et al. (2006), the Systemic Risk Indicator, with the latter being based on n-th to default baskets of CDS contracts.

As explained above, the calculation of the various probability measures boils down to integrating over the regions of the posterior distribution, where particular default cases occur. With regard to the SFM, this means that we have to sum up all posterior regions, where at least two entities default. For example, in a 3-dimensional system with entities A, B and C,[4] we need to sum up the following *unconditional* joint probabilities:

$$SFM = P\left(\neg A, B, C\right) + P\left(A, \neg B, C\right) + P\left(A, B, \neg C\right) + P\left(A, B, C\right), \qquad (3)$$

where the "¬" sign indicates that the respective entity does not default.

The remaining subsections introduce probability measures that aim to capture *complex* scenarios - the effect of the default of a particular entity or subset of entities on the distress risk vulnerability of the remaining institutions in the financial system.

[4]The extension to the multivariate case is trivial.

3.2 Probability of A Defaulting Given B Defaults

We start with the simplest extension beyond the *unconditional* joint probability framework: the probability of default of entity A given entity B defaults (*P(A|B)*). Deriving *P(A|B)* is a direct application of the Bayes rule:

$$P(A|B) = \frac{P(A, B)}{P(B)}, \tag{4}$$

where *P(A, B)* is the *joint* probability of default of entities A and B, while *P(B)* is the *marginal* probability of default of entity B.

Note that in a system with N entities, the calculation of this measure effectively involves decreasing the dimensionality from N to 2 (for the calculation of *P(A, B)*) and to 1 (for the calculation of *P(B)*). Both ingredients of the conditional probability 4 could be derived by integrating the multivariate joint density over the remaining N-2 or N-1 entities. Moreover, integration over N-1 entities is in fact equivalent to using the CDS-derived marginal probabilities of default of the N-th entity. Therefore, only the integration over the N-2 entities not involved in the respective conditional probability suffices for the calculation of 4.

3.3 Probability of A Defaulting Given B and C Default

The next indicator measures the conditional probability of default of an entity, given two other entities default simultaneously. In the Bayes' framework, mentioned before, this probability of default is defined as

$$P(A|B, C) = \frac{P(A, B, C)}{P(B, C)}, \tag{5}$$

with *P(A, B, C)* and *P(B, C)* being respectively the joint probabilities of entities A, B and C, and of entities B and C defaulting.

The procedure for calculation of the measure is similar to the one in the previous subsection, but this time it involves decreasing the N-dimensional joint probability of default to its 3– and 2–dimensional alternatives.

3.4 Probability of at Least N-1 Additional Entities Defaulting Given Entity A Defaults

Our final (and most complex) probability measure is the probability of at least N-1 entities (banks or sovereigns) defaulting, given a particular entity defaults.[5] This measure is a generalization of the probability of at least one (PAO) bank defaulting,[6] introduced in Goodhart and Segoviano (2009) and aims at addressing the expected severity of a crisis stemming from a particular entity, hence the rate of *contagion penetration* in the financial system.

To define the measure, let us consider again a system of three banks, A, B and C.[7] The probability of at least one additional bank defaulting given a particular bank (say C) defaults is then

$$PNBD(\text{at least } 1|C) = P(A|C) + P(B|C) - P(A,B|C), \tag{6}$$

where $P(A|C)$, $P(B|C)$ and $P(A,B|C)$ are the respective conditional probabilities for all possible default contingencies. Using this intuition, it is easy to proceed one step further – to the probability of at least *two* banks (in this case A and B) defaulting given bank C defaults:

$$PNBD(\text{at least } 2|C) = P(A,B|C), \tag{7}$$

[5] In the text, we abbreviate the measure related to banks as PNBD and the one related to sovereigns as PNSD.

[6] PAO is also sometimes referred to as a probability of spillover effects (PSE).

[7] The extension to higher dimensions, although more involving, is straightforward, as long as one keeps account of the default contingencies to be added or subtracted.

Hence, in the limit (i.e. for N-1 additional entities defaulting) the PNBD/PNSD converges to the Conditional Joint Probability of Default (CoJPoD) measure, introduced in Radev (2013).

4 Data and Estimation Strategy

We recover marginal probabilities of default using CDS premia for contracts with maturities from 1 to 5 years for the period 01.01.2007 and 31.12.2011. The probabilities of default bootstrapping procedure that we employ requires as additional inputs refinancing interest rates, which we choose to be the AAA euro area government bond yields for maturities from 1 to 5 years. The CDS spreads and the government bond yields are at daily frequency, which is also the frequency of the resulting probabilities of default. Our analysis covers 10 euro area (EA) sovereigns (Austria, Belgium, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain) and 15 EA large and complex banking groups (EA LCBG).[8] The bank groups are chosen such that their individual total asset value is above EUR 200 Billion.

The reasons why we rely on CDS data are, first, because bank and sovereign defaults are a relatively rare events and it is difficult to arrive at meaningful actual default frequencies even for entities at the brink of insolvency. The problem can be circumvented by using CDS-derived expected probabilities of default. This brings us to our second motive for using bank and sovereign CDS spreads: we would like to analyze market expectations about *individual* default risk and to develop a model that transforms them into *joint* bank and/or sovereign default risk perceptions. The third reason why we focus mainly on the CDS market is that due to the numerous interventions on the EA bond market in recent times, bond prices are not suitable for analyzing expectations about individual or joint default risk. No interventions

[8] The banks used in our analysis are listed in Table 1.

are undertaken or planned on the CDS market, so the CDS premia are the true market consensus prices of default risk.

5 Empirical Results

5.1 Marginal Probability of Default Results

This section presents the results for the individual probabilities of default for our samples of banks and sovereigns.

5.1.1 Individual Bank Results

In figures 1 and 2, we present the 5-year annualized CDS-implied probabilities of default of the 15 banks in our sample. The series reveal that the LCBGs were considered relatively default-free before the outbreak of the subprime crisis and experienced the first peak of marginal default risk before the bailout of Bear Stearns in the spring of 2008. We notice that, with the exception of Dexia, the individual default risk was stable around 2 % during the financial crisis and the following global recession. Two subperiods can be discerned from the PoD dynamics in the second half of the sample period: between the exascerbation of the sovereign debt crisis at the end of the first quarter of 2010 and the second quarter of 2011, and the period thereafter.

5.1.2 Individual Sovereign Results

The sovereign marginal PoD results are presented in figure 3. As already observed in Radev (2013), before Lehman Brothers' debacle EA sovereign bonds were perceived as relatively riskless. The individual default risk rises in the following global recession, but still remains at relatively low levels. After the outbreak of the sovereign debt crisis in late 2009 and early 2010, we observe strong divergence in investors' perceptions about EA sovereigns' individual default risk. Apart from the outlier Greece, we can distinguish several other "bundles" of countries that are perceived equally risky by the end of the sample period: Ireland and Portugal; Belgium, Italy and Spain; Austria, France, Germany and Netherlands.

Our next sections present the results from our multivariate analysis, based on the individual entities' empirical information.

5.2 Systemic Fragility Measure Results

Figures 12 and 13 present the banking and sovereign Systemic Fragility Measures, plotted against major regulatory and systemic events since the beginning of 2007.[9] The results for the 15 LCBGs show a significant increase in the EA banking system fragility since the onset of the subprime crisis. The various rescue packages throughout the period seem to temporarily relieve the default pressure in our banking sample. The joint default risk in our 15-dimensional system oscillates around 5% for a large part of the sample period, but sharply increases after the bailout request by the Portoguese government in the spring of 2011. The negative investor default risk sentiments continue until the end of the sample and reach its maximum soon after the private sector involvement (PSI) agreement for Greece in October 2011. We notice that, the market joint default expectations seem to react positively to regulatory intervention announcements, but these effects appear to be of *transitory nature*.

In Figure 13, we depict the SFM results for the 10 sovereigns in our sample.[10] We notice that until Lehman Brothers' debacle, markets did not expect that the subprime crisis would affect the solvency of the EA member governments. The sovereign

[9]In both cases we use empirical correlation, derived between changes of 5-year CDS spreads of the respective entities.

[10]Please note that the levels of SFM between figures 12 and 13 are not directly comparable, because of the different number of dimensions of the underlying distributions.

SFM increases significantly during the global recession period in the beginning of 2009, but subsides gradually thereafter. Although the levels of the banking and sovereign SFM cannot be compared directly, one should notice the much steeper increase of sovereign systemic risk from the beginning of 2010 on. We document the same temporary positive effect of regulatory interventions as in the banks case, even after the sharp decline following the EU summit on 21 July 2011.

5.3 Conditional Probabilities Results

Figures 6 and 7 depict the univariate banking (sovereign) probabilities of default, given a particular bank (sovereign) defaults. We notice very different patterns in the banking measures. Apparently, a distressed bank as Dexia (throughout most of the period) had lower impact on the probability of default perceptions with regard to a safer counterpart (Intesa Sanpaolo or Unicredit) than the effect of a default of the latter counterpart on Dexia. This is intuitive, since if the safer bank in a particular couple actually defaulted, investors would expect that it is more likely for the more riskier to follow suit. The bottom two plots deserve special attention, as they present same-country bank couples: Societe Generale - BNP Paribas (France) and Unicredit - Intesa Sanpaolo (Italy). We notice that the conditional default perceptions of the particular banks narrowly trace each other. However, it appears that the French banks couple is perceived generally as the riskier one throughout the sample period.

Turning to the sovereign couples (figure 7), we notice that investors perceive Greece to be very sensitive to default expectations concerning other EA countries (Portugal and Spain). While we find a reciprocal effect for Portugal, given an expected Greek government default, the effect on the Spanish default perceptions is minimal. Conditioning on a default of Spain provides a different picture, with the perceived conditional probability of default of Portugal rising steadily since November 2009. The strongest feedback effects are present for the Italy - Spain couple, with the default perceptions of both sovereigns being tightly linked since the mid-2007.

Overall, we confirm that the perceived feedback effects in the banking sector start occurring with the beginning of the subprime crisis, while the major peaks in the sovereign couples appear to be after the bankruptcy of Lehman Brothers in the autumn of 2008. In either cases, the events during the sovereign debt crisis, especially after July 2011, seem to have strong effect on conditional default risk perceptions.

In figures 8 and 9, we present univariate banking and sovereign probability results, conditional on two entities defaulting. The top plot of figure 8 reveals that a joint default of Dexia and DZ Bank has a relatively low effect on the default perceptions with regard to Unicredit. On the other end of the spectrum is the perceived probability of default of Societe Generale given BNP Paribas and Unicredit jointly default. Considering Deutsche Bank, international investors seem to expect that it is less vulnerable to Dexia and DZ bank defaulting than to a joint default of Societe Generale and Unicredit.

Turning to perceptions about sovereign default given two states go bankrupt (figure 9), we note that investors expect Spain to react relatively more intensively to a default of Italy and Portugal (second plot from the top) than to a simultaneous default of Greece and Portugal. Nonetheless, both contingencies seem to have a very high overall effect on expectations about a Spanish default. The same holds for the perceptions about Italy defaulting given that Portugal and Spain jointly default (second plot from the bottom). Here we note a significant increase in the expected Italian conditional probability of default since mid-2011. Finally, the last plot in figure 9 shows that the joint default of Italy and Spain seems to have limited effect on the expected probability of default of Germany.

Our last measure, the probability of at least N-1 additional entities defaulting given a particular entity defaults, is presented in figure 10 (for banks) and figure 11 (for sovereigns). As mentioned before, for convenience, we call the measure for banks PNBD and the one for sovereigns PNSD. An additional detail is that, for presentation purposes, each curve in figures 10 and 11 presents the cross-sectional median values of the respective probability. Although we still cannot compare the magnitudes of the probabilities of banks and sovereigns, one could expect that if the dimensions of the system's distribution are higher, there would be a higher likelihood of at least one entity defaulting given another one defaults. Therefore, in general one would expect that the conditional probability values for the 15-dimensional banks distribution would be higher than the corresponding values for the 10-dimensional sovereign distribution.

The results confirm the impression from all our previous measures that the distress in the banking system started already in mid-2007, while we witness a slow build-up of sovereign systemic risk up to the default of Lehman Brothers. A surprising finding is that, at least when medians are compared, the probabilities of default of at least 1, 2 and 3 sovereigns given a particular sovereign defaults are higher than the corresponding PNBD values for most of the second half of the sample period. What can be also noted is that the conditional probability of *at least one* entity defaulting rises very fast to the unity limit of the probability domain (the unreported maximum values are even closer to 1), making the dynamics of this measure (introduced in Goodhart and Segoviano, 2009; and often referred to as the probability of spill-over effects) relatively uninformative. Therefore, we believe that our generalization provides a richer picture of the depth of penetration of default spill-over effects within the financial system.

6 Robustness Checks

In this section, we outline a panel of robustness checks that examines the sensitivity of our approach to changes in the underlying parameters. For brevity, only the results for marginal PoDs and for the banking and sovereign SFM are presented. The remaining results are available upon request.

Figure 3 in Gorea and Radev (2014) examines the sensitivity of the marginal probabilities of default of Greece from 01.01.2007 to 31.08.2011 to different recovery rates assumptions, starting from 40% and reaching 85%. The figure reveals that assuming higher recovery rates provides *more conservative* probabilities of default. The relationship is monotonous - higher recovery rates lead to higher PoDs. The effect is most noticeable in the middle of the sample period and less prominent in 2007 and 2011. These results suggest that changing the recovery rates assumption has a predictable effect on our probability measures and would alter their level, but not their dynamics.

We proceed with an analysis of the Systemic Fragility Measure's sensitivity to different dependence structure assumptions (figures 12 and 13). Figure 12 presents the banking SFM results for both the cases where zero correlation (red) and empirical correlation (blue) is assumed.[11] The measures are juxtaposed to major events during the subprime and the sovereign debt crises. We document that both indicators have similar dynamics throughout the sample period and the SFM with empirical correlation is usually above the one with zero correlation. We observe a steady upward trend in the last five years, since the outbreak of the subprime crisis.

The indicator peaks at 15 % for the correlation case and at 25 % for the zero-correlation case. The crossing point of both versions of the indicator is surprising, as one would expect that when positive correlation between banks' asset is taken into

[11] Please, note that in a CIMDO context, assuming a joint standard normal distribution as a prior distribution is equivalent to assuming independence between its underlying entities. Moreover, it can be shown that this independence transfers to the CIMDO posterior distribution (for details, see Radev, 2012).

account, banks would tend to default jointly more frequently. In order to explain this result, we need to realize that high correlation between banks means not only that they would most likely default together, but also that they might jointly survive. At some level of default probability on, this transfer of probability mass to the regions not included in equation 3 leads the correlation SFM having a lower value than the benchmark zero-correlation case.

Similar observations can be made in figure 13 for the case of sovereigns. The usually lower zero-correlation series of the sovereign SFM, compared to the correlation case, cross the latter ones around the European Summit on 21 July 2011. Apparently, as previously observed in figure 12, this has been a milestone date both on the banking and the sovereign debt markets.

Our last robustness check concerns the prior distribution assumption, underlying the CIMDO distribution. We recall that the main purpose of the CIMDO approach is to adjust the tails of the prior distribution, such that they represent the market consensus about the unobserved system's asset distribution. The resulting CIMDO distribution is fat-tailed by construction, no matter what the prior distribution is. This intuition is confirmed when we compare the sovereign SFM results when a normal distribution and a Student-t distribution (5 degrees of freedom) are used as priors. Our CIMDO-derived measures are almost identical. Although the Student-t-based measure is always above the Gaussian-based one, the latter has the same dynamics. Hence, for practical purposes assuming Normal prior distribution might suffice in a CIMDO setting.[12]

[12]The insensitivity of the CIMDO distribution to changing the prior distribution assumption is analytically shown in Segoviano (2006).

7 Conclusion

This paper outlines a procedure for consistent estimation of individual and joint default risk within the euro area financial system. We apply our method to calculate several measures of systemic fragility, first of euro area banks, and then with regard to euro area sovereigns. In addition, we undertake a number of robustness checks with respect to the major parameter assumptions in our methodology.

Our analysis documents that although the fragility of the EA banking system had started to deteriorate before Lehman Brothers' file for bankruptcy, investors did not expect the crisis to affect EA sovereigns' solvency before September 2008. Since then, and especially after November 2009, joint sovereign default risk has outpaced the rise of systemic risk within the banking system.

The procedure and systemic risk measures that we propose should extend the policy makers' toolkit for analysis of systemic default risk in the euro area financial system.

References

[1] Avesani, R., A. Garcia Pascual, and J. Li, 2006, "A New Risk Indicator and Stress Testing Tool: A Multifactor Nth-to-Default CDS Basket," IMF Working Paper, No. 06/105.

[2] EBA, 2011, "2011 EU-Wide Stress Test Aggregate Report," European Banking Authority, July 2011.

[3] ECB, 2010, Financial Stability Review, European Central Bank, Frankfurt am Main, December 2010.

[4] Giglio, S., 2011, "Credit default swap spreads and systemic financial risk," Proceedings, Federal Reserve Bank of Chicago: 104–141, May 2011.

[5] Goodhart, C., and M. Segoviano, 2009, "Banking Stability Measures," IMF Working Paper 09/4.

[6] Gorea, D., and D. Radev, 2014, "Euro Area Sovereign Debt Crisis: Can Contagion Spread from the Periphery to the Core?," *International Review of Economics and Finance*, 30: 78-100.

[7] Gramlich, D., and M. Oet, 2011, "The structural fragility of financial systems: Analysis and modeling implications for early warning systems," *The Journal of Risk Finance*, 12(4): 270-290.

[8] Hull, J., and A. White, 2000, "Valuing credit default swaps I: No counterparty default risk," *Journal of Derivatives*, 8: 29-40

[9] IMF, 2011, Global Financial Stability Report, International Monetary Fund, Washington, DC, September 2011.

[10] Kullback, S., 1959, "Information Theory and Statistics," John Wiley, New York.

[11] Lehar, A, 2005, "Measuring Systemic Risk: A Risk Management Approach," *Journal of Banking and Finance*, 29(10): 2577-2603.

[12] Merton, R., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *The Journal of Finance*, 29: 449-470.

[13] Radev, D., 2013, "Systemic Risk and Sovereign Debt in the Euro Area," SAFE Working Paper No. 37, Research Center SAFE, Goethe University Frankfurt, available at SSRN: http://ssrn.com/abstract=2368283.

[14] Segoviano, M., 2006, "Consistent Information Multivariate Density Optimizing Methodology," FMG Discussion Papers dp557, Financial Markets Group.

[15] Sturzenegger, F. and J. Zettelmeyer, 2005, "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998-2005," IMF Working Paper, WP/05/137.

[16] Zhang, X., B. Schwaab, and A. Lucas, 2012, "Conditional probabilities and contagion measures for Euro Area sovereign default risk," Working Paper, VU University Amsterdam, January 12, 2012.

A Figures

Figure 1: 5-year annualized CDS-implied bootstrapped probabilities of default for 8 banks: BBVA, Banco Santander, Bayerische LB, BNP Paribas, Commerzbank, Credit Agricole, Deutsche Bank and Dexia. Euro-denominated CDS spreads are used. Period: 01.01.2007 - 31.12.2011. Source: own calculations.

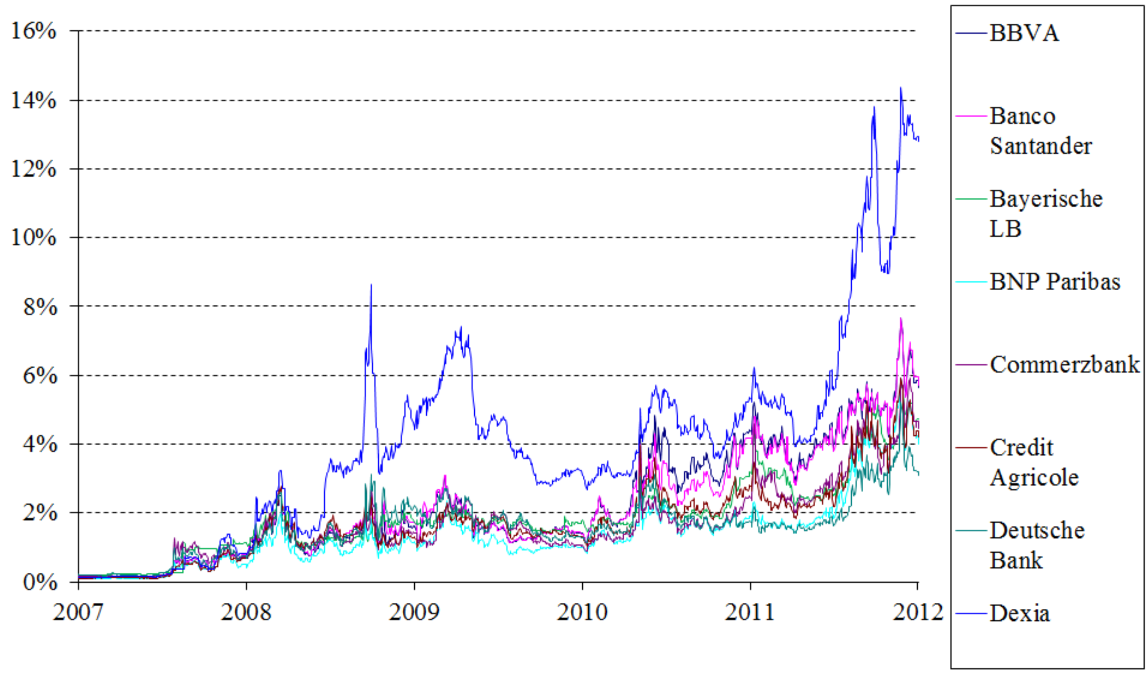


Figure 2: 5-year annualized CDS-implied bootstrapped probabilities of default for 7 banks: DZ Bank, ING, Intesa Sanpaolo, LB Baden-Wrttemberg, Rabobank, Societe Generale and Unicredit. Euro-denominated CDS spreads are used. Period: 01.01.2007 - 31.12.2011. Source: own calculations.

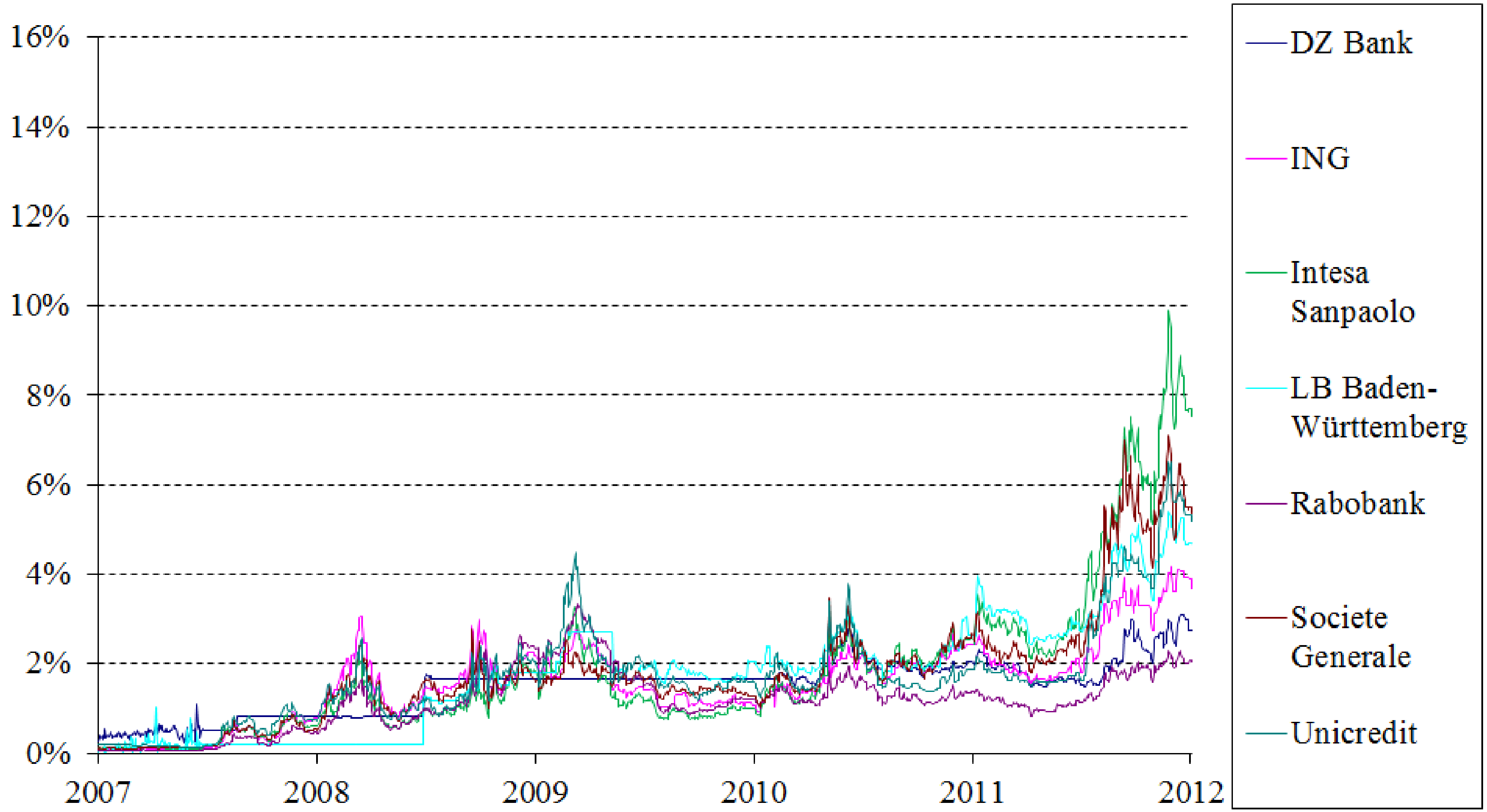


Figure 3: 5-year annualized CDS-implied bootstrapped probabilities of default for 10 sovereigns: Austria (AT), Belgium (BE), France (FR), Germany (GE), Greece (GR), Ireland (IE), Italy (IT), Netherlands (NL), Portugal (PT), Spain (ES). Euro-denominated CDS spreads are used. Period: 01.01.2007 - 31.12.2011. Source: own calculations.

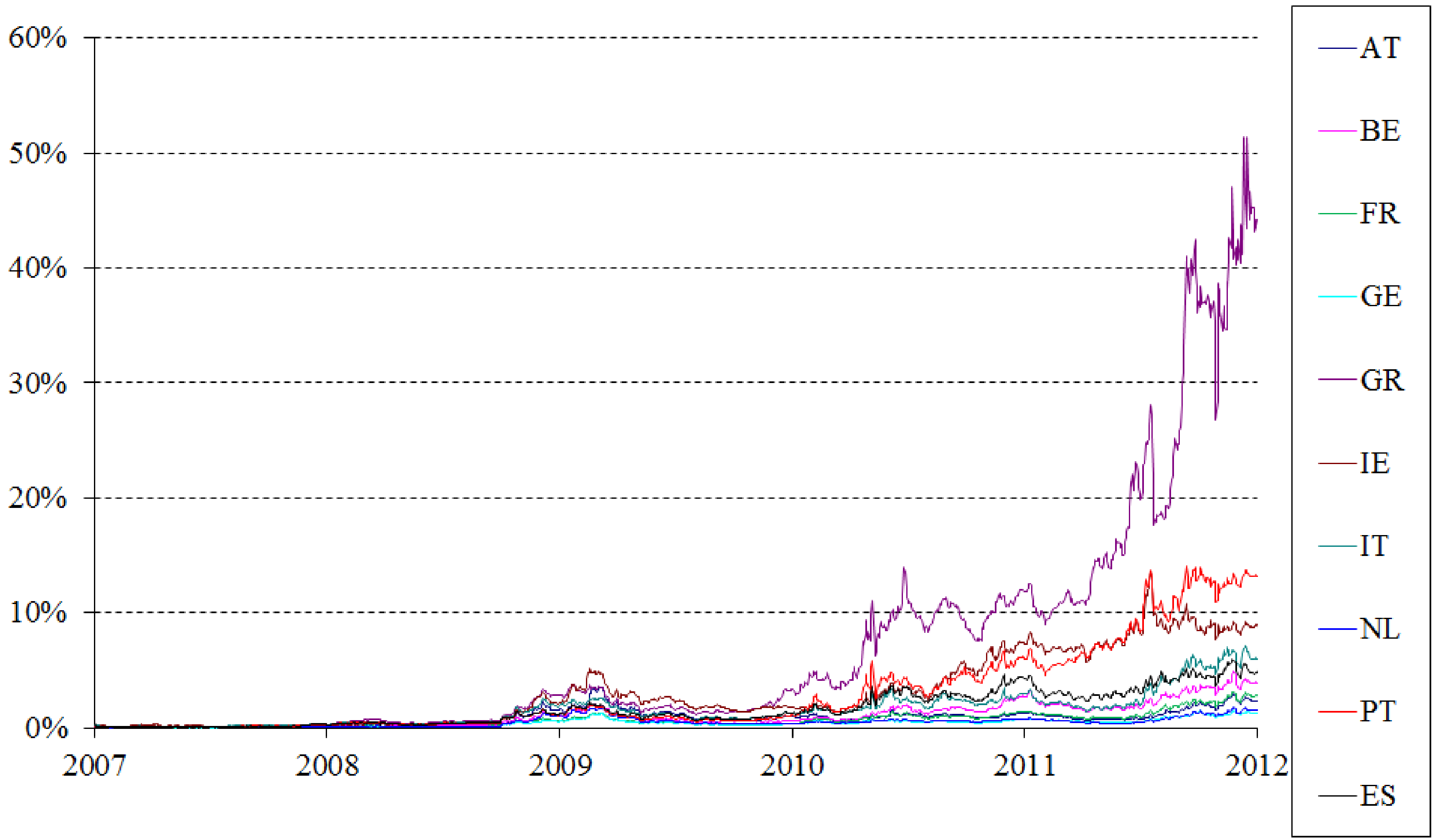


Figure 4: **Bank Systemic Fragility Measure** (probability of at least two *banks* jointly defaulting) involving 15 large and complex banking groups in the period 01.01.2007 - 31.12.2011. The measure is derived incorporating empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads. Source: ECB FSS and own calculations.

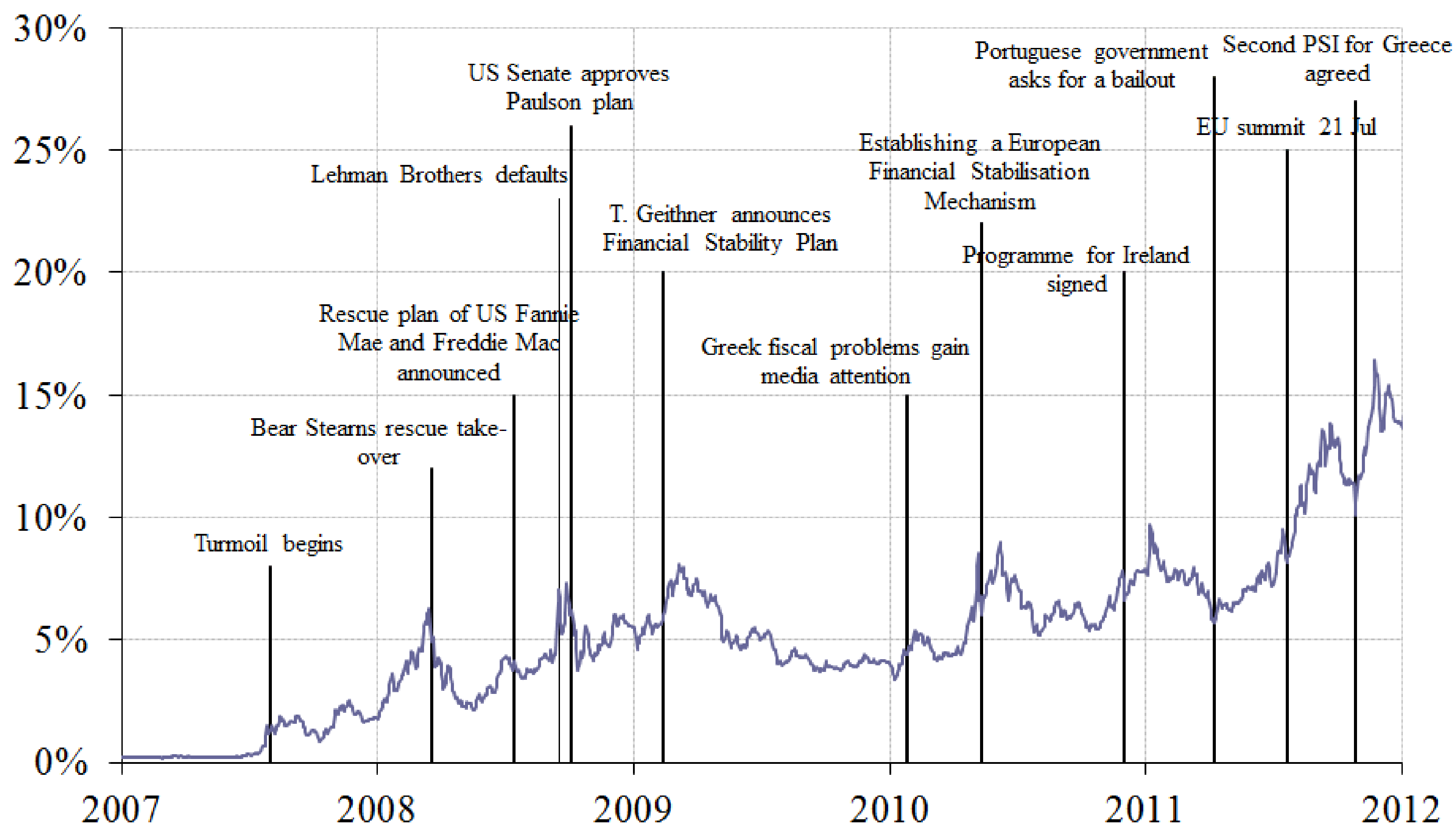


Figure 5: **Sovereign Systemic Fragility Measure** (probability of at least two *sovereigns* jointly defaulting) involving 10 EA countries in the period 01.01.2007 - 31.12.2011. The measure is derived incorporating empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads. Source: ECB FSS and own calculations.

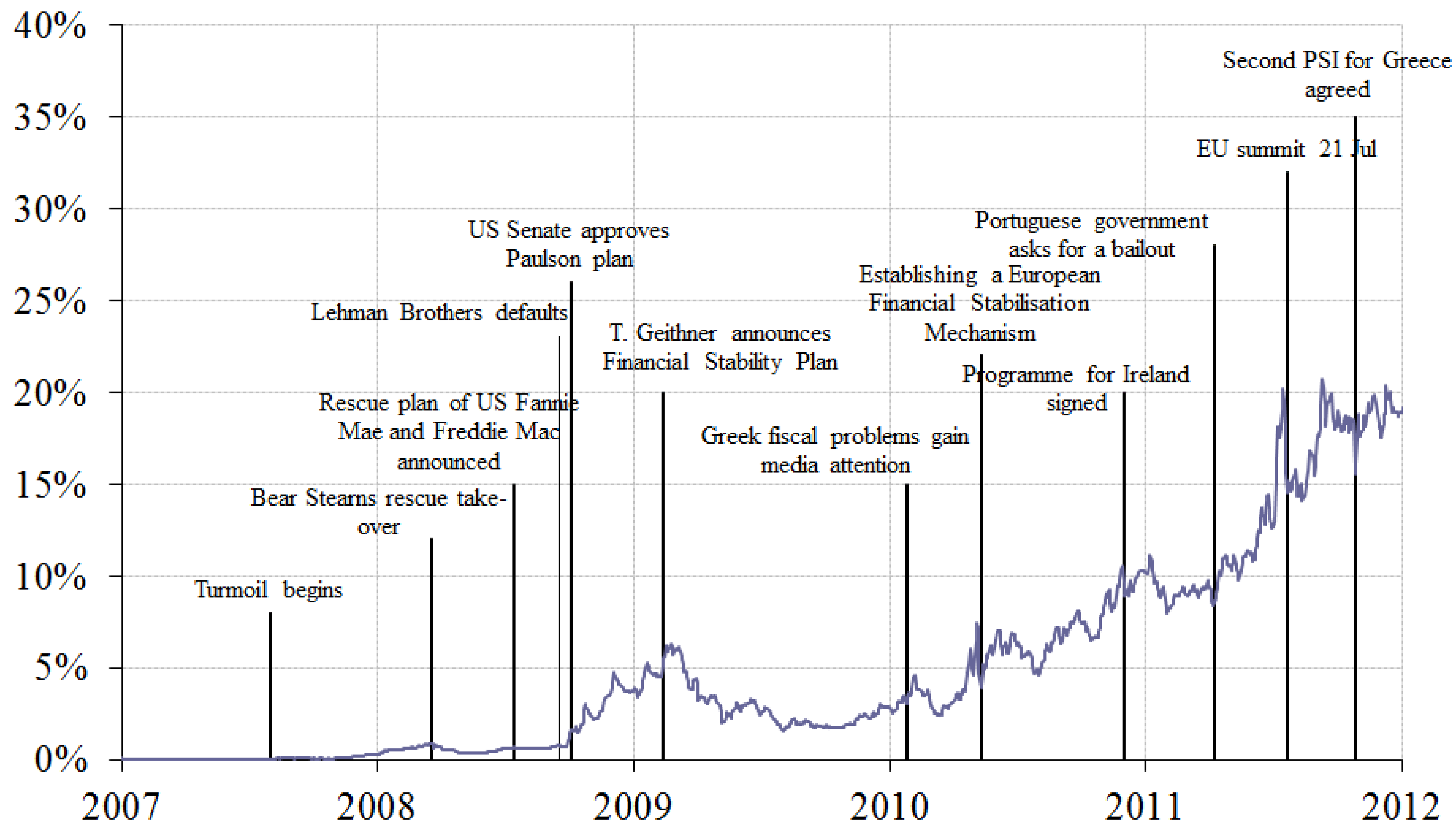


Figure 6: **Banking conditional probability of default given a particular bank defaults:** 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 31.12.2011. The blue (red) line corresponds to the probability of default of the first (second) bank listed in the couple, given the second (first) bank defaults. E.g. the blue line in the top plot represents the probability of default of Intesa Sanpaolo given Dexia defaults, while the red line corresponds to the probability of default of Dexia given Intesa Sanpaolo defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads. Source: own calculations.

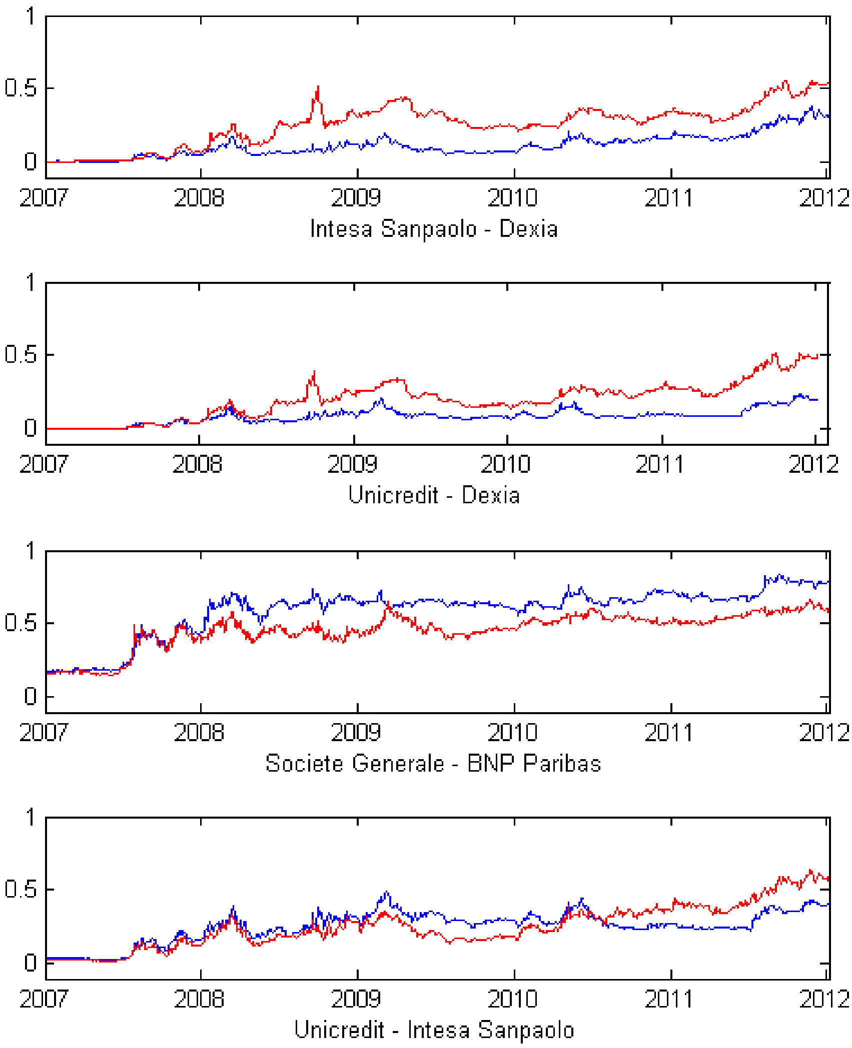


Figure 7: **Sovereign conditional probability of default given a particular country defaults:** 5-year annualized conditional probabilities of default of selected sovereigns in the period 01.01.2007 - 31.12.2011. The blue (red) line corresponds to the probability of default of the first (second) country listed in the couple, given the second (first) state defaults. E.g. the blue line in the top plot represents the probability of default of Portugal given Greece defaults, while the red line corresponds to the probability of default of Greece given Portugal defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads. Source: own calculations.

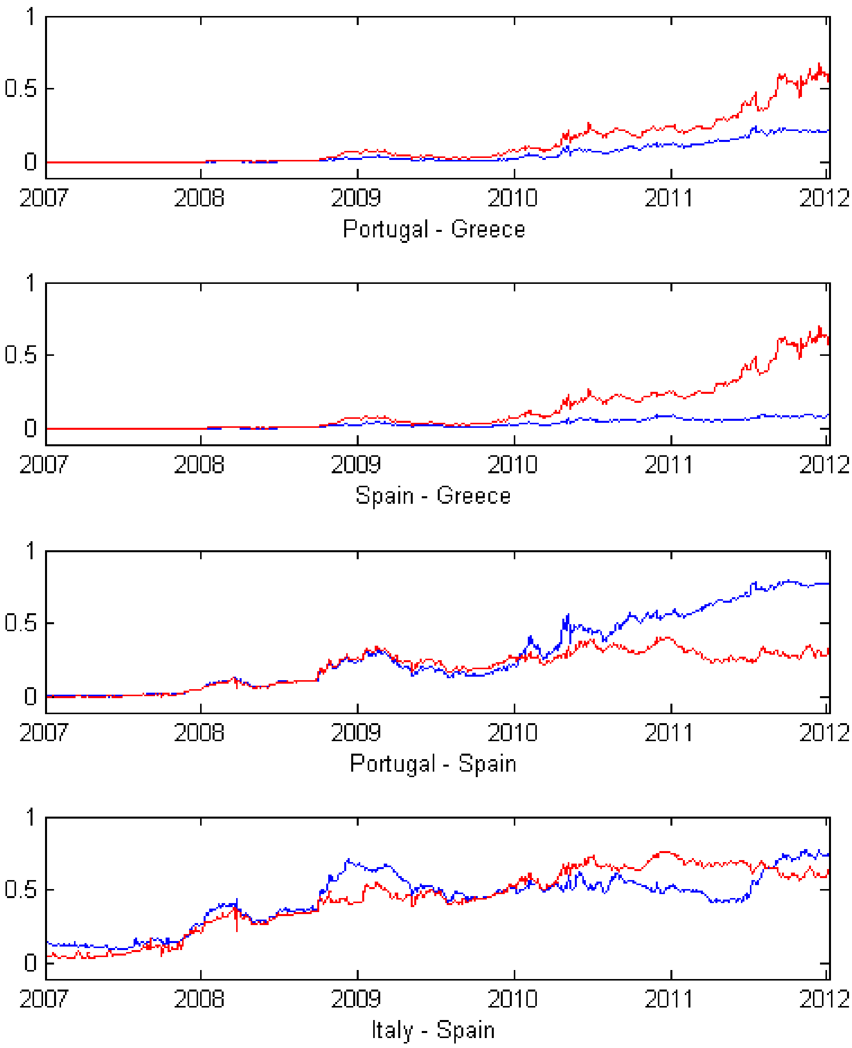


Figure 8: **Banking conditional probability of default given two banks default:** 5-year annualized conditional probabilities of default of selected complex banking groups in the period 01.01.2007 - 31.12.2011. The blue line corresponds to the probability of default of the first bank listed in the couple, given the remaining two listed banks default simultaneously. E.g. the blue line in the top plot represents the probability of default of Unicredit given Dexia and DZ Bank default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads. Source: own calculations.

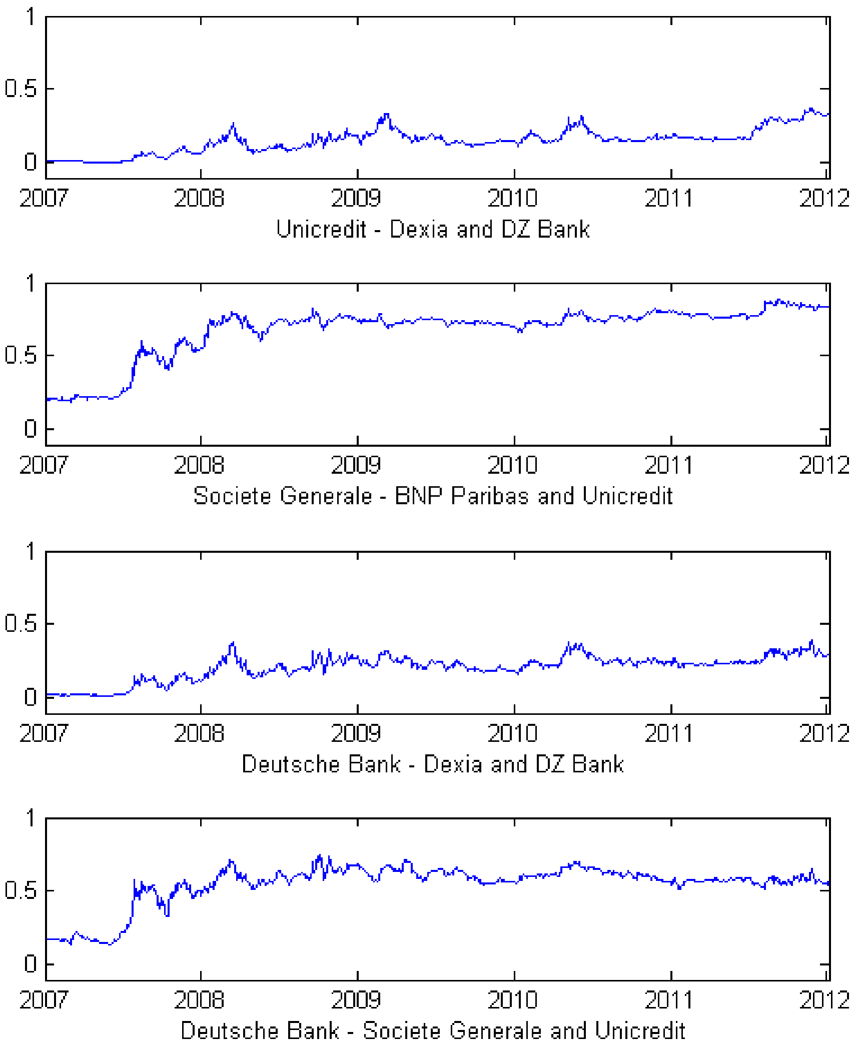


Figure 9: **Sovereign conditional probability of default given two states default:** 5-year annualized conditional probabilities of default of selected sovereigns in the period 01.01.2007 - 31.12.2011. The blue line corresponds to the probability of default of the first sovereign listed in the couple, given the remaining two listed sovereigns default simultaneously. E.g. the blue line in the top plot represents the probability of default of Spain given Greece and Portugal default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads. Source: own calculations.

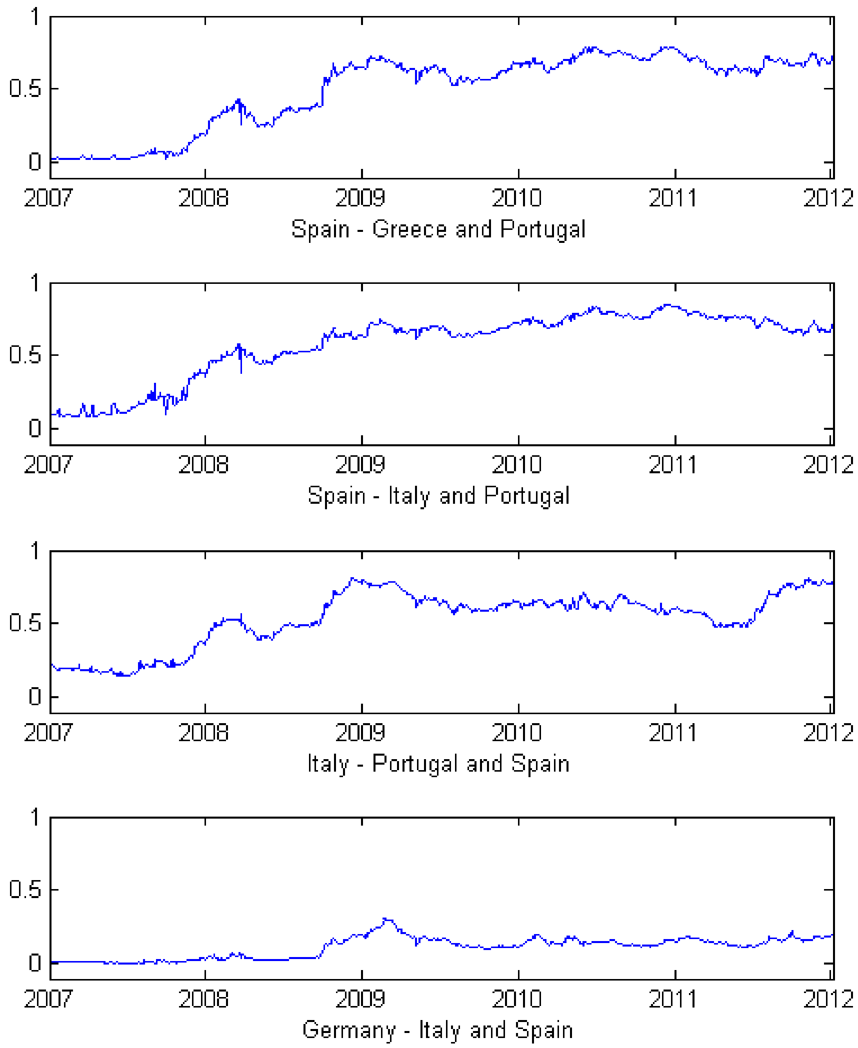


Figure 10: **Probability of *at least* N-1 additional banks defaulting given a particular bank defaults**, involving 15 large and complex banking groups in the period 01.01.2007 - 31.12.2011. The median values across the cross-section of the respective probabilities are reported. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads. Source: ECB FSS and own calculations.

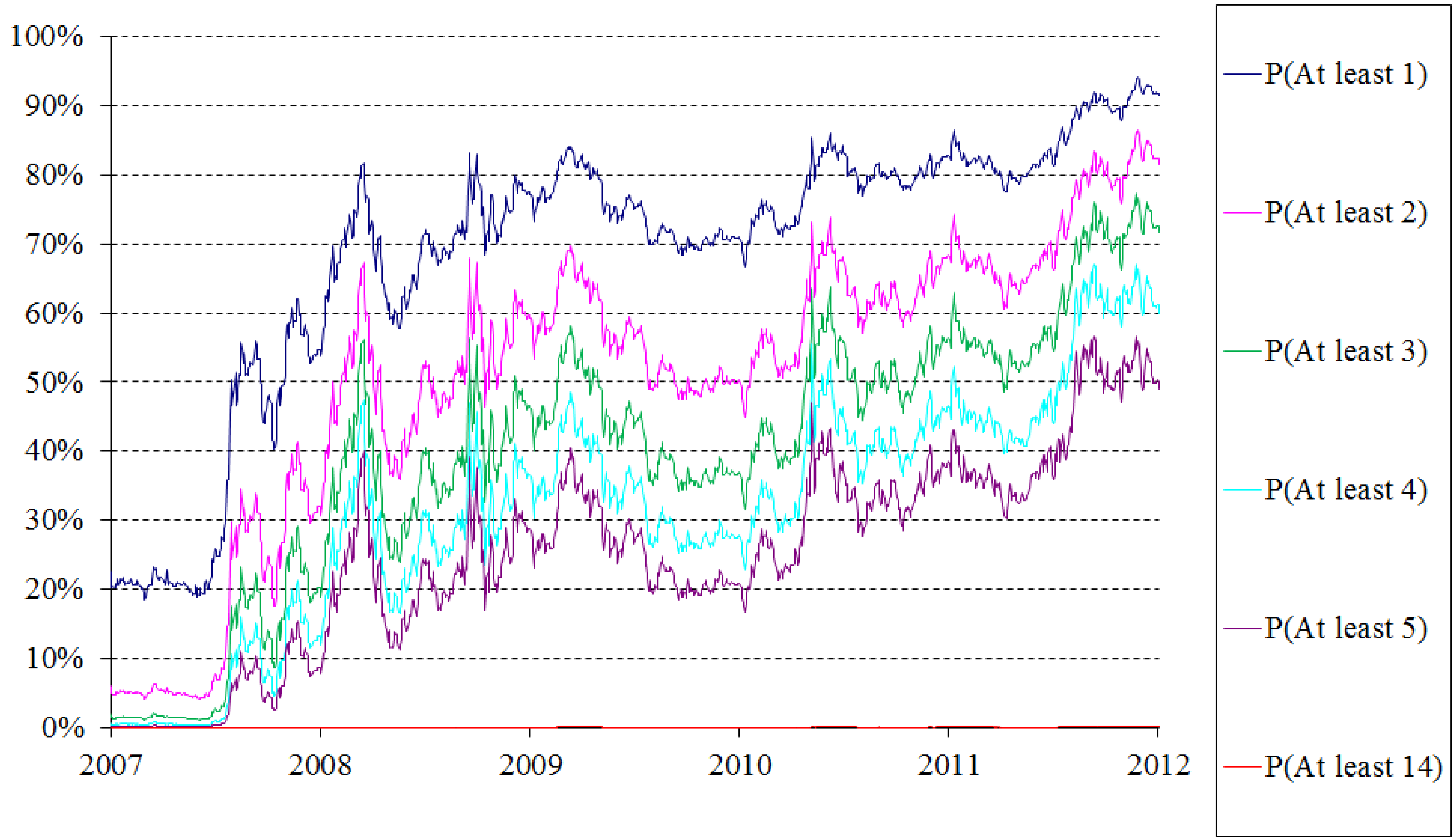


Figure 11: **Probability of *at least* N-1 additional sovereigns defaulting given a particular country defaults**, involving 10 EA countries in the period 01.01.2007 - 31.12.2011. The median values across the cross-section of the respective probabilities are reported. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads. Source: ECB FSS and own calculations.

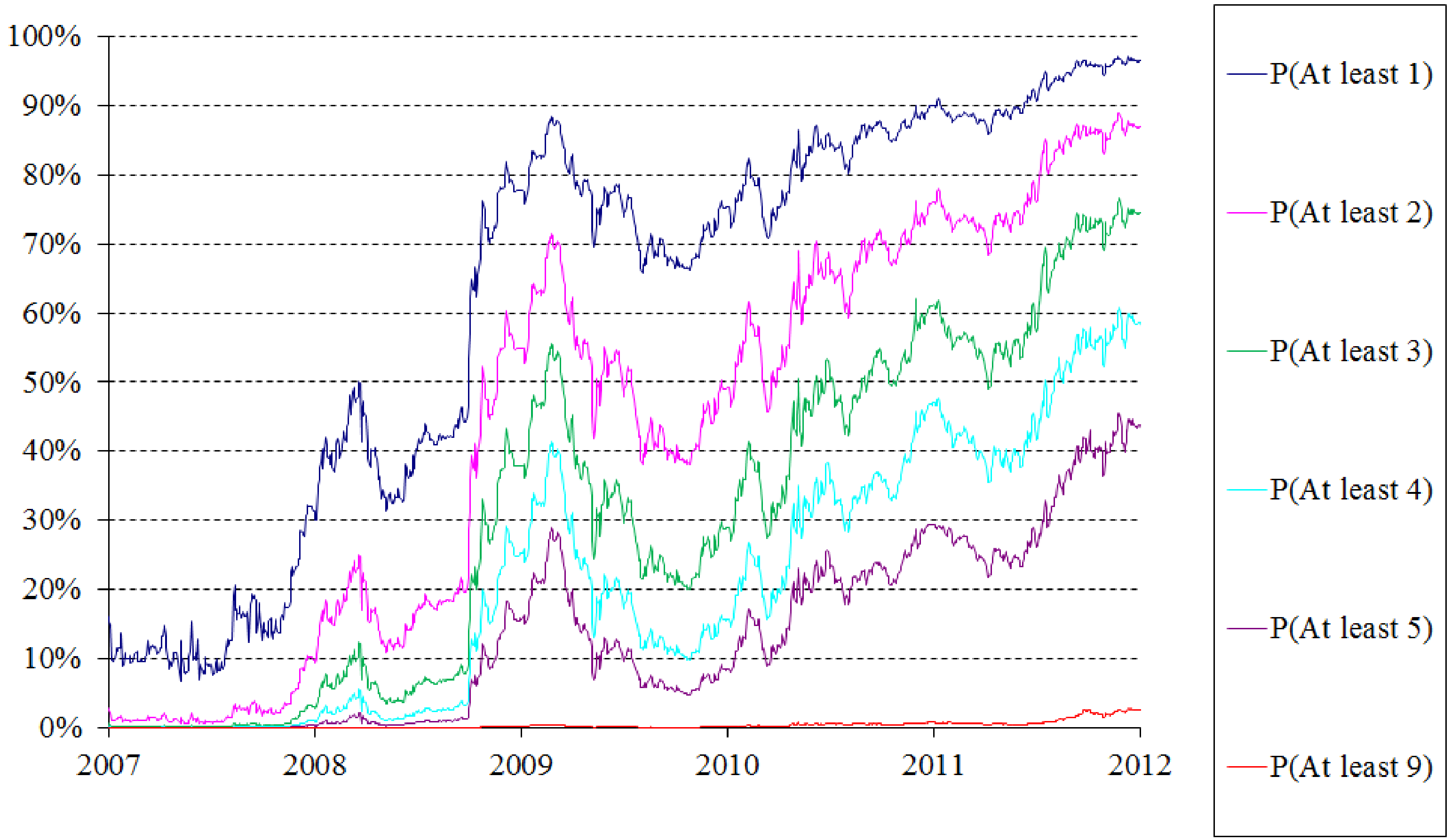


Figure 12: **Bank Systemic Fragility Measure** (probability of at least two *banks* jointly defaulting) involving 15 large and complex banking groups in the period 01.01.2007 - 31.12.2011. The red line presents the results when zero correlation is assumed between banks. The blue line is calculated incorporating empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads. Source: ECB FSS and own calculations.

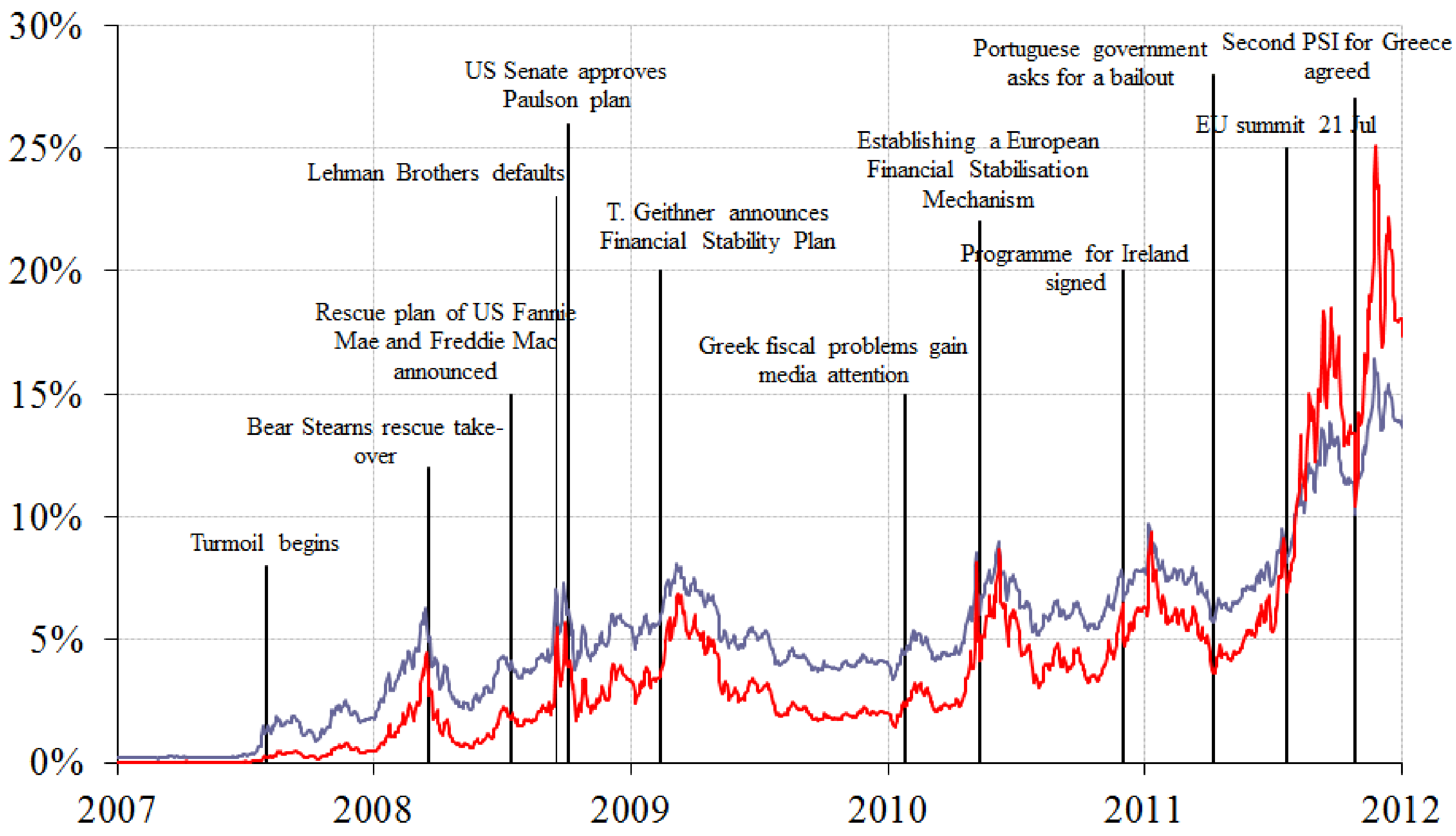


Figure 13: **Sovereign Systemic Fragility Measure** (probability of at least two *countries* jointly defaulting) involving 10 euro area sovereigns (listed in Figure 3) in the period 01.01.2007 - 31.12.2011. The red line presents the results when zero correlation is assumed between sovereigns. The blue line is calculated incorporating empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads. Source: ECB FSS and own calculations.

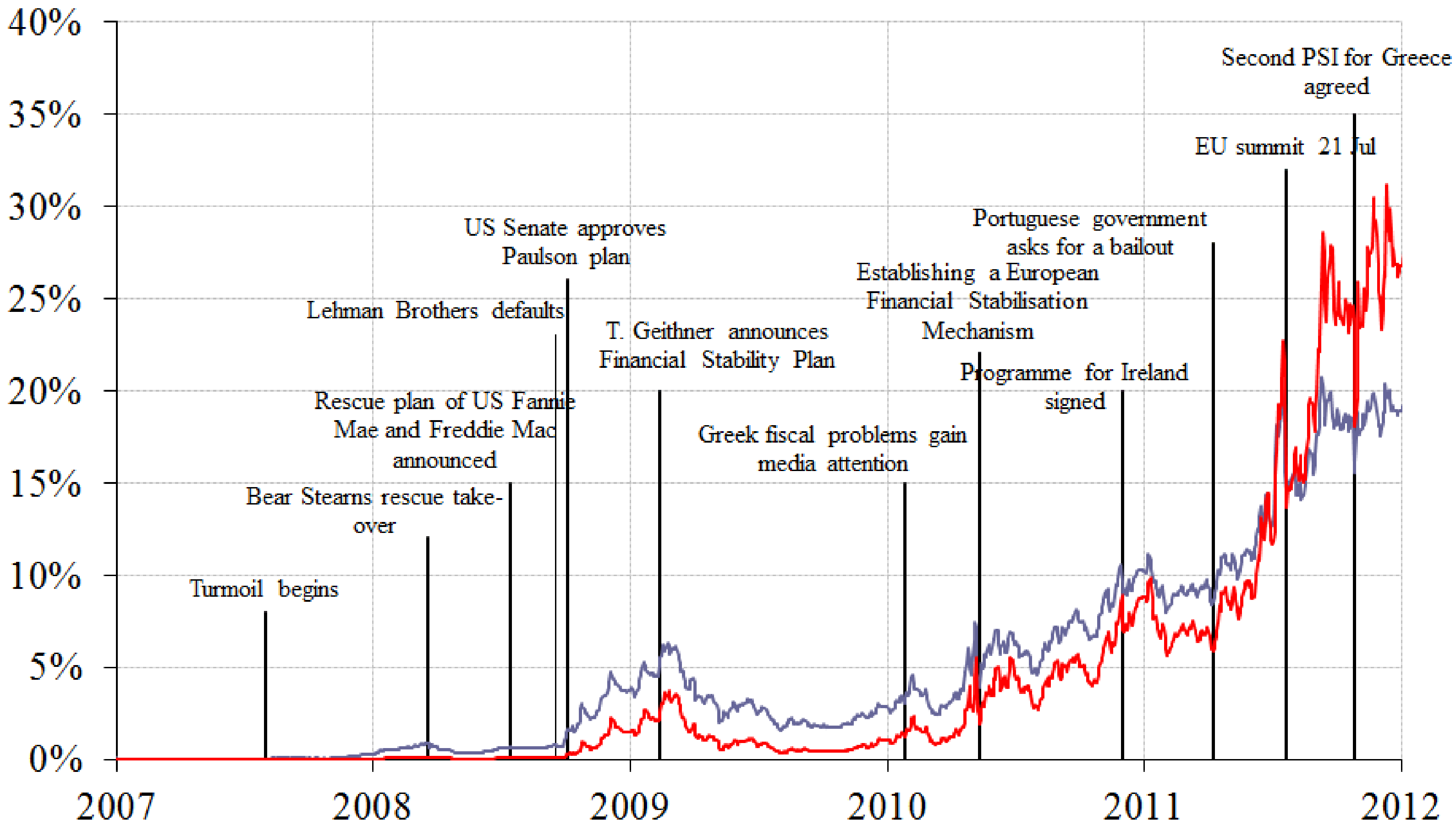
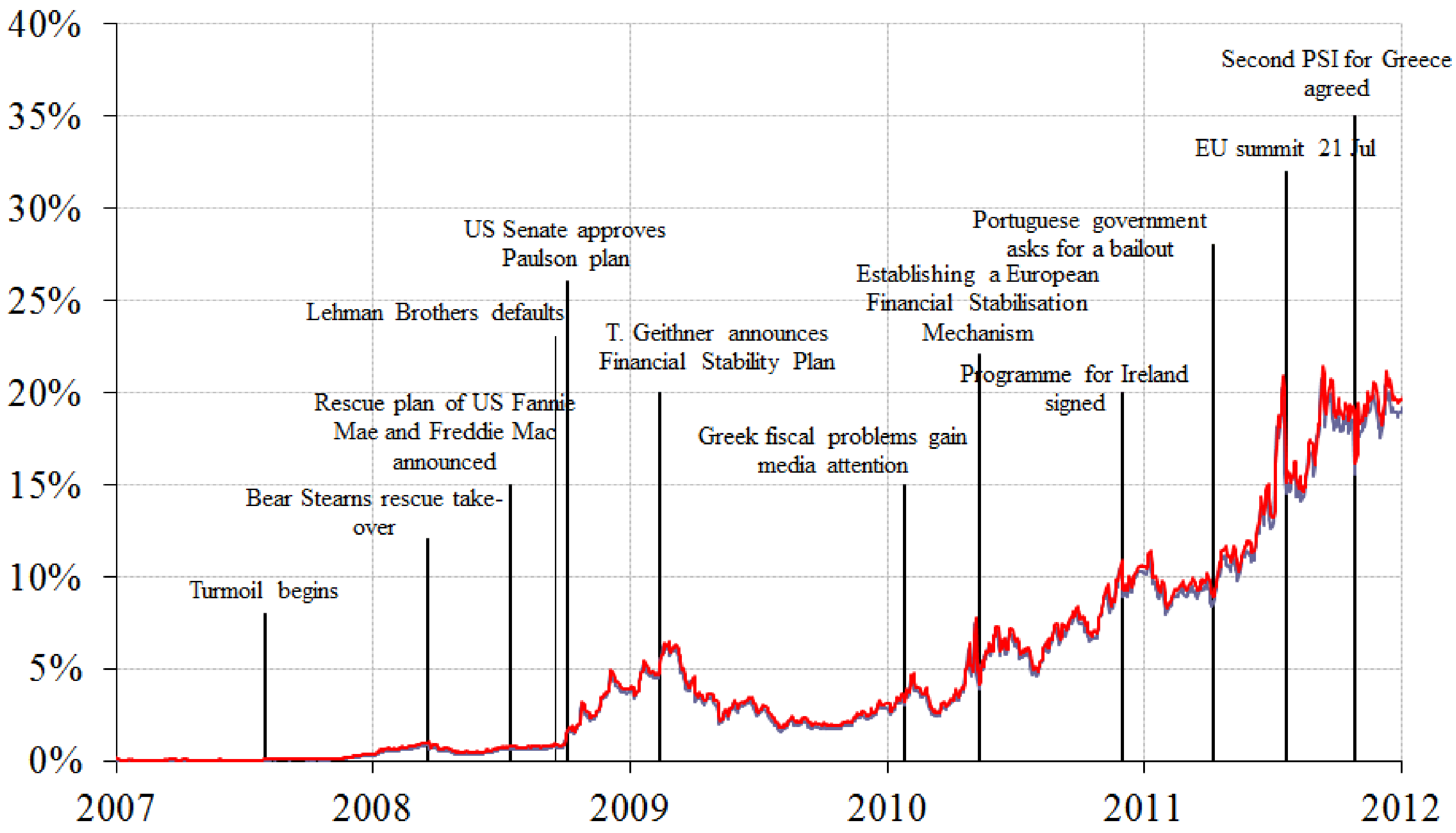


Figure 14: **Sovereign Systemic Fragility Measure: Normal vs. Student-T Prior Distribution.** The figure plots the probability of at least two *countries* jointly defaulting for 10 euro area sovereigns (listed in Figure 3) using Normal (blue line) and Student-T (red line) distributions as prior density assumptions. The measures are derived incorporating empirical correlation, calculated between changes of the respective banks' 5-year CDS spreads. Period 01.01.2007 - 31.12.2011. Source: ECB FSS and own calculations.



B Tables

Table 1: List of euro area large and complex banking institutions used in our analysis.

Euro Area Large and Complex Banking Groups		
	Country code	Name
1	BE	Dexia SA
2	DE	Bayerische Landesbank
3	DE	Commerzbank AG
4	DE	Deutsche Bank AG
5	DE	DZ Bank AG
6	DE	Landesbank Baden-Wrttemberg
7	ES	Banco Bilbao Vizcaya Argentaria
8	ES	Banco Santander SA
9	FR	BNP Paribas
10	FR	Credit Agricole SA
11	FR	Societe Generale
12	IT	Intesa Sanpaolo SpA
13	IT	UniCredit SpA
14	NL	ING Groep NV
15	NL	Rabobank

HOUSE OF FINANCE

SAFE

WORKING PAPER SERIES

Recent Issues

No. 69	Nina Biljanovska, Spyros Palligkinis	**Control Thyself: Self-Control Failure and Household Wealth**
No. 68	Tobias Tröger	**How Special Are They? – Targeting Systemic Risk by Regulating Shadow Banking**
No. 67	Sascha Baghestanian, Paul J. Gortner, Joel van der Weele	**Peer Effects and Risk Sharing in Experimental Asset Markets**
No. 66	Tobias Tröger	**Corporate Groups**
No. 65	Elia Berdin, Helmut Gründl	**The Effects of a Low Interest Rate Environment on Life Insurers**
No. 64	Daniel Herbold	**A Repeated Principal-Agent Model with On-the-Job Search**
No. 63	Nicola Fuchs-Schündeln, Michael Haliassos	**Does Product Familiarity Matter for Participation?**
No. 62	Patrick Behr, Alejandro H. Drexler, Reint Gropp, Andre Guettler	**Financial Incentives and Loan Officers Behavior: Multitasking and Allocation of Effort Under an Incomplete Contract**
No. 61	Iñaki Aldasoro, Mike Seiferling	**Vertical Fiscal Imbalances and the Accumulation of Government Debt**
No. 60	Stefano Colonnello, Giuliano Curatola, Ngoc Giang Hoang	**Executive Compensation Structure and Credit Spreads**
No. 59	Daniel Harenberg, Alexander Ludwig	**Social Security and the Interactions Between Aggregate and Idiosyncratic Risk**
No. 58	Michael Haliassos, Thomas Jansson, Yigitcan Karabulut	**Incompatible European Partners? Cultural Predispositions and Household Financial Behavior**
No. 57	Brigitte Haar	**Financial Regulation in the EU – Cross-Border Capital Flows, Systemic Risk and the European Banking Union as Reference Points for EU Financial Market Integration**
No. 56	Giuliano Curatola, Michael Donadelli, Alessandro Gioffré, Patrick Grüning	**Austerity, Fiscal Uncertainty, and Economic Growth**