Interbank Networks and Backdoor Bailouts: Benefiting from other Banks' Government Guarantees*

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Abstract

This paper explains why banks derive a benefit from being highly interconnected. We show that when banks are protected by government guarantees they can significantly increase their expected returns by channeling funds through the interbank market before these funds are invested in real assets. If banks that are protected by implicit or explicit government guarantees act as intermediaries between other banks and real investments, there is the possibility that these intermediary banks will be rescued by their governments if the real assets fail. This additional hedge increases the likelihood that banks and their creditors are repaid relative to a direct investment in those same real assets. We show that this incentive to exploit the government guarantees of *other* banks leads to long intermediation chains and a degree of interconnectedness that is above the welfare-optimal level, which justifies regulatory intervention.

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1 Introduction

The 2008-2009 financial crisis vividly revealed how shocks can propagate through the financial system, which raised concerns that financial institutions are "too interconnected to fail" and elevated systemic risk as a priority for policymakers. Given these concerns about the system risk in the financial sector, the question arises as to why market solutions did not emerge to eliminate these concerns.

This paper provides a theoretical underpinning for why it benefits banks to be highly interconnected on the interbank market. In particular, we show that, when banks are protected by government guarantees, they can significantly increase their expected returns by channeling funds through the interbank market before these funds are invested in real assets. If banks that are protected by implicit or explicit government guarantees act as intermediaries between other banks and real investments, there is the possibility that these intermediary banks will be rescued by their governments if the real assets fail. This additional hedge increases the likelihood that banks and their creditors are repaid compared to a direct investment in the same real assets.

The bailout of American International Group (AIG) provides a good illustration of this mechanism. Just two days after allowing Lehman Brothers to collapse on September 15, 2008, U.S. authorities decided to rescue AIG, which ultimately led to a government bailout of $170 billion for the insurer.[1] The authorities stated that the reason for AIG's bailout involved its complex ties with banks around the world, which meant that its failure would have entailed high systemic risk. Many commentators, however, have called it a "backdoor bailout," as a large fraction of the bailout funds was directly funneled to AIG's counterparties.[2] U.S. banks that received bailout funds injected into AIG included Goldman Sachs ($12.9 billion), Merrill Lynch ($6.8 billion), Bank of America ($5.2 billion), Citigroup ($2.3 billion), and Wachovia ($1.5 billion).[3] Foreign banks also benefited tremendously from AIG's bailout. For example, Societe Generale received $11.9 billion, Deutsche Bank $11.8 billion, and Barclays $8.5 billion.[4] However, not only were the bailout funds funneled to direct counterparties of AIG, but some of these counterparties also acted as intermediaries between AIG and other market participants. Goldman Sachs, for example, sold credit protection worth $14 billion to its clients and then entered into contracts with AIG to hedge these positions, while Merrill Lynch undertook roughly $6 billion of these deals.[5] Furthermore, some of AIG's counterparties attempted to limit their direct exposure to AIG by building in additional intermediaries. For example, Goldman Sachs used Societe Generale as an additional intermediary, and as a result, a portion of the $11.9 billion that AIG paid to Societe Generale was subsequently transferred to Goldman Sachs.[6] Therefore, some of the bailout funds injected into AIG were passed on to Societe Generale, then to Goldman Sachs and, finally, to Goldman Sachs' clients.

Why were so many intermediaries built in between AIG and the final investor? Our paper shows

[1] "A.I.G. Lists Banks It Paid With U.S. Bailout Funds" by Mary Williams Walsh, *NY Times*, March 15, 2009.
[2] "Did Goldman Benefit The Most From AIG Bailout?" by Liz Moyer, *Forbes*, January 26, 2010.
[3] "The Big Money: How AIG fell apart" by Adam Davidson, *Reuters*, September 19, 2008.
[4] "AIG ships billions in bailout abroad" by Eamon Javers, *Politico*, March 15, 2009.
[5] "Goldman Fueled AIG Gambles" by Serena Ng and Carrick Mollenkamp, *Wall Street Journal*, December 12, 2009.
[6] "Testy Conflict With Goldman Helped Push A.I.G. to Edge" by Gretchen Morgenson and Louise Story, *NY Times*, February 2, 2010.

that channeling funds through other government-protected banks before investing in real assets (i.e., exploiting the government bailout subsidies of *other* banks) increases the repayment probability of banks and their creditors since even when the real investments fail, the intermediary banks may still be rescued by their governments. As a result, higher interbank exposure (i.e., channeling more funds through other government-protected banks) increases the banks' expected returns due to a higher repayment probability and lower funding costs.

This result directly follows from the Modigliani-Miller intuition. As interbank exposure increases the value of bailout subsidies without affecting the total amount invested in real investments, banks can increase their total firm value (i.e., the sum of real investment returns and the value of the bailout subsidies) by increasing their interbank exposure. If banks have the bargaining power vis-a-vis their creditors, they appropriate all of the value increase in the government bailout subsidies and thus higher interbank exposure increases their expected returns.

Although channeling funds through government-protected intermediaries is especially valuable for non-insured entities (hedge funds, mutual funds, etc.), banks that are themselves protected by implicit bailout guarantees can also increase their repayment probability by channeling their funds through other implicitly-insured banks. This behavior potentially leads to long intermediation chains. Furthermore, our analysis shows that the incentive to channel funds through implicitly-insured intermediaries even exists in the case where banks' public guarantees are limited due to bounded government bailout budgets. Market participants only lose the incentive to channel additional funds through an implicitly-insured bank when its interbank exposure is already increased to a level such that the bank's total liabilities exceed the available bailout budget of its government.

Moreover, we show that the incentive to establish interbank connections increases with the bailout probability of the bank's counterparties and decreases with a bank's own bailout probability. First, the value of the additional hedge provided by funneling funds through a government-protected intermediary increases with the intermediary's bailout probability. Second, if a bank itself has a comparatively low bailout probability, establishing interbank connections with banks that are very likely to be bailed out allows the bank to significantly increase insurance coverage for its creditors' funds, which, in turn, lowers its funding costs. Hence, we help to explain the formation of core-periphery network structures because borrowing and lending extensively on the interbank market makes a bank larger and more interconnected, which increases the likelihood that the bank is considered "too-big-to-fail" and "too-interconnected-to-fail." This classification, in turn, increases the bank's likelihood of being rescued by the government, which reinforces other banks' incentive to use this bank as an intermediary.

Furthermore, we document that when banks decide to be highly interconnected on the interbank market to exploit government bailout subsidies of other banks, they can maximize the government subsidy per invested unit of capital by investing in correlated assets. This finding is related to the existing literature on the time-inconsistency of bank rescue decisions, which has pointed out that this time-inconsistency might give banks an incentive to herd, a result shown, for example, in Acharya and Yorulmazer (2007, 2008a) and Farhi and Tirole (2012). In our model, this herding incentive results from the following mechanism: The possibility that banks will be rescued by their governments reduces the downside risk for their creditors. Therefore, their required risk premia become less sensitive to banks' liquidation value in default states. Hence, given public guarantees, banks benefit less from co-insuring one another by engaging in interbank lending and investing in negatively correlated assets, which would allow them to repay their creditors in states in which their real investments fail but their interbank investments are repaid. Since, at the same time, higher portfolio correlation between banks increases their cash flows in success states (i.e., interbank loans are repaid in the same states in which their real investments are successful), banks have an incentive to invest in similar portfolios.

Finally, we show that banks have an incentive to become excessively interconnected even when we allow the interbank market to exist for welfare-improving reasons, providing a justification for regulatory intervention. Hence, we discuss possible measures that aim at reducing banks' incentive to create excessive interbank exposures, including (i) increasing the risk weights for interbank liabilities, (ii) lowering bailout expectations, (iii) limiting a bailout to domestic counterparties, (iv) introducing a financial transaction tax, and (v) introducing a bank levy.

2 Related Literature

Our paper contributes to the literature on bank bailouts. Freixas (1999) shows that if the social cost of a bank's bankruptcy is too high, it might be optimal for the government to rescue the bank, leading to a "too-big-to-fail" problem. Diamond and Rajan (2002) show that bailouts alter available liquidity in the economy and distinguish between well-targeted bailouts (which can be beneficial) and poorly-targeted bailouts that can lead to systemic crisis. Gorton and Huang (2004) argue that there is a potential role for governments to provide liquidity through bailouts to reduce the problem of agents hoarding liquidity inefficiently. Leitner (2005) and David and Lehar (2011) show that interbank linkages can be optimal ex ante because they act as a commitment device to facilitate mutual private sector bailouts. Similarly, Rogers and Veraart (2013) analyze the incentive and ability of banks to rescue failing banks to avoid interbank contagion. By contrast, we investigate the effect of public government bailouts on the incentives of banks to create such liabilities. Finally, Niepmann and Schmidt-Eisenlohr (2013) study the incentives of governments to bail out banks when there are international spillover effects.

Moreover, our paper adds to the literature on interbank network formation and contagion. Pioneering work by Allen and Gale (2000) shows that banks can co-insure one another through an interbank market against liquidity shocks as long as these shocks are not perfectly correlated. This theme has since been explored by many other papers. Brusco and Castiglionesi (2007) show that interbank markets, while leading to an increase of the expected social welfare, may also decrease financial stability due to risk-shifting incentives. Dasgupta (2004) and Babus (2013) determine the optimal level of interconnectedness when interbank deposits can be used by banks to hedge against shocks but simultaneously expose them to the risk of contagion. Freixas and Holthausen (2005) analyze the scope of international interbank market integration when cross-border information about banks is less precise than home country information. Zawadowski (2013) analyzes how banks use OTC contracts to hedge their portfolio risks. Finally, Farboodi (2015) develops a model in which excessive interbank exposure (compared with the socially optimal exposure) emerges as a result of incentives to capture intermediation profits. In particular, each intermediary receives a fraction of the positive net present value of the investment opportunities of the final borrower bank. In our model, banks also benefit from intermediation. However, this intermediation benefit is attributable to the increased value of the banks' government guarantees and is not the result of a redistribution of the returns of real investments.

Furthermore, our paper is related to the literature on the effects of government guarantees on bank behavior, which shows that the time-inconsistency of bank rescue decisions might give banks an incentive to herd by investing in highly correlated portfolios to increase their bailout probability. In particular, Acharya and Yorulmazer (2007, 2008a) show that banks can increase their own bailout probability when they invest in a manner such that they will jointly default. In this case, there are no survivor banks that could acquire the failed banks. Therefore, if banks have a high portfolio correlation, the government is incentivized to rescue failed banks to avoid inefficient bank bankruptcies.[7] Farhi and Tirole (2012) describe a similar mechanism. In their model policymakers are content to incur the "fixed cost" associated with an intervention only when a sufficient number of financial institutions are simultaneously exposed to a shock, which again incentivizes banks to correlate their risk exposures. Related to these findings, we show that banks also have an incentive to become highly interconnected and to herd because this allows them to optimally exploit the government guarantees of *other* banks.

Moreover, herding incentives can also arise from mechanisms not related to government guarantees. Acharya (2009) shows that herding can result from a reduction in the aggregate supply of capital during a recession when banks are protected by limited liability. Acharya and Yorulmazer (2008b) show that banks might have an incentive to herd because herding minimizes the impact of information contagion. Furthermore, herding can occur due to reputation concerns of managers (see, for example, Rajan 1994 and Scharfstein and Stein 1990).

Our paper is also related to several empirical contributions. Iyer and Peydro (2011) find evidence that interbank linkages can lead to financial contagion, which validates the "too-interconnected-to-fail" concerns. Moreover, in line with the predictions of our model, there is ample evidence that banking networks are both highly dense and highly concentrated and resemble a core periphery network (see Soramäki et al. 2007, Minoiu and Reyes 2013, Mueller 2006, Wells 2004).[8] Furthermore, as predicted by our model, superfluous interbank liabilities can be observed both bilaterally between banks (e.g., Craig and Von Peter 2014, Wetherilt et al. 2010) and throughout the entire financial system (e.g., Heijmans et al. 2008).[9] Consistent with our results, empirical studies analyzing the extent to which banks engage in herding behavior find that banks tend to herd more when economic conditions are less favorable, when the health of the banking industry is rather weak, and when they are systemically

[7] More generally, banks can increases the probability of a joint default either through a coordinated investment in certain assets or due to diversification motives, which also leads to situations in which banks have a very high portfolio correlation (see Wagner 2010).

[8] In addition, there is also high interconnectedness on other interbank markets. See, e.g., Markose et al. (2012) for the Credit Default Swaps (CDS) market.

[9] Superfluous interbank liabilities are interbank transactions that do not reallocate funds but only serve to increase total bank exposure.

important (Bonfim and Moshe 2012, Liu 2011, Stever and Wilcox 2007). Cai et al. (2014) show that, in fact, a larger overlap of banks' loan portfolios makes them greater contributors to systemic risk, which highlights the importance of analyzing banks' incentives to increase their portfolio correlations.

3 Setup

We consider an economy that consists of two time periods $t = 0$ and $t = 1$ and N different countries, which are denoted by the index i, where $i \in \{1, ..., N\}$. In each country, there is a bank (protected by limited liability) with an equity endowment of e at $t = 0$ and a government which potentially bails out its domestic bank in case of a bank failure. Furthermore, each country contains a continuum of creditors that can either lend to the domestic bank at $t = 0$ or invest in a risk-free asset that transfers one unit of capital to the next period. In total, the creditors in each country are endowed with c units of capital at $t = 0$. For simplicity, we normalize $e + c$ to one. The contract between the creditors and the bank in country i takes the form of a standard debt contract, that is, it specifies the interest payment C_i, and it cannot be made contingent on either the realization of the investment or the realization of the state of nature. Furthermore, we assume that banks have all the bargaining power vis-a-vis the creditors.[10] All parties are assumed to be risk neutral.

Moreover, we assume that the banks can borrow and lend on an interbank market at $t = 0$ and that interbank loans must be repaid at $t = 1$. In Section 4, we consider the case in which banks are located on a line, i.e., bank i can lend to bank $i + 1$ at $t = 0$. Hence, the first bank can only lend – and the last bank can only borrow – on the interbank market. This specification allows us to clearly illustrate and analyze the banks' incentive to inefficiently channel funds through the interbank markets. In Section 5, we then extend the model to a fully symmetric setup in which banks are located on a circle and, thus, all banks can borrow and lend funds on the interbank market. The size of the interbank loan extended from bank i to bank $i + 1$ is denoted as $b_{i,i+1}$ with an interest rate of $B_{i,i+1}$. Although we consider only interbank loans in our model, the mechanisms presented in this paper also hold for other types of interbank exposures, such as credit derivatives, etc.[11]

Furthermore, at $t = 0$, each bank can invest in a risky, scalable investment possibility. This real asset generates a risky return $\widetilde{A}$, that is, a high return A per unit of invested capital with probability λ_a, and zero with probability $(1 - \lambda_a)$. The asset matures at $t = 1$ and has a positive NPV, i.e., $\lambda_a A > 1$. The investment of bank i in the risky real asset is denoted a_i.

Regarding the banks' investment in the real asset, we assume that the banks can alter their mutual portfolio correlation (i.e., the probability that the banks' real investments are successful or unsuccessful at the same time). In particular, bank i can choose the joint probability, $\rho_{i,i+1} \in [0, \lambda_a]$, that bank i

[10] Shifting the bargaining power to the creditors does not affect bank behavior qualitatively, it only changes the distribution of the gains from exploiting the government guarantees by artificially channeling funds through the interbank market. If creditors have the bargaining power, they will increase their interest rate until the bank owners just break even in expectations.

[11] In fact, creating interbank exposure with derivatives posts even lower requirements, as the banks do not necessarily need liquidity to lend to one another to create interbank exposures but rather can create exposure "out of thin air".

and bank $i+1$ both have a successful risky asset at the same time. Because $\rho_{i,i+1}$ is the joint success probability, it follows that $(\lambda_a - \rho_{i,i+1})$ is the joint probability that only one of the banks is successful, and $(1 - 2\lambda_a + \rho_{i,i+1})$ is the joint probability that both banks' investments fail at the same time.[12] Hence, for $\rho_{i,i+1} = \lambda_a$, the banks' portfolios are perfectly positively correlated, for $\rho_{i,i+1} = \lambda_a^2$, they are uncorrelated, and for $\rho_{i,i+1} = 0$, they are strongly negatively correlated (i.e., both banks cannot be successful simultaneously). Table 1 depicts the respective joint probabilities.

		$\widetilde{A}_i$	
		A	0
$\widetilde{A}_{i+1}$	A	$\rho_{i,i+1}$	$\lambda_a - \rho_{i,i+1}$
	0	$\lambda_a - \rho_{i,i+1}$	$1 - 2\lambda_a + \rho_{i,i+1}$

Table 1: Joint probabilities for the banks' return realizations of the risky asset (i.e., bank i and bank $i+1$)

Moreover, we assume that the governments in the N different countries provide implicit public guarantees for their respective domestic bank. In particular, if bank i defaults on its debt liabilities, we assume that the government of country i settles the bank's liabilities with probability $\alpha \in (0,1)$. Furthermore, we assume that the goverment becomes the residual claimant and thus receives any payments paid to the insolvent bank after having bailed out its domestic bank, which gives a lower bound for the banks' incentive to channel funds through the interbank market. If the equityholders would remain residual claimants and thus receive any payments that the bank receives after the bailout, this would reinforce the banks' incentive to increase their interbank exposure.

If several banks are illiquid and/or insolvent simultaneously, we assume that insolvent banks are rescued before illiquid banks and if more than one bank is illiquid or insolvent simultaneously, the banks are saved in a random order (each bank with equal probability of being the first to be considered for a bailout).[13] Furthermore, in the main analysis, we assume that the bankruptcy costs for the banks are equal to zero.[14] Finally, if bank i is not able to meet all its debt liabilities, but it has a positive liquidation value, we assume that its liquidation value is shared pro-rata, that is, the creditors of bank i receive the share $\delta_{i,c} = cC_i/(cC_i + b_{i-1,i}B)$ and the lender bank $\delta_{i,b} = b_{i-1,i}B/(cC_i + b_{i-1,i}B)$ of bank i's liquidation value.[15]

In the main analysis in Sections 4 and 5 we consider the case in which the governments' bailout budgets are unlimited and consider α to be exogenous and constant. These assumptions also imply that the banks' bailout probabilities are independent.

[12] Restricting the success probability to $\lambda_a \leq 1/2$ ensures that all joint probabilities are greater than zero for all $\rho_{i,i+1} \in [0, \lambda_a]$.

[13] Illiquid banks are banks that still have claims outstanding that can enable them to fully settle their creditors' claims, while insolvent banks are banks that are definitively not able to fully repay their creditors.

[14] In Section 6.4, we consider the consequences of bankruptcy costs for the banks' incentive to become interconnected and in Section 8.5 in the Online Appendix, we discuss the implications of introducing private bankruptcy costs for the banks' herding incentive.

[15] The pro-rata sharing rule is the common procedure in bankruptcy proceedings.

Anecdotal and empirical evidence suggests that the likelihood of a bailout actually increases after a bank defaults (within the same country and across countries). In March 2008, U.S. authorities decided to support Bear Stearns by guaranteeing creditors' claims worth $30 billion during its acquisition by J.P. Morgan, while in September of the same year, U.S. authorities decided to allow Lehman Brothers to fail. Then, only two days later, they decided to bailout AIG with a massive rescue package, as outlined in the introduction. Even more importantly, Hett and Schmidt (2015) show empirically that bailout expectations actually increased significantly following the failure of Lehman Brothers. In particular, their study shows that market participants believed that there was a higher probability that failing U.S. banks would be bailed out after the U.S. government decided against a bailout of Lehman Brothers, compared with the perceived bailout probabilities prior to the default of Lehman Brothers. Cross-country examples include the bailouts of Dexia (headquartered in Belgium), Fortis (headquartered in the Netherlands and Belgium), the Royal Bank of Scotland (headquartered in the UK), and HSH Nordbank, Landesbank Baden-Wuerttemberg, and BayernLB (all headquartered in Germany). All these banks had significant exposure to Lehman Brothers and were bailed out by their governments shortly after the U.S. government decided not to rescue Lehman Brothers.[16]

We relax the assumptions of unlimited bailout budgets and independent exogenous bailout probabilities in Section 6. First, in Section 6.1, we relax the assumption that the banks' bailout probabilities are independent and show that if the bailout probability increases for a bank's counterparties after the bank fails, as suggested by the empirical and anecdotal evidence, the banks' incentive to become interconnected would be reinforced. If, however, the likelihood of being rescued would decrease for the counterparties of a bank after the bank fails and is not rescued by its government, banks' incentive to become interconnected decreases (but does not disappear). Second, in Section 6.2, we relax the assumption that governments bailout budgets are unlimited and consider the case in which the governments' bailout budget is fixed to a maximum of g. Third, in Section 6.3, we relax the assumption that the bailout probabilities are symmetric and consider asymmetric bailout probabilities. Finally, in the welfare analysis in Section 6.4, we present a micro-foundation for the bailout probability α by endogenizing the governments' decision whether to rescue a bank.

4 Channeling funds through the interbank market

We begin by showing that, given the existence of government guarantees, banks have an incentive to channel funds through the interbank network. In this section, for ease of illustration, we will consider the case in which banks are located on a line, that is, bank i can lend to bank $i+1$ at $t=0$. Therefore, bank 1 can only lend and bank N can only borrow on the interbank market, while all other banks can both borrow and lend at the same time. Regarding the negotiation of the interbank interest rate, we assume that the lender bank has the bargaining power.[17] These assumptions allows us to clearly

[16] See Pisani-Ferry et al. (2010), "RBS admits £1bn gross exposure to Lehman Brothers" by Peter Taylor, *Telegraph*, September 18, 2008, and "Lehman-Pleite trifft Landesbanken hart" by Peter Koehler, *Handelsblatt*, September 19, 2008.

[17] All our results are qualitatively similar for all bargaining power distributions between the banks. Only the distribution of the gains from exploiting the government guarantees changes.

analyze banks' incentives to inefficiently channel funds through the interbank markets. In Section 5, we then extend the model to a setup in which banks are located on a circle, which allows all banks to both borrow and lend funds on the interbank market.

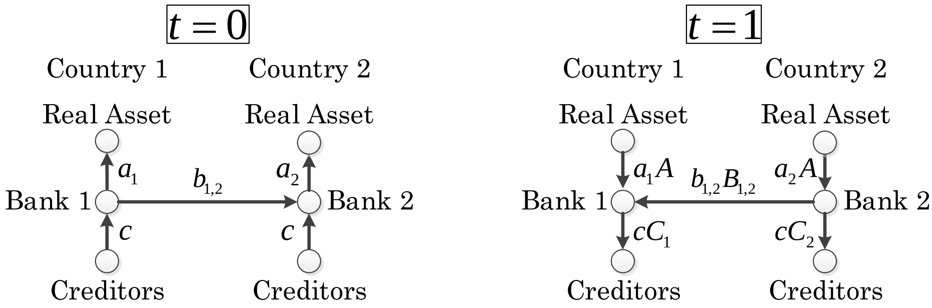

Figure 1: Cash flows for the case in which two banks are located on a line

4.1 $N = 2$ banks

First, we consider the two-country case, that is, $N = 2$. Figure 1 depicts the cash flows at $t = 0$ and the potential cash flows at $t = 1$ for this case. In the first step, we determine the expected return of bank 1 for the case in which it invests directly in the risky asset (i.e., the autarky case). In the second step, we then show that bank 1 can increase its expected return by instead investing its funds in bank 2, and determine whether bank 2 has an incentive to borrow from bank 1. In Section 4.2, we extend this analysis to a N country setting.

4.1.1 Bank 1 - Investment in the real asset.

As described above, bank 1 has an equity endowment of e and can also take on debt from its creditors. Therefore, if bank 1 decides to directly invest its funds in the real asset (i.e., $a_1 = e + c$), its expected return becomes

$$\Pi_{1,a} = \lambda_a \left[(e + c)A - cC_{1,a} \right]. \tag{1}$$

The investment in the risky asset is successful with probability λ_a, in which case the bank receives the residual asset return after having repaid its creditors. To borrow the creditors' endowment c, the bank must offer an interest rate that makes the creditors at least indifferent between lending to the bank and investing in their outside option.[18] When the bank's investment in the risky asset is unsuccessful, the government of country 1 settles the creditors' claims with probability α. Hence, the participation constraint of bank 1's creditors becomes

$$\lambda_a cC_{1,a} + (1 - \lambda_a)\alpha cC_{1,a} \geq c. \tag{2}$$

[18]Borrowing the maximum possible amount from the creditors is optimal for the bank because their required expected return is lower than the expected return of the risky asset.

As the participation constraint must be binding in the optimum, it follows that

$$C_{1,a}^* = \frac{1}{\lambda_a + (1 - \lambda_a)\alpha}. \tag{3}$$

Plugging the binding creditors' participation constraint from Eq. (2) into Eq. (1) yields

$$\Pi_{1,a}^* = \lambda_a A + \underbrace{(1 - \lambda_a)\alpha c C_{1,a}^*}_{=G_{1,a}^*} - c, \tag{4}$$

where we already incorporated that $e + c = 1$. Eq. (4) consist of the following terms: The bank earns in expectations $\lambda_a A$ from the real investment (first term) and repays its creditors in expectations their initial investment c (third term). Moreover, as the bank has the bargaining power vis-a-vis its creditors, it appropriates all the bailout subsidy (second term). As government 1 repays the creditors of bank 1 with probability α in case the bank fails (which happens with probability $(1 - \lambda_a)$), the expected value of the bailout subsidy for bank 1 is equal to $G_{1,a}^*$.

Plugging the interest rate from Eq. (3) into Eq. (4) and simplifying yields the maximized expected bank return in the case where bank 1 invests all its funds in the real asset

$$\Pi_{1,a}^* = \lambda_a \left[A - \frac{c}{\lambda_a + (1 - \lambda_a)\alpha} \right]. \tag{5}$$

Therefore, investing in the interbank market (i.e., lending its funds to bank 2) dominates the direct investment in the real asset if bank 1's expected return associated with investing in bank 2 equals or is greater than $\Pi_{1,a}^*$.

4.1.2 Bank 2.

Like bank 1, bank 2 is also endowed with e units of equity capital and can borrow c units of capital from the creditors in its country. If bank 2 decides to operate in autarky, that is, without borrowing on the interbank market, its maximum expected return is equal to $\Pi_{2,a}^* = \Pi_{1,a}^*$ and the value of government 2's bailout subsidy for bank 2 is equal to $G_{2,a}^* = G_{1,a}^*$ (due to the symmetric setup).

If, however, bank 2 borrows funds (denoted $b_{1,2}$) from bank 1 on the interbank market, its total investment in the real asset becomes $a_2 = e + c + b_{1,2}$. The loan size, $b_{1,2}$, is limited to bank 1's total budget, i.e., $b_{1,2} \leq 1$. Hence, when borrowing $b_{1,2} = 1$ from bank 1, bank 2's expected return becomes

$$\Pi_{2,b} = \lambda_a \left[(e + c + b_{1,2})A - cC_{2,b} - b_{1,2}B_{1,2} \right], \tag{6}$$

where $B_{1,2}$ is the interbank interest rate offered by bank 1 to bank 2. The participation constraint of bank 2's creditors becomes

$$\lambda_a c C_{2,b} + (1 - \lambda_a)\alpha c C_{2,b} \geq c \quad \Rightarrow C_{2,b}^* = \frac{1}{\lambda_a + (1 - \lambda_a)\alpha}, \tag{7}$$

where with probability α government 2 rescues bank 2 if its investment in the real asset fails. Plugging the binding creditors' participation constraint from Eq. (7) into Eq. (6) and simplifying yields bank 2's expected return and its participation constraint

$$\Pi_{2,b} = \lambda_a \left[(1 + b_{1,2})A - b_{1,2}B_{1,2}\right] + \underbrace{(1 - \lambda_a)\alpha c C^*_{2,b}}_{=G^*_{2,b}} - c \geq \Pi^*_{2,a}, \tag{8}$$

where $\Pi^*_{2,a}$ is bank 2's maximum possible expected return in autarky (which is equal to $\Pi^*_{1,a}$) and $G^*_{2,b}$ the value of the government bailout subsidy for bank 2 (where $G^*_{2,b} = G^*_{2,a}$). From Condition (8), it directly follows that bank 2 has an incentive to maximize the funds borrowed from bank 1 (i.e., to maximize $b_{1,2}$) for all $B_{1,2} \leq A$.

4.1.3 Bank 1 - Investment in the interbank market.

Next, we determine whether bank 1 can increase its expected profits by lending its funds to bank 2 instead of directly investing in the real asset (i.e., $b_{1,2} = e + c$). In this case, bank 1's expected profit becomes

$$\Pi_{1,b} = \lambda_a \left[(e + c)B_{1,2} - cC_{1,b}\right] + (1 - \lambda_a)\alpha \left[(e + c)B_{1,2} - cC_{1,b}\right]. \tag{9}$$

Therefore, bank 1's repayment probability increases if it lends its funds to bank 2 instead of directly investing in the risky asset since even if the real investment fails, bank 1 receives the interbank repayment if bank 2 is bailed out (i.e., $\lambda_a + (1 - \lambda_a)\alpha > \lambda_a$ for $\alpha > 0$).

Furthermore, if bank 1 decides to invest its funds in the interbank market, its creditors' participation constraint becomes

$$\lambda_a c C_{1,b} + (1 - \lambda_a)\left[\alpha c C_{1,b} + (1 - \alpha)\alpha c C_{1,b}\right] \geq c \quad \Rightarrow C^*_{1,b} = \frac{1}{\lambda_a + (1 - \lambda_a)\left[\alpha + (1 - \alpha)\alpha\right]}. \tag{10}$$

In particular, the creditors of bank 1 are fully repaid if either the real investment is successful (first term), or unsuccessful but either bank 1 or bank 2 is bailed out by its respective government (second term). As a result, as $\alpha \in (0, 1)$, it follows that $C^*_{1,b} < C^*_{1,a}$. Hence, if banks are partially protected by government guarantees, bank 1 can lower its financing costs by investing in the interbank market instead of investing directly in the real asset. When bank 1 lends the funds it borrowed from its creditors to bank 2, these funds are not only protected by bank 1's government guarantee but are also covered by the government guarantee of bank 2.

Bank 1 has an incentive to invest in the interbank market (i.e., lend its funds to bank 2), whenever

$$\Pi_{1,b} \geq \Pi^*_{1,a}. \tag{11}$$

Plugging the creditors' interest rate from Eq. (10) into bank 1's expected return $\Pi_{1,b}$ and solving

Condition (11) for the lowest interbank interest rate that still satisfies this condition yields

$$\underline{B}_{1,2} = \frac{\lambda_a}{\lambda_a + (1-\lambda_a)\alpha}\left[A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha}\right] + \frac{c}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\alpha\right]}. \tag{12}$$

Comparing $\underline{B}_{1,2}$ and the risky investment's return A yields the following lemma.

Lemma 1 *If banks are protected by implicit government guarantees (i.e., $\alpha > 0$), it follows that $\underline{B}_{1,2} < A$.*

Proof. Proof For $\alpha = 0$ it holds that $\underline{B}_{1,2} = A$. As both terms on the right-hand side of Eq. (12) decrease with α, it follows that $\underline{B}_{1,2} < A$ if $\alpha > 0$. ■

As for $\alpha > 0$ it holds that $\underline{B}_{1,2} < A$, bank 1 is willing to lend to bank 2 at an interest rate that is lower than the return on its real investment when bank 2 is protected by government guarantees. As bank 2 has an incentive to borrow the maximum possible amount (i.e., $b_{1,2} = e + c = 1$) from bank 1 for all $B_{1,2} \leq A$ (follows from $d\Pi_{2,b}/db_{1,2} \geq 0$ for $B_{1,2} \leq A$), both banks agree to proceed with the interbank loan for all $B_{1,2} \in [\underline{B}_{1,2}, A]$. Because the lender bank has the bargaining power, bank 1 will set the interest rate to $B_{1,2} = A$ such that bank 2's expected return is equal to its expected return in the autarky case (i.e., $\Pi^*_{2,b} = \Pi^*_{2,a}$).

Plugging the binding creditors' participation constraint from Eq. (10) and $B_{1,2} = A$ into Eq. (9) yields for bank 1's expected return

$$\Pi^*_{1,b} = \lambda_a A + \underbrace{(1-\lambda_a)\alpha A + (1-\lambda_a)(1-\alpha)\alpha c C^*_{1,b}}_{=G^*_{1,b}} - c. \tag{13}$$

That is, when lending its funds to bank 2, bank 1 earns in expectations $\lambda_a A$ from the investment of its funds into the real asset made by bank 2 (first term of Eq. (13)) and repays its creditors in expectations their initial investment c (last term of Eq. (13)). Moreover, in addition to the bailout subsidy provided by government 1, bank 1 now also benefits from the bailout subsidy provided by government 2 through its investment in bank 2. In particular, even if the real investment fails, but bank 2 is bailed out by government 2, bank 1 receives the interbank payment (second term in Eq. (13)). If bank 2 is not bailed out, but bank 1 is bailed out by government 1, the creditors of bank 1 are again fully repaid (third term in Eq. (13)). Hence, the value of the bailout subsidies for bank 1 with interbank exposure is given by $G^*_{1,b}$, which is larger than $G^*_{1,a}$ (as $A > cC^*_{1,a}$). Since the banks have the bargaining power vis-a-vis their creditors and bank 1 vis-a-vis bank 2 regarding the interbank loan interest rate, bank 1 appropriates any value increase of the bailout subsidies induced by the banks' interbank exposure.

Comparing the total value of the banks' bailout subsidies in autarky and with interbank lending shows that

$$G^*_{1,b} + G^*_{2,b} > G^*_{1,a} + G^*_{2,a} \tag{14}$$

$$(1-\lambda_a)\alpha\left[A + \frac{(1-\alpha)c}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\alpha\right]} + \frac{c}{\lambda_a + (1-\lambda_a)\alpha}\right] > (1-\lambda_a)\alpha\frac{2c}{\lambda_a + (1-\lambda_a)\alpha}, \tag{15}$$

which holds as $A > c/(\lambda_a + (1-\lambda_a)\alpha)$. Therefore, channeling bank 1's funds through bank 2 before they are invested in the real asset increases the total value of the government bailout subsidies.

Plugging $cC^*_{1,b}$ from Eq. (10) into Eq. (13) and simplifying yields for bank 1's expected return

$$\Pi^*_{1,b} = (\lambda_a + (1-\lambda_a)\alpha)\left[A - \frac{c}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\alpha\right]}\right] > \Pi^*_{1,a}. \tag{16}$$

Hence, if bank 2 is rescued with a positive probability $\alpha > 0$, lending to bank 2 increases bank 1's repayment probability compared to directly investing in the risky asset. In addition, lending to bank 2 also increases the repayment probability for bank 1's creditors and thereby decreases bank 1's financing costs. As a result, bank 1's expected return when it channels its funds through bank 2 before they are invested in the real asset (i.e., $\Pi^*_{1,b}$) is higher than in the case in which it directly invests in the real asset (i.e., $\Pi^*_{1,a}$).

This result directly follows from the Modigliani-Miller intuition. Due to the absence of bankruptcy costs and as banks have all the bargaining power vis-a-vis their creditors (i.e., they appropriate all of the value of the government bailout subsidies), banks can maximize their expected return by maximizing the total firm value (i.e., the sum of the value generated by the banks' investment in the real asset and the value of the government bailout subsidies). As interbank exposure does not affect the total amount that both banks combined invest in the real asset, but increases the value of the government bailout subsidies, bank 1 can increase its expected return by investing its funds into bank 2 compared to direct investment in the real asset (i.e., the autarky case). These findings are summarized in the following proposition.

Proposition 2 *If banks are protected by public guarantees, they have an incentive to channel their funds through the interbank market as this increases the value of the government bailout subsidies and, in turn, the banks' expected returns.*

4.2 N banks

Next, we consider the case of N banks. For brevity, we directly consider the case where banks can borrow and lend on the interbank market at the interest rate $B_{i,i+1} = A$. First, we analyze the incentives of the last bank in the lending chain (i.e., bank N). Similar to the expected return of bank 2 in Section 4.1, the expected return of bank N is given by

$$\Pi_{N,b} = \lambda_a \left[(1 + b_{N-1,N})A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha} - b_{N-1,N}A\right], \tag{17}$$

when borrowing funds from bank $N-1$. From Eq. (17), it follows that bank N is willing to borrow the maximum possible funds from bank $N-1$. When bank $N-1$ invests all its available funds into bank N (i.e., $b_{N-1,N} = 1 + b_{N-2,N-1}$), its expected return therefore becomes

$$\Pi_{N-1,b} = (\lambda_a + (1-\lambda_a)\alpha)\left[(1 + b_{N-2,N-1})A - \frac{c}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\alpha\right]} - b_{N-2,N-1}A\right], \quad (18)$$

which is larger than its expected return in autarky, $\Pi_{N-1,a} = \Pi^*_{1,a}$. The same argument holds for all other banks in the chain. Therefore, all banks can increase their expected return by borrowing the maximum possible amount from their predecessor bank and lend all funds to their successor bank, before bank N finally invests the funds in the real asset. Hence, when lending to its successor bank, the expected returns for banks $1, ..., N-1$ are

$$\Pi^*_{i,b} = \lambda_i \left[(1 + i - 1)A - \frac{c}{\lambda_i + (1-\lambda_i)\alpha} - (i-1)A\right] = \lambda_i \left[A - \frac{c}{\lambda_i + (1-\lambda_i)\alpha}\right], \quad (19)$$

with

$$\lambda_i = \lambda_a + (1-\lambda_a)\sum_{k=i+1}^{N}(1-\alpha)^{N-k}\alpha, \text{ for } i \in \{1, ..., N-1\}. \quad (20)$$

Because $\Pi^*_{i,b} > \Pi^*_{i,a}$ for all $i \in \{1, ..., N-1\}$, investing in the interbank market strictly dominates the direct investment in the real asset for all banks $1, ..., N-1$. Moreover, bank N's expected return becomes

$$\Pi^*_{N,b} = \lambda_a \left[(1 + N - 1)A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha} - (N-1)A\right] = \lambda_a \left[A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha}\right] = \Pi^*_{N,a}. \quad (21)$$

From Eqs. (19) and (20), it follows that if banks are protected by implicit government guarantees, the repayment probability of bank i increases with the number of government-protected intermediary banks that are between bank i and the bank that finally invests the funds in the real asset (i.e., bank N). Each implicitly-insured bank through which the funds are channeled adds additional insurance coverage due to its implicit government guarantee, which gives banks an incentive to become highly interconnected and to create long intermediation chains. These findings yield the following proposition.

Proposition 3 *If governments provide banks with implicit government guarantees through bailout possibilities, banks can increase their expected return by increasing the number of implicitly-insured banks through which their funds are channeled before being invested in a real asset.*

5 Banks located on a circle

In this section, we extend the model to a fully symmetric case in which all banks are able to borrow and lend funds on the interbank market, that is, two identical banks located on a circle.[19] For simplicity, we assume that banks can borrow and lend on the interbank market at the interest rate $B_{1,2} = B_{2,1} = B$

[19]We consider the case in which more than two banks are located on a circle in Section 8.1 in the Online Appendix and show that the results derived in this section also hold for longer intermediation circles.

and we assume from now on an investment limit equal to one for the real asset. Hence, bank i's budget constraint becomes

$$e + c + b_{j \neq i,i} = a_i + b_{i,j \neq i}, \tag{22}$$

where $a_i \leq 1$. The left hand side of Condition (22) is the banks' sources of funds (i.e., equity and loans from the creditors and the other bank) and the right hand side the uses of funds (i.e., the investment in the real asset and the loan to the other bank). Figure 2 depicts the cash flows at $t = 0$ and the potential cash flows at $t = 1$.

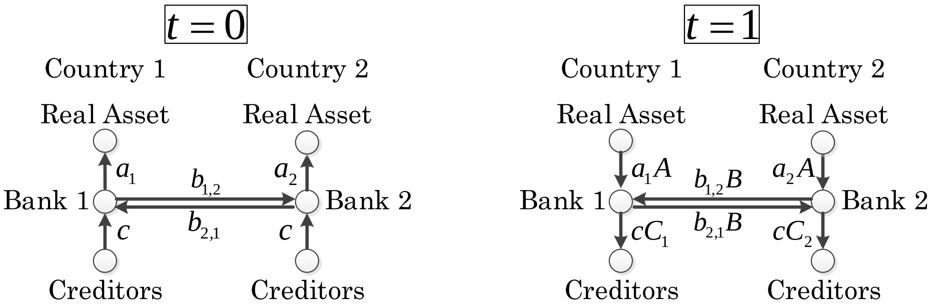

Figure 2: Cash flows for the case in which two banks are located on a circle

In the following, we consider the case in which a bank always remains solvent as soon as its real investment is successful and fails otherwise. In Section 8.2 in the Online Appendix, we show that the results and intuition derived in this section hold for all possible cases. With a positive interbank exposure, there are nine different outcomes (depending on the investment returns and whether the banks are bailed out or not), depicted in Table 2.

	Probability	$\widetilde{A}_1$	$\widetilde{A}_2$	*Bailout*	*Creditor* 1	*Creditor* 2	*Bank* 1	*Bank* 2
S_1	$\rho_{1,2}$	A	A	No bailout needed	$cC_{1,b}$	$cC_{2,b}$	solvent	solvent
S_2	$(1-2\lambda_a+\rho_{1,2})\alpha^2$	0	0	Both banks are bailed out	$cC_{1,b}$	$cC_{2,b}$	insolvent	insolvent
S_3	$(1-2\lambda_a+\rho_{1,2})(1-\alpha)\alpha$	0	0	Only bank 1 is bailed out	$cC_{1,b}$	$\delta_{2,c}b_{2,1}B$	insolvent	insolvent
S_4	$(1-2\lambda_a+\rho_{1,2})(1-\alpha)\alpha$	0	0	Only bank 2 is bailed out	$\delta_{1,c}b_{1,2}B$	$cC_{2,b}$	insolvent	insolvent
S_5	$(1-2\lambda_a+\rho_{1,2})(1-\alpha)^2$	0	0	No bank is bailed out	0	0	insolvent	insolvent
S_6	$(\lambda_a-\rho_{1,2})\alpha$	A	0	Bank 2 is bailed out	$cC_{1,b}$	$cC_{2,b}$	solvent	insolvent
S_7	$(\lambda_a-\rho_{1,2})\alpha$	0	A	Bank 1 is bailed out	$cC_{1,b}$	$cC_{2,b}$	insolvent	solvent
S_8	$(\lambda_a-\rho_{1,2})(1-\alpha)$	A	0	Bank 2 is not bailed out	$cC_{1,b}$	$\delta_{2,c}b_{2,1}B$	solvent	insolvent
S_9	$(\lambda_a-\rho_{1,2})(1-\alpha)$	0	A	Bank 1 is not bailed out	$\delta_{1,c}b_{1,2}B$	$cC_{2,b}$	insolvent	solvent

Table 2: Possible states for the case in which banks are located on a circle

With two identical banks located on a circle, bank i's expected return is given by

$$\begin{aligned}\Pi_{i,b} = \rho_{1,2} &\left[a_i A + b_{i,j\neq i}B - cC_{i,b} - b_{j\neq i,i}B\right] \\ &+ (\lambda_a - \rho_{1,2})\left[a_i A + \alpha b_{i,j\neq i}B + (1-\alpha)\delta_{j\neq i,b}b_{j\neq i,i}B - cC_{i,b} - b_{j\neq i,i}B\right].\end{aligned} \tag{23}$$

As described in Section 3, the coefficients $\delta_{i,b}$ and $\delta_{i,c}$ are the result of country i's sharing rule during bankruptcy proceedings. In particular, the creditors of bank i receive the share $\delta_{i,c} = cC_{i,b}/(cC_{i,b} +$

$b_{j\neq i,i}B$) and the lender bank receives $\delta_{i,b} = b_{j\neq i,i}B/(cC_{i,b} + b_{j\neq i,i}B)$ of bank i's liquidation value.

Eq. (23) consist of the following parts. The first line represents the case where both banks are successful, which happens with probability $\rho_{1,2}$. In this case, bank i receives the return from its real investment and the loan repayment from bank $j \neq i$ and has to repay its creditors and bank $j \neq i$. With probability $(\lambda_a - \rho_{1,2})$, bank i's investment in the real asset is successful, but bank $j \neq i$'s investment in the real asset fails. In this case, which is given in the second line of Eq. (23), bank i receives the full loan repayment of bank $j \neq i$ only if this bank is rescued by government $j \neq i$, which occurs with probability α. If bank $j \neq i$ fails and is not rescued (which happens with probability $(1 - \alpha)$), bank i receives only the fraction $\delta_{j\neq i,b}$ of its own loan payment $b_{j\neq i,i}B$ that it paid to bank $j \neq i$, as these funds are divided among all creditors of bank $j \neq i$ on a pro-rata basis.

From Eq. (23), it follows that the banks' expected cash flow in success states increases with $\rho_{1,2}$ for $\alpha \in (0, 1)$. In particular, if the banks' portfolio correlation increases, the likelihood that the banks receive an interbank repayment in the states in which they are solvent (i.e., when their real investment is also successful) also increases. Hence, the banks' portfolio correlation has a positive effect on the banks' expected returns through this channel.

Next, we determine the interest rate that is required to incentivize the banks' creditors to lend their endowment to the respective bank. The participation constraint of the creditors of bank i is given by

$$\begin{aligned}
&\rho_{1,2}cC_{i,b} + (\lambda_a - \rho_{1,2})\, cC_{i,b} + (\lambda_a - \rho_{1,2})\left[\alpha cC_{i,b} + (1-\alpha)\delta_{i,c} b_{i,j\neq i}B\right] \\
&+(1 - 2\lambda_a + \rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{i,b} + (1-\alpha)\alpha\delta_{i,c}b_{i,j\neq i}B\right) + \frac{1}{2}\left(\alpha^2 cC_{i,b} + (1-\alpha)\alpha cC_{i,b} + \alpha(1-\alpha)\delta_{i,c}b_{i,j\neq i}B\right)\right] \\
&\geq c. \qquad (24)
\end{aligned}$$

The creditors of bank i receive full repayment in all states in which either bank i's real investment is successful (first two terms of Constraint (24)) or bank i is rescued. If only bank i fails (third term), bank i is rescued with probability α by government i, in which case its creditors are fully repaid. If bank i is not rescued, its creditors are only partially repaid as they receive the fraction $\delta_{i,c}$ of the interbank repayment $b_{i,j\neq i}B$ from bank $j \neq i$.

Moreover, with probability $(1 - 2\lambda_a + \rho_{1,2})$, both banks' real investments fail and both banks are thus insolvent as a consequence (fourth term in Eq. (24)). Therefore, governments decide upon bailing out their respective bank in a random order and thus the probability of being the first to be considered for a bailout is 1/2 for both banks. If bank i is considered first for a bailout (first half of fourth term), the creditors of bank i are only fully repaid if bank i is bailed out. If bank i is not bailed out, but bank $j \neq i$ is bailed out, the creditors of bank i again receive a partial repayment due to the interbank repayment (i.e., $\delta_{i,c}b_{i,j\neq i}B$). That is, bank $j \neq i$ repays $b_{i,j\neq i}B$ to bank i, of which the creditors of bank i receive $\delta_{i,c}b_{i,j\neq i}B$ and $\delta_{i,b}b_{i,j\neq i}B$ is paid back to bank $j \neq i$ (and thus received by government $j \neq i$, which is now the residual claimant of bank $j \neq i$ after the bailout).[20] If bank $j \neq i$ is considered

[20] This assumption yields a lower bound for the incentive to channel funds through the interbank market. If the equityholders would receive any payments that the bank receives after the bailout, the banks' incentive to increase their

first for a bailout (second half of fourth term), the creditors of bank i are fully repaid if either both banks are bailed out or if only bank i is bailed out. If bank i is not bailed out, but bank $j \neq i$ is bailed out, the creditors of bank i again receive the fraction $\delta_{i,c}$ of the interbank repayment $b_{i,j\neq i}B$.

Comparing Constraint (24) with the autarky case (see Constraint (2)) shows that the interbank loan provides an additional hedge for bank i's creditors. Simplifying the binding Condition (24) yields

$$C^*_{i,b} = \frac{1}{\lambda_a + (1-\lambda_a)\alpha} - \frac{(1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha\right]}{c(\lambda_a + (1-\lambda_a)\alpha)}\delta_{i,c} b_{i,j\neq i}B < C^*_{i,a}, \tag{25}$$

which is lower than $C^*_{i,a}$ due to the additional hedge for the bank's creditors provided by the interbank network.

Next, we first determine the banks' optimal portfolio correlation given interbank exposure and then, in a second step, their optimal amount of interbank borrowing and lending.

5.1 Portfolio correlation

Taking the implicit derivative of $C^*_{i,b}$ with respect to $\rho_{1,2}$ yields[21]

$$\frac{dC^*_{i,b}}{d\rho_{1,2}} = \frac{1}{c}\frac{(1-\alpha)^2\delta_{i,c} b_{i,j\neq i}B}{\lambda_a + (1-\lambda_a)\alpha + (1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha\right]\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b} + b_{j\neq i,i}B)^2}} > 0. \tag{26}$$

As the portfolio correlation between banks increases, the value of the creditors' additional hedge provided by the interbank loan decreases. As a result, the creditors' interest rate $C^*_{i,b}$ increases with $\rho_{1,2}$ because with a higher portfolio correlation, it is less likely that bank i's creditors receive at least a partial repayment in the event that bank i's investment in the real asset fails. Therefore, through this second channel, the banks' portfolio correlation has a negative effect on the banks' expected return because a higher portfolio correlation leads to a higher creditor interest rate and, thus, to higher financing costs.

Bank i's optimization problem is thus to maximize Eq. (23) such that the bank's budget constraint from Eq. (22) and the participation constraint of the bank's creditors from Eq. (24) are satisfied. Incorporating the binding budget and participation constraint into Eq. (23) and taking the derivative of $\Pi_{i,b}$ with respect to $\rho_{1,2}$ yields[22]

$$\frac{d\Pi_{i,b}}{d\rho_{1,2}} = \frac{(1-\alpha)\alpha\left[1 + (1-\lambda_a)(1-\alpha)\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b} + b_{j\neq i,i}B)^2}\right]}{\lambda_a + (1-\lambda_a)\alpha + (1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha\right]\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b} + b_{j\neq i,i}B)^2}}\delta_{i,c} b_{i,j\neq i}B > 0. \tag{27}$$

From Eqs. (27), it follows that, given implicit government guarantees (i.e., $\alpha \in (0,1)$) and interbank exposure, the banks will maximize their portfolio correlation by choosing $\rho^*_{1,2} = \lambda_a$.

interbank exposure would be reinforced.

[21] Please see Section 9.1 in the Online Appendix for the derivation of Eq. (26).

[22] Please see Section 9.2 in the Online Appendix for the derivation of Eq. (27).

This result also follows from analyzing the effect of a change in the banks' portfolio correlation on the value of the governments' bailout subsidies. As the banks' have the bargaining power vis-a-vis their creditors (i.e., they appropriate all of the value of the government bailout subsidies) and the portfolio correlation does not affect the value generated by the banks' real investments, the portfolio correlation that maximizes the value of the government bailout subsidies also maximizes the banks' expected returns. Plugging the binding creditors' participation constraints from Condition (24) into the banks' combined expected return yields, after simplifying,

$$\Pi_{1,b} + \Pi_{2,b} = \lambda_a(a_1 + a_2)A + \underbrace{(1-\lambda_a)\alpha(cC^*_{1,b} + cC^*_{2,b}) + (1 - 2\lambda_a + \rho_{1,2})\alpha(\delta_{1,c}b_{1,2}B + \delta_{2,c}b_{2,1}B)}_{=G^*_{1,b}+G^*_{2,b}} - 2c. \tag{28}$$

That is, the banks earn in expectations both $\lambda_a a_i A$ from their investment into the real asset and repay their creditors in expectations their initial investment (i.e., $2c$). Furthermore, when bank i defaults, it is bailed out with probability α by government i, in which case its creditors are repaid (second term in Eq. (28)). Finally, if both banks default at the same time and only one bank is bailed out (third term in Eq. (28)), the other bank' creditors also receive a partial repayment due to the banks' interbank exposure.

From Eq. (28) it follows that the total value of the bailout subsidies increases with $\rho_{1,2}$ as $C^*_{1,b}$ and $C^*_{2,b}$ (see Eq. (26)) and the third term in Eq. (28) increase with $\rho_{1,2}$. These findings yield the following proposition.

Corollary 1 *If banks are protected by implicit government guarantees and given interbank exposure, they have an incentive to maximize their portfolio correlation, that is, choose $\rho^*_{1,2} = \lambda_a$, since this maximizes the value of the government bailout guarantees, and, in turn, their expected return.*

The intuition for this result is as follows: Increasing the portfolio correlation has both benefits and costs (i.e., higher gross returns in success states vs. higher financing costs). Without government guarantees, these benefits and costs exactly offset each other ($d\Pi_{i,b}/d\rho_{1,2} = 0$ for $\alpha = 0$).

This changes when bank creditors are at least partially protected by government guarantees. The benefit of increasing the portfolio correlation, that is, bank i's higher interbank repayment probability in success states, is the same with and without government guarantees for bank i's debt liabilities. However, with government guarantees, the disadvantage of higher portfolio correlation in the form of higher financing costs is mitigated. In particular, because the creditors of bank i receive full repayment in the case of a bailout of bank i in any case, they do not value interbank repayments in states in which bank i fails and is rescued.

Hence, implicit public guarantees decrease the value of the additional hedge for the banks' creditors provided by possible interbank repayments and, as a result, having negatively correlated portfolios does not reduce the creditors' interest rate as much as it does without government guarantees. Because the advantage of higher portfolio correlation remains the same as it is with no government guarantees, and the disadvantage is mitigated, banks' incentives shift toward more portfolio correlation. Therefore, given interbank exposure and public guarantees, banks benefit less from co-insuring each other by investing in negatively correlated assets and hence choose perfectly correlated investments, that is, $\rho^*_{1,2} = \lambda_a$.[23]

This finding is related to Acharya and Yorulmazer (2007, 2008a) and Farhi and Tirole (2012), who have pointed out that the time-inconsistency of bank rescue decisions might give banks an incentive to invest in correlated asset. In their models, the welfare loss is convex in the number of jointly defaulting banks and thus herding behavior increases the likelihood of a government intervention during crisis episodes.

With $\rho^*_{1,2} = \lambda_a$ the binding participation constraint of bank i's creditors becomes

$$\lambda_a c C^*_{i,b} + (1-\lambda_a)\left[\alpha c C^*_{i,b} + (1-\alpha)\alpha\delta_{i,c} b_{i,j\neq i} B\right] = c, \tag{29}$$

and plugging $\rho^*_{1,2} = \lambda_a$ and the creditors' interest rate $C^*_{i,b}$ into Eq. (23) yields for bank i's expected return

$$\Pi_{i,b} = \lambda_a \left[a_i A + b_{i,j\neq i} B - \frac{c}{\lambda_a + (1-\lambda_a)\alpha} - b_{j\neq i,i} B\right] + \frac{\lambda_a(1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha}\delta_{i,c} b_{i,j\neq i} B. \tag{30}$$

5.2 Interbank exposure

Next, we determine the level of interbank exposure that maximizes the banks' expected return. Incorporating the banks' budget constraints from Eq. (22) and taking the derivative of Eq. (30) with respect to bank i's interbank loan size to bank $j \neq i$ yields[24]

$$\frac{d\Pi_{i,b}}{db_{i,j\neq i}} = \frac{\lambda_a(1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha + (1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i} B b_{j\neq i,i} B}{(cC_{i,b} + b_{j\neq i,i} B)^2}}\delta^2_{i,c} B > 0. \tag{31}$$

From Eq. (31) it follows that the banks' expected return always increases with the banks' interbank exposure. This is due to the fact that interbank exposure increases the expected repayment of a bank's creditors by exploiting the other bank's implicit bailout guarantee. In particular, when only one bank is bailed out by its government, the creditors of the bank that was not rescued by its government also receive a fraction of these bailout funds. Moreover, the amount paid to the creditors of the bank that was not rescued increases with the banks' interbank exposure. As a result, banks always have an incentive to lend and borrow more on the interbank market and for any positive interbank exposure it holds that $\Pi_{i,b} > \Pi^*_{i,a}$.[25]

This result also follows from analyzing the effect of interbank exposure on the value of the bailout

[23] Note that, when banks choose perfectly correlated investments, they always default in the same states, which eliminates the risk of contagious defaults. A potential counter-incentive can arise from bankruptcy costs since such costs create the opportunity for diversification benefits. We consider this possibility in Section 8.5 in the Online Appendix.

[24] Please see Section 9.3 in the Online Appendix for the derivation of Eq. (31).

[25] As shown in Section 6.2, in the case where government guarantees are limited, banks only have an incentive to increase their interbank exposure as long as the governments bailout budgets are not fully exhausted.

subsidies provided by the governments in countries 1 and 2. Again, while the banks' interbank exposure does not affect the expected return generated by the banks' real investments, it affects the value of the government bailout subsidies. Since banks are the residual claimants and have the bargaining power vis-a-vis their creditors, the level of interbank exposure that maximizes the value of the government bailout subsidies thus also maximizes the banks' expected return. Using the binding creditors' participation constraints from Eq. (29) and the banks' budget constraints from Eq. (22), the banks' expected returns from Eq. (30) can be rewritten as

$$\Pi_{i,b} = \lambda_a A + \underbrace{(1-\lambda_a)\alpha c C^*_{i,b} + (1-\lambda_a)\alpha\delta_{i,c} b_{i,j\neq i} B}_{=G^*_{i,b}} - c. \tag{32}$$

That is, bank i earns in expectations $\lambda_a A$ from the investment of its funds into the real asset and repays its creditors in expectations their initial investment c. In case the banks' real investment fails, government i bails out bank i with probability α, in which case the creditors of bank i are fully repaid (second term in Eq. (32)). In addition, due to its interbank exposure, bank i also benefits from the government bailout guarantee of bank $j \neq i$ (third term in Eq. (32)). Comparing the value of the government bailout subsidies for bank i in the case where the banks' engage in interbank lending (i.e., $G^*_{i,b}$) to its value in the autarky case, that is, without interbank exposure (i.e., $G^*_{i,a}$) shows that

$$G^*_{i,b} = (1-\lambda_a)\alpha c C^*_{i,b} + (1-\lambda_a)\alpha\delta_{i,c} b_{i,j\neq i} B > G^*_{i,a} = (1-\lambda_a)\alpha\frac{c}{\lambda_a + (1-\lambda_a)\alpha}, \tag{33}$$

which is true as $\lambda_a + (1-\lambda_a)\alpha^2 > 0$. Hence, the value of the government guarantees and thus the banks' expected return is higher with interbank exposure. These findings yield the following proposition.

Proposition 4 *If banks are located on a circle (and thus all banks are able to borrow and lend on the interbank market), they always have an incentive to lend and borrow more on the interbank market when they are protected by unlimited implicit government guarantees as this increases the value of these guarantees. Moreover, if banks are located on a circle there is no endogenous limit for their interbank exposure.*

While banks also always have an incentive to channel more funds through the interbank market when being located on a line, the banks' interbank exposure is limited by their budget constraints in this case and thus interbank borrowing is limited to the total endowment of the preceding banks on the lending chain. When banks' are located on a circle and are thus all able to lend and borrow on the interbank market, the banks' budget constraint does not limit their interbank exposure anymore. Hence, in theory, banks can increase their interbank exposure to infinity.

6 Extensions

In the following, we provide four extensions to our main model. First, we consider the case where the bailout decisions are dependent. Second, we analyze the case in which governments' bailout budgets are limited. Third, we introduce transaction costs and analyze the implications of asymmetric bailout probabilities and changes in the banks' bailout probabilities on their incentives to become interconnected. Finally, we provide a micro-foundation for the bailout probability α and analyze the banks' incentive to become interconnected if the interbank market can also be used for welfare-improving purposes. Based on this analysis, we then derive welfare implications.

6.1 Dependent government rescue decisions

In the following, we analyze the banks' incentive to inefficiently channel funds through the interbank market when the banks' bailout probabilities are not independent. We closely follow the analysis in Section 4.1. However, with regard to the banks' bailout probability, we assume from this point forward that if bank i does not lend funds to another bank, its bailout probability is equal to $\alpha_i = \alpha_{i,a}$ (autarky case). If, however, bank i lends funds to another bank, the conditional probability that bank i is rescued given that the borrower bank fails and is not bailed out becomes $\alpha_{i,b}$. Hence, if $\alpha_{i,b} > \alpha_{i,a}$ ($\alpha_{i,b} < \alpha_{i,a}$), it becomes more (less) likely that bank i is rescued if it becomes interconnected with another bank. Therefore, if bank 1 invests solely in the real asset, its expected return becomes

$$\Pi_{1,a} = \lambda_a \left[(e + c)A - cC_{1,a}\right], \tag{34}$$

and the creditors' participation constraint becomes

$$\lambda_a cC_{1,a} + (1 - \lambda_a)\alpha_{1,a} cC_{1,a} \geq c. \tag{35}$$

Plugging the binding creditors' participation constraint from Eq. (35) into Eq. (34) yields

$$\Pi^*_{1,a} = \lambda_a \left[A - \frac{c}{\lambda_a + (1 - \lambda_a)\alpha_{1,a}}\right] = \lambda_a A + \underbrace{(1 - \lambda_a)\alpha_{1,a} \frac{c}{\lambda_a + (1 - \lambda_a)\alpha_{1,a}}}_{=G^*_{1,a}} - c, \tag{36}$$

In contrast, if bank 1 decides to lend its funds to bank 2, its expected return changes to

$$\Pi_{1,b} = \lambda_a \left[(e + c)A - cC_{1,b}\right] + (1 - \lambda_a)\alpha_{2,a} \left[(e + c)A - cC_{1,b}\right], \tag{37}$$

where we already incorporated that $B_{1,2} = A$. Moreover, the creditors' participation constraint changes to

$$\lambda_a cC_{1,b} + (1 - \lambda_a) \left[\alpha_{2,a} cC_{1,b} + (1 - \alpha_{2,a})\alpha_{1,b} cC_{1,b}\right] \geq c. \tag{38}$$

Plugging the binding creditors' participation constraint from Eq. (38) into Eq. (37) yields

$$\Pi^*_{1,b} = (\lambda_a + (1-\lambda_a)\alpha_{2,a}) \left[A - \frac{c}{\lambda_a + (1-\lambda_a)\left[\alpha_{2,a} + (1-\alpha_{2,a})\alpha_{1,b}\right]} \right] \tag{39}$$

$$= \lambda_a A + \underbrace{(1-\lambda_a)\alpha_{2,a}A + (1-\lambda_a)(1-\alpha_{2,a})\alpha_{1,b}\frac{c}{\lambda_a + (1-\lambda_a)\left[\alpha_{2,a} + (1-\alpha_{2,a})\alpha_{1,b}\right]}}_{=G^*_{1,b}} - c. \tag{40}$$

Comparing Eqs. (36) and (39) shows that if $\alpha_{1,b} \geq \alpha_{1,a}$ or $\alpha_{2,a} \geq \alpha_{1,a}$, banks always have an incentive to invest in the interbank market instead of investing directly in the real asset. Hence, if the governments' rescue decisions are either unaffected by banks' interconnectedness, the bailout probability increases as banks become more interconnected, or the borrower bank has a higher bailout probability than the lender bank in autarky, banks always have an incentive to become interconnected.

The same result follows from comparing the value of the government bailout subsidies for bank 1 in the case where the banks' engage in interbank lending (i.e., $G^*_{1,b}$) to its value in the autarky case (i.e., $G^*_{1,a}$):

$$G^*_{i,b} \geq G^*_{i,a} \tag{41}$$

$$(1-\lambda_a)\left[\alpha_{2,a}A + \frac{(1-\alpha_{2,a})\alpha_{1,b}c}{\lambda_a + (1-\lambda_a)\left[\alpha_{2,a} + (1-\alpha_{2,a})\alpha_{1,b}\right]}\right] \geq \frac{(1-\lambda_a)\alpha_{1,a}c}{\lambda_a + (1-\lambda_a)\alpha_{1,a}}. \tag{42}$$

That is, if either $\alpha_{1,b} \geq \alpha_{1,a}$ or $\alpha_{2,a} \geq \alpha_{1,a}$ it always holds that $G^*_{i,b} \geq G^*_{i,a}$. Moreover, Condition (42) shows that the expected bailout subsidy in the case where banks engage in interbank lending increases with $\alpha_{i,b}$. Hence, the more negatively correlated the banks' bailout probabilities are, the higher is the value of the government bailout subsidies and thus the banks' incentives to be interconnected.

Furthermore, comparing Eqs. (36) and (39) shows that a necessary (although not sufficient) condition for $\Pi_{1,b} < \Pi_{1,a}$ is

$$\lambda_a + (1-\lambda_a)\left[\alpha_{2,a} + (1-\alpha_{2,a})\alpha_{1,b}\right] < \lambda_a + (1-\lambda_a)\alpha_{1,a} \Leftrightarrow \alpha_{2,a} + (1-\alpha_{2,a})\alpha_{1,b} < \alpha_{1,a}. \tag{43}$$

Hence, banks can only (potentially) lose the incentive to become interconnected if the probability that at least one of the banks in the intermediation chain is rescued is lower than bank 1's probability of receiving a bailout in autarky, i.e., in the case in which it does not have interbank connections.

Finally, solving $\Pi_{1,b} \geq \Pi_{1,a}$ for $\alpha_{1,b}$ yields

$$\alpha_{1,b} \geq \underline{\alpha}_{1,b} = \frac{c}{(1-\lambda_a)(1-\alpha_{2,a})A - \frac{\lambda_a(1-\lambda_a)(1-\alpha_{2,a})}{\lambda_a+(1-\lambda_a)\alpha_{2,a}}\left[A - \frac{c}{\lambda_a+(1-\lambda_a)\alpha_{1,a}}\right]} - \frac{\lambda_a + (1-\lambda_a)\alpha_{2,a}}{(1-\lambda_a)(1-\alpha_{2,a})}, \tag{44}$$

which is the threshold $\underline{\alpha}_{1,b}$ for the bailout probability above which the banks have an incentive to become interconnected. This implies that banks lose the incentive to invest in banks with significantly lower bailout probabilities when their bailout probabilities are sufficiently positively correlated (i.e., $\alpha_{1,b} < \underline{\alpha}_{1,b}$). Hence, in this case, only the bank with the lower bailout probability has an incentive to invest in the bank with the higher bailout probability but not vice versa. Moreover, solving Condition (42) for $\alpha_{1,b}$ yields the same threshold as in Condition (44). Therefore, in line with the Modigliani-Miller intuition, as soon as $G^*_{i,b} \geq G^*_{i,a}$ it holds that $\Pi_{1,b} \geq \Pi_{1,a}$. These results yield the following proposition.

Proposition 5 *Banks always have an incentive to lend to banks with higher bailout probabilities than their own (even if their bailout probabilities are perfectly positively correlated) and to banks that have independent or negatively correlated bailout probabilities. If the bailout probabilities between two banks are sufficiently positively correlated (i.e., $\alpha_{1,b} < \underline{\alpha}_{1,b}$), only the bank with the lower bailout probability has an incentive to invest in the bank with the higher bailout probability but not vice versa.*

6.2 Limited government bailout budgets

In this section, we relax the assumption that the governments' bailout budgets are unlimited. For simplicity, we again consider the two-country case where banks are located on a line, as discussed in Section 4.1. However, now we assume that the bailout budget of each government in the two countries is limited to g (which is exogenously determined by a country's spending capacity). In Section 8.4 in the Online Appendix, we analyze the case in which the banks are located in the same country and where the bailout funds that the respective government is able to provide are again limited to g, which yields qualitatively the same results as in the two-country case.

To investigate how the banks' incentive to become interconnected changes when increasing the interbank exposure potentially leads to a situation where governments are no longer able to fully bail out all banks in the economy, we again compare the banks' maximum expected return when they engage in interbank lending to the autarky case. For the comparison, we focus on the case where the bailout funds of the governments would be sufficient to bail out their respective domestic banks in autarky, i.e., $g \geq cC_{1,a} = cC_{2,a}$.

For the the two-country case where banks are located on a line, the banks' budget constraints for bank 1 and 2, respectively, are given by

$$e + c = a_1 + b_{1,2}. \tag{45}$$

$$e + c + b_{1,2} = a_2. \tag{46}$$

Moreover, for positive interbank exposure, we have to distinguish two possible cases:

- *Case (a) - $cC_{2,b} + b_{1,2}B_{1,2} \leq g$:* the interbank exposure is low enough that government 2's budget is still sufficient to fully bailout bank 2 and
- *Case (b) - $cC_{2,b} + b_{1,2}B_{1,2} > g$:* the interbank exposure is so high that bank 2's total liabilities exceed government 2's bailout budget.[26]

[26] Due to the symmetry of the setup and the additional interbank hedge for the creditors of bank 1, it always holds that $C_{1,b} < C_{1,a}$ and thus government 1 can always fully bail out bank 1 as $g \geq cC_{1,a} = cC_{2,a}$.

6.2.1 Case (a) - $cC_{2,b} + b_{1,2}B_{1,2} \leq g$:

In which states bank 1 remains solvent depends on its portfolio choice (i.e., the investment size in the real asset and the interbank loan) and the real asset's return. In particular, we have to distinguish four different sub-cases, that is, bank 1 remains solvent (a.i) only if its real investment is successful; (a.ii) only if the interbank loan is repaid; (a.iii) only if both the real asset and the interbank loan are successful; and (a.iv) as soon as one of the bank's investments (interbank loan or real asset) is successful.

For brevity, we again focus our main analysis on the case where bank 1 remains solvent only if its real investment is successful, i.e., Case (a.i). In Section 8.3 in the Online Appendix, we show that the results and the intuition derived in this section hold for all possible cases. For Case (a.i) the expected return for bank 1 becomes

$$\Pi_{1,b} = \rho_{1,2}\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (\lambda_a - \rho_{1,2})\left[a_1 A + \alpha b_{1,2}B_{1,2} - cC_{1,b}\right], \tag{47}$$

and the participation constraint of its creditors is given by

$$\begin{aligned}
&\rho_{1,2}cC_{1,b} + (\lambda_a - \rho_{1,2})\,cC_{1,b} + (\lambda_a - \rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)b_{1,2}B_{1,2}\right] \\
&+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha b_{1,2}B_{1,2}\right) + \frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)b_{1,2}B_{1,2} + (1-\alpha)\alpha cC_{1,b}\right)\right] \\
&\geq c.
\end{aligned} \tag{48}$$

With probability $\rho_{1,2}$, both banks' investment in the real asset are successful, bank 2 repays bank 1, bank 1 is solvent, and thus the creditors of bank 1 are fully repaid (first term in Eqs. (47) and (48)). With probability $(\lambda_a - \rho_{1,2})$ only the real investment of bank 1 is successful (second term in Eqs. (47) and (48)). In this state, bank 1 remains solvent (and its creditors are thus fully repaid), but it only receives the interbank repayment if bank 2 is bailed out by its government, which happens with probability α. Moreover, again with probability $(\lambda_a - \rho_{1,2})$, only bank 2's real investment is successful, in which case bank 1 always defaults (third term in Eq. (48)). The creditors of bank 1 are hence only fully repaid if government 1 bails out bank 1, which again happens with probability α. Otherwise, the creditors just receive the interbank repayment of bank 2.

Finally, with probability $(1 - 2\lambda_a + \rho_{1,2})$, both banks' real assets fail and both banks are thus insolvent (fourth term in Eq. (48)). Hence, the governments decide whether to bail out their respective bank in a random order (i.e., the banks' likelihood of being considered first for a bailout is 1/2). If bank 1 is considered first for a bailout (first half of fourth term), its creditors are only fully repaid if bank 1 is bailed out. If bank 1 is not bailed out, but bank 2 is rescued, the creditors of bank 1 receive the interbank repayment $b_{1,2}B_{1,2}$. If bank 2 is considered first for a bailout (second half of fourth term), the creditors of bank 1 are fully repaid if bank 1 is bailed out as well. If bank 2 is bailed out, but bank 1 is not, the creditors of bank 1 again receive the interbank repayment $b_{1,2}B_{1,2}$. If bank 2 is not rescued, but bank 1 is bailed out, the creditors of bank 1 are again fully repaid.

Moreover, the participation constraint of bank 2's creditors becomes

$$\begin{aligned}&\rho_{1,2}cC_{2,b}+(\lambda_a-\rho_{1,2})cC_{2,b}+(\lambda_a-\rho_{1,2})\alpha cC_{2,b}\\&+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\alpha cC_{2,b}+\frac{1}{2}\left(\alpha^2cC_{2,b}+(1-\alpha)\alpha cC_{2,b}\right)\right]\geq c\Rightarrow C^*_{2,b}=\frac{1}{\lambda_a+(1-\lambda_a)\alpha}.\end{aligned}\tag{49}$$

The creditors of bank 2 are fully repaid if bank 2 is successful (first and second term) or bank 2 fails but is bailed out (third and fourth term). Furthermore, bank 2's expected return and participation constraint is given by

$$\Pi_{2,b}=\lambda_a\left[(1+b_{1,2})A-cC^*_{2,b}-b_{1,2}B_{1,2}\right]\geq\Pi_{2,a}=\lambda_a\left[A-\frac{c}{\lambda_a+(1-\lambda_a)\alpha}\right].\tag{50}$$

From Conditions (49) and (50) it directly follows that bank 1 will set the interest rate to $B_{1,2}=A$. Next, we determine the banks' optimal portfolio correlation by taking the derivative of bank 1's expected return with respect to $\rho_{1,2}$, which yields[27]

$$\frac{d\Pi_{1,b}}{d\rho_{1,2}}=\frac{(1-\alpha)\alpha}{\lambda_a+(1-\lambda_a)\alpha}b_{1,2}A>0,\tag{51}$$

where we already incorporated that the participation constraint of the creditors of bank 1 will be binding in the optimum. Therefore, the banks will choose perfectly positively correlated portfolios, i.e., $\rho^*_{1,2}=\lambda_a$, to maximize their expected return.

Plugging $\rho^*_{1,2}=\lambda_a$ and the binding creditors' participation constraint from Eq. (48) into bank 1's expected return from Eq. (47) yields, after rearranging,

$$\Pi_{1,b}=\lambda_a\left[a_1A+b_{1,2}B_{1,2}\right]+\underbrace{(1-\lambda_a)\left[\frac{\alpha c}{\lambda_a+(1-\lambda_a)\alpha}+\frac{\lambda_a(1-\alpha)\alpha b_{1,2}B_{1,2}}{\lambda_a+(1-\lambda_a)\alpha}\right]}_{=G_{1,b}}-c.\tag{52}$$

Finally, taking the derivative of Eq. (52) with respect to $b_{1,2}$ yields

$$\frac{d\Pi_{1,b}}{db_{1,2}}=\frac{\lambda_a(1-\lambda_a)(1-\alpha)\alpha A}{\lambda_a+(1-\lambda_a)\alpha}>0,\tag{53}$$

where we incorporated already that $B_{1,2}=A$. Taking the derivative of the value of the bailout subsidies $G_{1,b}$ from Eq. (52) with respect to $b_{1,2}$ and comparing the results to Eq. (53) shows that $dG_{1,b}/db_{1,2}=d\Pi_{1,b}/db_{1,2}>0$. Hence, for Case (a), the banks have an incentive to increase their interbank exposure, that is, increase $b_{1,2}$ as long as the interbank exposure is still low enough such that $cC_{2,b}+b_{1,2}B_{1,2}\leq g$.

The intuition for this result is as follows: As long as the bailout budget of the government of the country where the borrower bank is located (i.e., government 2) is still sufficient to fully bailout the bank, the banks can increase the value of the bailout subsidies by channeling more funds through this

[27] Please see Section 9.4 in the Online Appendix for the derivation of Eq. (51).

bank. Thereby, the banks increase the potential (bailout funds) injection into the banking system in case of a default of the borrower bank (i.e., bank 2).

This yields for bank 1's expected return

$$\Pi^*_{1,b} = \lambda_a \left[A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha} \right] + \frac{\lambda_a(1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha} b_{1,2} A > \Pi^*_{1,a}, \tag{54}$$

where $\Pi^*_{1,b}$ is larger than $\Pi^*_{1,a}$ for all $b_{1,2} > 0$, while bank 2's expected return is again equal to the expected return in the autarky case, i.e., $\Pi^*_{2,b} = \Pi^*_{2,a}$.

Hence, for Case (a), the banks always choose a positive interbank exposure as bank 1's expected return with interbank exposure always exceeds its return in the autarky case due to the increased value of the government bailout subsidies.

6.2.2 Case (b) - $cC_{2,b} + b_{1,2}B_{1,2} > g$:

Next, we analyze Case (b), that is, the case in which the banks' increase the interbank exposure to a level such that bank 2's total liabilities exceed the bailout budget of government 2 (i.e., $cC_{2,b} + b_{1,2}B_{1,2} > g$). Again, we have to distinguish between four possible sub-cases. Depending on bank 1's portfolio choice and the real asset's return, bank 1 remains solvent (b.i) only if its real investment is successful; (b.ii) only if at least part of the interbank loan is repaid; (b.iii) only if both the investment in the risky asset and the interbank investment are successful; and (b.iv) as soon as one of the investments is successful.

Again, for brevity, we focus our analysis on the case where bank 1 remains solvent only if its real investment is successful, i.e., Case (b.i), and show in Section 8.3 in the Online Appendix that the results derived in this section also hold for all remaining cases. For Case (b.i), the expected return for bank 1 is given by

$$\Pi_{1,b} = \rho_{1,2} \left[a_1 A + b_{1,2} B_{1,2} - cC_{1,b} \right] + (\lambda_a - \rho_{1,2}) \left[a_1 A + \alpha \delta_{2,b} g - cC_{1,b} \right], \tag{55}$$

while bank 2's expected return and participation constraint is again the same as in Eq. (50). Moreover, the creditors' participation constraint of bank 1 becomes

$$\begin{aligned} &\rho_{1,2} cC_{1,b} + (\lambda_a - \rho_{1,2})\, cC_{1,b} + (\lambda_a - \rho_{1,2}) \left(\alpha cC_{1,b} + (1-\alpha) b_{1,2} B_{1,2} \right) \\ &+ (1 - 2\lambda_a + \rho_{1,2}) \left[\frac{1}{2} \left(\alpha cC_{1,b} + (1-\alpha)\alpha \delta_{2,b} g \right) + \frac{1}{2} \left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\delta_{2,b} g + (1-\alpha)\alpha cC_{1,b} \right) \right] \geq c, \end{aligned} \tag{56}$$

and bank 2's creditors' participation constraint is

$$\begin{aligned} &\rho_{1,2} cC_{2,b} + (\lambda_a - \rho_{1,2}) cC_{2,b} + (\lambda_a - \rho_{1,2}) \alpha \delta_{2,c} g \\ &+ (1 - 2\lambda_a + \rho_{1,2}) \left[\frac{1}{2} \alpha \delta_{2,c} g + \frac{1}{2} \left(\alpha^2 \delta_{2,c} g + (1-\alpha)\alpha \delta_{2,c} g \right) \right] \geq c. \end{aligned} \tag{57}$$

This case is similar to Case (a.i), but now if bank 2 is rescued, bank 1 is only paid $\delta_{2,b}g$ instead of the full interbank repayment. Next, we determine the banks' optimal portfolio correlation for Case (b). Incorporating the binding participation constraints and taking the derivative of bank 1's expected returns with respect to $\rho_{1,2}$ yields[28]

$$\frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha \frac{b_{1,2}B_{1,2} - \alpha\delta_{2,b}g}{\lambda_a + (1-\lambda_a)\alpha} > 0. \tag{58}$$

Therefore, the banks will again choose $\rho^*_{1,2} = \lambda_a$. Next, we determine the banks' optimal interbank exposure. Plugging $\rho^*_{1,2} = \lambda_a$ into bank 1's expected return yields

$$\Pi_{1,b} = \lambda_a \left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{59}$$

$$\textbf{PC creditors 1: } \lambda_a cC_{1,b} + (1-\lambda_a)\left[\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b}g\right] \geq c \tag{60}$$

$$\textbf{PC creditors 2: } \lambda_a cC_{2,b} + (1-\lambda_a)\alpha\delta_{2,c}g \geq c. \tag{61}$$

Incorporating the binding participation constraints and taking the derivative of Eq. (59) with respect to $b_{1,2}$ yields[29]

$$\frac{d\Pi_{1,b}}{db_{1,2}} = -\frac{\frac{\lambda_a(1-\lambda_a)\alpha^2}{\lambda_a+(1-\lambda_a)\alpha}\frac{\delta_{2,c}gA}{cC_{2,b}+b_{1,2}B_{1,2}}}{\lambda_a + (1-\lambda_a)\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}} < 0. \tag{62}$$

Hence, for Case (b), that is, the case where banks increase their interbank exposure to a level such that bank 2's total liabilities exceed the government budget, increasing the interbank exposure even further than this threshold reduces the banks' expected return. As in Case (a), the derivative of the value of the bailout subsidies with respect to $b_{1,2}$ again equals the derivative of the expected returns, given in Eq. (62). That is, also for Case (b) it holds that $dG_{1,b}/db_{1,2} = d\Pi_{1,b}/db_{1,2}$.[30]

The intuition for this result is as follows. If the total liabilities of the borrower bank already exceed the bailout budget of the respective government, channeling more funds through this bank has no effect on the size of the bailout injection in case of a default of the borrower bank. However, it shifts bailout funds from the borrower to the lender bank (i.e., from bank 2 to bank 1). Hence, for Case (b), higher interbank exposure decreases the amount of bailout funds received by the creditors of bank 2 and thus increases the interest rate $C_{2,b}$ of the creditors of bank 2. However, the increase in $C_{2,b}$ has no effect on the value of the bailout subsidy provided by government 2 since its bailout budget is already maxed out.

Moreover, an increase in the interbank exposure increases the amount received by the creditors of bank 1 and thus lowers $C_{1,b}$, which lowers the value of the bailout subsidy provided by government 1 (a lower $C_{1,b}$ implies a smaller bailout injection when bank 1 is bailed out by government 1). Overall, channeling more funds through bank 2 when the bailout budget of government 2 is already maxed out

[28]Please see Section 9.8 in the Online Appendix for the derivation of Eq. (58).
[29]Please see Section 9.16 in the Online Appendix for the derivation of Eq. (62).
[30]See Section 9.16 in the Online Appendix for the derivation of $dG_{1,b}/db_{1,2}$.

thus decreases the value of the total government bailout subsidies and reduces the banks' expected return.

Taken together, the analysis of Cases (a) and (b) shows that, with limited government bailout budgets, increasing their interbank exposure still allows banks to increase the value of the implicit bailout guarantee provided to the borrower bank (i.e., bank 2) as long as the government's bailout budget is still sufficient to settle all of the borrower bank's liabilities. These results are summarized in the following proposition.

Proposition 6 *If banks are protected by limited public guarantees, they have an incentive to channel funds through the interbank market until the bailout budget of the government providing guarantees for the borrower bank is completely exhausted.*

6.3 Transaction costs and asymmetric bailout probabilities

In this section, we analyze the implications of asymmetric bailout probabilities and determine how a bank's incentives to establish interbank connections are affected by changes in its own bailout probability and the bailout probability of its counterparty. For this analysis, we again build on the case discussed in Sections 4.1, that is, the two-country case where banks are located on a line. However, to be able to analyze marginal effects of changes in the bailout probabilities on the banks' incentive to engage in interbank lending, we introduce costs on interbank transactions, which allows for interior solutions for the banks' interbank exposure. For brevity, we take the banks' decision to invest in perfectly correlated portfolios as given and focus on the case where bank 1 remains solvent only if its investment in the risky asset is successful.

In particular, we consider now the case where the likelihood that government 1 bails out bank 1 in case of a failure is α_1 and the probability that government 2 bails out bank 2 is α_2. Moreover, we now assume that bank i incurs transaction costs $\tau(b_{i,i+1})$, where $\tau(0) = \tau'(0) = 0$ and $\tau'' > 0$, when lending funds on the interbank market. These costs include a variety of expenses associated with trading funds, such as brokerage, CHIPS or Fedwire transaction fees, as well as the costs of searching for banks with matching liquidity needs. The convex form of $\tau(\cdot)$ represents the increasing marginal costs of searching for trade partners and those resulting from the need to split large interbank transactions into many small transactions to work around credit lines (e.g., Neyer and Wiemers, 2004). With transaction costs, bank 1's expected return thus becomes

$$\Pi_{1,b} = \lambda_a \left[a_1 A + b_{1,2} B_{1,2} - cC_{1,b}\right] - \tau(b_{1,2}). \tag{63}$$

and the participation constraint of bank 1's creditors becomes

$$\begin{aligned} &\lambda_a cC_{1,b} + (1-\lambda_a)\frac{1}{2}\left[\alpha_1 cC_{1,b} + (1-\alpha_1)\alpha_2 b_{1,2} B_{1,2}\right] \\ &+(1-\lambda_a)\frac{1}{2}\left[\alpha_2\alpha_1 cC_{1,b} + \alpha_2(1-\alpha_1) b_{1,2} B_{1,2} + (1-\alpha_2)\alpha_1 cC_{1,b}\right] \geq c. \end{aligned} \tag{64}$$

Moreover, bank 2's expected return and participation constraint is given by

$$\Pi_{2,b} = \lambda_a \left[(1+b_{1,2})A - cC_{2,b} - b_{1,2}B_{1,2}\right] \geq \Pi_{2,a} = \lambda_a \left[A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha_2}\right], \tag{65}$$

while the participation constraint of its creditors becomes

$$\lambda_a cC_{2,b} + (1-\lambda_a)\left[\frac{1}{2}\alpha_2 cC_{2,b} + \frac{1}{2}\left(\alpha_1\alpha_2 cC_{2,b} + (1-\alpha_1)\alpha_2 cC_{2,b}\right)\right] \geq c. \tag{66}$$

From the binding Conditions (65) and (66) it follows that $B_{1,2} = A$. Incorporating $B_{1,2} = A$, bank 1's budget constraint, and the participation constraint of its creditors and taking the derivative of Eqs. (63) with respect to the interbank loan size yields the first-order condition for the banks' optimal interbank exposure[31]

$$\frac{d\Pi_{1,b}}{db_{1,2}} = \frac{\lambda_a(1-\lambda_a)(1-\alpha_1)\alpha_2 A}{\lambda_a + (1-\lambda_a)\alpha_1} - \tau'(b^*_{1,2}) = 0. \tag{67}$$

Next, we analyze the implications of a change in the banks' bailout probabilities on bank 1's incentive to lend funds to bank 2. Taking the implicit derivative of $b^*_{1,2}$ with respect to bank 1's own bailout probability (α_1) and its counterparty's bailout probability (α_2) yields

$$\frac{db^*_{1,2}}{d\alpha_1} = -\frac{\lambda_a(1-\lambda_a)\alpha_2 A}{(\lambda_a + (1-\lambda_a)\alpha_1)^2\tau''(b_{1,2})} < 0 \tag{68}$$

$$\frac{db^*_{1,2}}{d\alpha_2} = \frac{\lambda_a(1-\lambda_a)(1-\alpha_1)A}{(\lambda_a + (1-\lambda_a)\alpha_1)\tau''(b_{1,2})} > 0. \tag{69}$$

These results yield the following proposition.

Proposition 7 *A bank's desire to invest on the interbank market increases with the bailout probability of its counterparty and decreases with the bank's own bailout probability.*

First of all, if bank i's counterparty has a comparatively high likelihood of being rescued in case of an investment failure, funneling funds through this intermediary bank significantly increases bank i's repayment probability. Second, if bank i has a comparatively low bailout probability, establishing interbank connections with banks that are very likely to be bailed out allows the bank to increase insurance coverage for its creditors' funds, which, in turn, lowers its funding costs. However, if bank i already has relatively high individual government insurance coverage, the additional insurance value of establishing interbank connections with other banks is comparatively low. Hence, bank i would not able to significantly lower its funding costs by increasing its interbank exposure. Taken together, the analysis shows that banks have an incentive to establish a large interbank exposure to counterparties with high government insurance coverage, which is even stronger if the bank itself has only low government insurance coverage.

This result can help to explain the formation of core-periphery network structures. Banks with a

[31] Since $\tau'' > 0$, the second-order condition for a maximum is satisfied.

high probability of being rescued attract substantial fund inflows and thereby become an important hub in the interbank network. These banks thus borrow and lend extensively on the interbank market, which makes them even larger and more interconnected. This, in turn, is associated with an increase in the likelihood that the bank is rescued in case of default based on “too-big-to-fail” and “too-interconnected-to-fail” concerns. Because an increase in the bank’s government insurance coverage reinforces the incentive of other banks to use this bank as an intermediary, it becomes a self-reinforcing mechanism.

6.4 Government decision and welfare implications

In the main analysis, we show that, due to implicit government guarantees, banks have an incentive to inefficiently channel funds through the interbank market. However, the literature on interbank markets has stressed that interbank networks are also able to improve welfare. For example, through interbank lending, banks can co-insure each other against liquidity shocks, that is, negative shocks resulting from sudden deposit withdrawals (e.g., Allen and Gale, 2000) or positive shocks resulting from emerging investment opportunities (e.g., Rochet and Tirole, 1996). In this section, we show that, when the interbank network additionally serves welfare-improving purposes, banks have an incentive to inefficiently increase their interbank exposure beyond the first-best level if they are protected by implicit government guarantees. Based on this analysis, we then derive policy implications and propose measures to counteract bank incentives to inefficiently channel funds through the interbank market in Section 7.

For this analysis, we add two features to the model: (i) a welfare-improving purpose of interbank networks, and (ii) an endogenous government rescue decision that trades-off the costs of letting a bank fail and rescuing it. In the following, we build on the case discussed in Section 5, that is, two banks located on a circle which are both able to invest up to one unit of capital in the real asset (i.e., $a_1 \leq 1$ and $a_2 \leq 1$). For brevity, we again focus on the case in which the banks invest in perfectly correlated portfolios and where a bank remains solvent only if its own investment in the real asset is successful.

We modify the setup as follows. Instead of two countries, we assume that the economy now consists of two regions 1 and 2, both located in the same country with a government that tries to maximize welfare. In each region, there is a bank, endowed with equity e and a continuum of households (creditors). The banks can only raise capital and invest in their own region. To add a welfare-improving purpose of the interbank network to the model, we assume that the total endowment of the households in region 1 is $c - \underline{\epsilon}$ (with $\underline{\epsilon} \in (0, c)$), while the total endowment of the households in region 2 is $c + \overline{\epsilon}$, with $\overline{\epsilon} > \underline{\epsilon}$. Hence, interbank lending can be used to transfer funds that bank 2 raised from its creditors to bank 1, which can then invest the funds in the real asset. Moreover, we consider the case where the interbank interest rate is equal to A, which ensures that both banks have an incentive to transfer excess funds from bank 2 to bank 1.

With regard to the government rescue decision, we assume that, when a bank defaults at $t = 1$, the government must decide whether to rescue the bank by settling its liabilities or to let the bank fail. In particular, the government must trade-off the costs of a bank failure against the costs of transferring funds from the public to the private sector (e.g., deadweight costs that originate from taxation, see, e.g., Ballard, Shoven, and Whalley, 1985 and Feldstein, 1999). For the taxation deadweight costs, we assume that when the government raises the funds necessary for the bailout (which it only can do from households that still have funds available, i.e., households that did not lend their funds to a bank), it causes the costs $\chi > 0$.[32]

To add costs of a bank failure to the model, we assume that, in addition to the real asset, bank 2 has another illiquid investment with a value of L where $L < c$.[33] When the bank defaults and is not rescued, we assume that the fraction βL is lost due to bankruptcy costs (where we assume that L is sufficiently low such that raising $\underline{\epsilon}$ from bank 2's creditors and lending it to bank 1 is welfare improving).[34] These costs can be interpreted as fire sale costs due to rapid asset liquidation, legal expenditures, or costs that result from breaking a loan originator-borrower relationship (e.g., Acharya and Yorulmazer, 2008a). Because the costs of a bank failure are driven by bank-specific factors (e.g., availability of outside investors, asset liquidity, and lending relationships with the non-financial sector) that are revealed only in times of distress, we assume that at $t = 0$, only the distribution of β is known, which is a uniform distribution between zero and some upper limit $\overline{\beta}$ (i.e., $\beta = \mathcal{U}(0, \overline{\beta})$), where $\overline{\beta} \leq 1$ and $\overline{\beta} L > \chi$. Note that only bank 2 is potentially bailed out in case of a failure since bank 1 does not own an asset whose value is impaired by bankruptcy.[35]

To capture the "too-big-to-fail" argument (e.g., Freixas, 1999), we assume that, at least initially, the upper limit of the bankruptcy costs $\overline{\beta}$ increases with a bank's size and thus interbank liabilities (i.e., $d\overline{\beta}/db_{1,2} \geq 0$ and for $b_{1,2} = 0$, $d\overline{\beta}/db_{1,2} > 0$). This assumption captures the fact that the complexity (and thus the costs) involved in breaking up a bank increases with the bank's size and thus also its interbank liabilities.

Overall, welfare consists of the bank and creditor returns, minus the costs that are incurred when a bank fails or when the government raises the taxes to rescue a bank. Therefore, at $t = 1$, it is optimal for the government to bail out bank 2 if

$$\chi \leq \beta L. \tag{70}$$

Hence, the ex-ante probability of bank 2 being bailed out at $t = 1$ is equal to $\alpha = 1 - \chi/(\overline{\beta} L)$ and, thus, increases with government's efficiency to raise bailout funds and with the costs of a bank failure. The first-best interbank exposure is thus given by

$$\left[b_{1,2}^{FB}, b_{2,1}^{FB}\right] = [0, \underline{\epsilon}]. \tag{71}$$

[32]This assumption models the fact that a bailout is a costly wealth redistribution from households that still have taxable wealth to households that lent funds to a failed bank. Moreover, we assume that $\overline{\epsilon}$ is large enough such that the government is able to raise sufficient funds to bail out a bank in default and that the taxes and costs χ are shared equally by the households that still have funds available.

[33]For simplicity, we assume that bank 2 can pledge this illiquid investment as collateral to its outside creditors such that they receive the liquidation value of this asset in case of bankruptcy.

[34]Note that bank 2 always fails when its investment a_2 in the real asset is unsuccessful since $L < c$.

[35]This assumption does not qualitatively affect our results, but it makes the analysis much more tractable.

That is, it is always beneficial for both, the banks and the government, that bank 2 raises the total amount $c + \underline{\epsilon}$ from its creditors and lends the amount $\underline{\epsilon}$ to bank 1. However, due to the mechanism described in the main analysis, banks might have an incentive to increase their interbank exposure beyond this level. Therefore, we next determine the level of interbank exposure banks choose at $t = 0$. With the additional model ingredients, the expected bank returns at $t = 0$ become

$$
\begin{aligned}
\Pi_{1,b} &= \lambda_a \left[A + b_{1,2}B - (c - \underline{\epsilon})C_{1,b} - b_{2,1}B\right] && (72)\\
\Pi_{2,b} &= \lambda_a \left[A + L + b_{2,1}B - (c + \underline{\epsilon})C_{2,b} - b_{1,2}B\right]. && (73)
\end{aligned}
$$

Moreover, the participation constraint of the banks' creditors becomes[36]

$$
\begin{aligned}
\lambda_a(c - \underline{\epsilon})C_{1,b} + (1 - \lambda_a)\alpha\delta_{1,c}b_{1,2}B &\geq (c - \underline{\epsilon}) && (74)\\
\lambda_a(c + \underline{\epsilon})C_{2,b} + (1 - \lambda_a)\left[\alpha(c + \underline{\epsilon})C_{2,b} + (1 - \alpha)\left(1 - \frac{\chi}{2L}\right)L\right] &\geq (c + \underline{\epsilon}), && (75)
\end{aligned}
$$

where $\chi/(2L)$ is the expected value of β conditional on bank 2 not being bailed out, and $\delta_{1,c} = (c - \underline{\epsilon})C_{1,b}/((c - \underline{\epsilon})C_{1,b} + b_{2,1}B)$. Determining the derivative of the banks' expected return with respect to $b_{1,2}$ at $b_{1,2} = 0$ yields:[37]

$$
\frac{d\Pi_{1,b}}{db_{1,2}}(b_{1,2} = 0) = (1 - \lambda_a)\,\alpha\delta_{1,c}B > 0 \qquad (76)
$$

$$
\frac{d\Pi_{2,b}}{db_{1,2}}(b_{1,2} = 0) = \frac{\lambda_a\,(1 - \lambda_a)\,\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}\left[(c + \underline{\epsilon})C_{2,b} - \left(1 - \frac{\chi}{2L}\right)L\right]}{\lambda_a + (1 - \lambda_a)\,\alpha} > 0. \qquad (77)
$$

Therefore, as a result of the government bailout subsidies, both banks have an incentive to increase their interbank exposure beyond the first-best level, which yields the following proposition.

Proposition 8 *When the interbank network serves welfare-improving purposes, banks always have an incentive to inefficiently increase their interbank exposure beyond the first-best level if they are protected by implicit government guarantees.*

In particular, bank 1 has an incentive to become highly interconnected with bank 2 to exploit bank 2's government bailout subsidies (due to the mechanism presented in this paper) and bank 2 has an incentive to become bigger to increase its own bailout probability (due to the well-known "too-big-to-fail" problem; see, e.g., Freixas, 1999), which reduces its funding costs. Hence, as pointed out by the literature on the time-inconsistency of government bailout decisions, it would be optimal for the government to commit to a no-bailout policy at $t = 0$ from an ex-ante perspective. However, this solution is inconsistent with the government's ex-post decision at $t = 1$, creating a commitment problem similar to the one in, for example, Freixas (1999) and Farhi and Tirole (2012). This result yields important policy implications, which are discussed in the next section below.

[36]Note that creditors that lent their funds to the banks do not incur the deadweight costs that originate from taxation as these costs are only borne by households that still have funds available (i.e., households that did not lend to the banks).

[37]Please see Section 9.17 in the Online Appendix for the derivations of Eqs. (76) and (77).

7 Conclusion and policy implications

This paper sheds light on the puzzle of why banks have an incentive to be highly interconnected on the interbank market. We show that banks have an incentive to inefficiently channel funds through the interbank market before these funds are invested in real assets because this increases the value of government bailout guarantees and, in turn, the banks' expected returns. In particular, if banks that are protected by implicit or explicit government guarantees act as intermediaries between other banks and real investments, there is the possibility that these intermediary banks are rescued by their governments when the real assets fail. This additional hedge increases the likelihood that banks and their creditors are repaid relative to a direct investment in the real assets. As a result, banks have an incentive to choose a higher interbank exposure than necessary in order to fully exploit welfare-improving possibilities provided by the interbank market. This behavior considerably increases expected bankruptcy costs, systemic risk, and leverage without altering the aggregate relation with the real economy, which justifies regulatory intervention.

Therefore, governments should introduce measures that aim at reducing banks' incentive to create excessive interbank exposures. A first possibility would be to increase the risk weights for interbank liabilities under the Basel accord, which would tighten the minimum capital requirements. If interbank liabilities receive a higher risk weight, banks are incentivized to reduce their excessive lending activities and, hence, reduce systemic risk in the interbank market. However, banks might potentially counter this regulatory measure by engaging in cross-equity holdings in addition to interbank debt liabilities. By mutually investing equity in other banks in the interbank network, banks can reach any desired equity ratio without being dependent on outside investors.

From the results discussed in Section 6.3, it follows that if a government lowers bailout expectations (e.g., by credibly committing to a no-bailout policy), this actually leads to higher interconnectedness when other governments do not make the same no-bailout commitment. The reason is that the incentive to be interconnected increases if a bank's own government insurance coverage is reduced. Hence, if the government insurance coverage of banks is reduced in only one country, these banks then aim to have more interbank exposure to banks in other countries to benefit from their government guarantees. Therefore, reducing interconnectedness on the interbank market by lowering bailout expectations can only be achieved when governments employ a coordinated approach and lower expectations in all countries simultaneously.

Another possible option might be to limit a bailout to domestic counterparties, which would reduce the incentive of foreign banks to establish interbank connections with domestic banks. However, this process would require an adjustment of the principles of national treatment and equality of competitive opportunity in the International Banking Act, which requires that domestic banks and branches of foreign banks are treated equally (see Baxter (2010) for more details). Additionally, national laws often prohibit regulators from treating foreign and domestic banks differently. For example, the principles embodied in Section 4 of the Federal Reserve Act require the Federal Reserve to treat member banks and other banks equally. Another reason why regulators might be reluctant to limit a possible bailout only to domestic banks might be the quid pro quo response of other countries (see Baker, 2009). Moreover, regulators might be concerned that committing to limit bailouts to domestic counterparties of a bailed-out bank might severely reduce the competitiveness of the country's banking system because foreign banks would be reluctant to lend funds to domestic banks. In general, limiting bailouts to domestic banks would also not completely eliminate the incentives of foreign banks to become interconnected with domestic banks to exploit their government guarantees, as they only have to incorporate additional domestic intermediaries. In this case, bailout funds are first funneled through other domestic banks before they are transferred to foreign banks (as in the AIG bailout, where bailout funds from AIG were paid to Goldman Sachs and then passed on to foreign counterparties).

Furthermore, one of the key topics in the current discussion in the European Union is whether to introduce a financial transaction tax to limit speculative trading activities. Because interconnectedness can also be created via derivatives, such as CDSs (in addition to interbank loans), such a tax might potentially reduce the high interconnectedness by adding transaction costs (which reduce bank incentives to become interconnected, as shown in Section 6.3). Another measure to mitigate the incentives to inefficiently channel funds through the interbank market is to introduce the widely discussed bank levy. Levying higher taxes against banks with large balance sheets (which can very well result from high interconnectedness) based on their systemic risk can potentially mitigate the incentive to create large interbank exposures in the first place.

With regard to the interbank network structure, it is important to note that, for interbank liabilities that exist bilaterally between two banks, regulators might be able to net gross exposures before deciding to conduct bank bailouts. If, however, these interbank flows involve more than two banks, regulators would have to know the entire network topology to be able to cancel out superfluous interbank exposures.[38] Because interbank exposures are highly complex, opaque, and layered across different countries, canceling out these flows before bailing out a bank is impossible in practice. However, creating a centralized clearing house for interbank activities can potentially mitigate the perverse incentives described in this paper. If all interbank activities are channeled through a clearing house, the regulator knows the complete interbank network topology and is thus able to cancel the matching interbank deposits of the various banks. However, this approach would require a global clearing house and, thus, a collaboration of all the bank regulators involved.

[38] In Section 8.1 in the Online Appendix, we show that banks also always have an incentive to lend and borrow more on the interbank market when they are located on a circle that involves more than two banks.

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For Online Publication

Online Appendix for

Interbank Networks and Backdoor Bailouts:
Benefiting from other Banks' Government Guarantees

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8 Robustness checks

8.1 Three banks located on a circle

For brevity, we take the banks' decision to invest in perfectly correlated portfolios as given and focus on the case where banks remain solvent only if their investment in the risky asset is successful. Similar to the setting in Section 5, with three banks located on a circle, the banks' expected returns become

$$\Pi_{1,b} = \lambda_a \left[a_1 A + b_{1,2} B - cC_{1,b} - b_{3,1} B \right] \tag{78}$$
$$\Pi_{2,b} = \lambda_a \left[a_2 A + b_{2,3} B - cC_{2,b} - b_{1,2} B \right] \tag{79}$$
$$\Pi_{3,b} = \lambda_a \left[a_3 A + b_{3,1} B - cC_{3,b} - b_{2,3} B \right], \tag{80}$$

while the banks' budget constraints become

$$e + c + b_{3,1} = a_1 + b_{1,2} \tag{81}$$
$$e + c + b_{1,2} = a_2 + b_{2,3} \tag{82}$$
$$e + c + b_{2,3} = a_3 + b_{3,1}. \tag{83}$$

Moreover, the creditors' participation constraints become

$$\lambda_a cC_{1,b} + (1-\lambda_a)\left[\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{1,c} b_{1,2} B\right] \geq c \tag{84}$$
$$\lambda_a cC_{2,b} + (1-\lambda_a)\left[\alpha cC_{2,b} + (1-\alpha)\alpha\delta_{2,c} b_{2,3} B\right] \geq c \tag{85}$$
$$\lambda_a cC_{3,b} + (1-\lambda_a)\left[\alpha cC_{3,b} + (1-\alpha)\alpha\delta_{3,c} b_{3,1} B\right] \geq c, \tag{86}$$

where we assume that all banks (and their creditors) expect that their respective successor bank invests the funds it borrowed on the interbank market in the real asset. If the banks would expect that their successor bank also invests these funds in the interbank market, their expected return would be even higher due to the additional insurance coverage of the additional borrower bank's government guarantee. Hence, this assumption will give lower bounds for the banks' expected returns. Incorporating the banks' budget constraints and their creditors' participation constraints and taking the derivatives of Eqs. (78) to (80) with respect to the banks' interbank exposure yields[39]

$$\frac{d\Pi_{1,b}}{db_{1,2}} = \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha + (1-\lambda_a)(1-\alpha)\alpha \frac{b_{1,2}Bb_{3,1}B}{(cC_{1,b}+b_{3,1}B)^2}} \delta_{1,c}^2 B > 0 \tag{87}$$
$$\frac{d\Pi_{2,b}}{db_{2,3}} = \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha + (1-\lambda_a)(1-\alpha)\alpha \frac{b_{2,3}Bb_{1,2}B}{(cC_{2,b}+b_{1,2}B)^2}} \delta_{2,c}^2 B > 0 \tag{88}$$
$$\frac{d\Pi_{3,b}}{db_{3,1}} = \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha + (1-\lambda_a)(1-\alpha)\alpha \frac{b_{3,1}Bb_{2,3}B}{(cC_{3,b}+b_{2,3}B)^2}} \delta_{3,c}^2 B > 0. \tag{89}$$

Hence, due to their implicit government guarantees, all banks are always incentivized to borrow and lend more on the interbank market.

[39] Please see Section 9.18 for the derivation of Eqs. (87) to (89).

8.2 Remaining case for Section 5

In the following, we analyze the case in which the banks fail if their interbank loan is not repaid. First, we again determine the banks' optimal portfolio correlation given interbank exposure and then their optimal amount of interbank borrowing and lending. For this case, the expected return of bank i is given by

$$\Pi_{i,b} = \left(\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\alpha\right)\left[a_i A + b_{i,j\neq i}B - cC_{i,b} - b_{j\neq i,i}B\right]. \tag{90}$$

The participation constraint of the creditors of bank i is given by

$$\begin{aligned}
&\rho_{1,2}cC_{i,b} + (\lambda_a - \rho_{1,2})\left[\alpha cC_{i,b} + (1-\alpha)\alpha cC_{i,b} + (1-\alpha)^2 a_i A\left[1 - \frac{b_{j\neq i,i}BC_{j\neq i,b}}{b_{i,j\neq i}BC_{i,b} + (cC_{i,b} + b_{j\neq i,i}B)C_{j\neq i,b}}\right]\right]\\
&+ (\lambda_a - \rho_{1,2})\left[\alpha cC_{i,b} + (1-\alpha)\alpha\delta_{i,c}b_{i,j\neq i}B + (1-\alpha)^2 a_{j\neq i}A\frac{b_{j\neq i,i}BC_{j\neq i,b}}{b_{i,j\neq i}BC_{i,b} + (cC_{i,b} + b_{j\neq i,i}B)C_{j\neq i,b}}\right]\\
&+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{i,b} + (1-\alpha)\alpha\delta_{i,c}b_{i,j\neq i}B\right) + \frac{1}{2}\left(\alpha^2 cC_{i,b} + (1-\alpha)\alpha cC_{i,b} + \alpha(1-\alpha)\delta_{i,c}b_{i,j\neq i}B\right)\right]\\
&\geq c.
\end{aligned} \tag{91}$$

The repayment that the creditors of bank i receive in the case where only bank i is successful follows from

$$\sum_{n=0}^{\infty} a_i A\delta_{i,c}\delta_{i,b}^n\delta_{j\neq i,b}^n = a_i A\frac{\delta_{i,c}}{1-\delta_{i,b}\delta_{j\neq i,b}} = a_i A\left[1 - \frac{b_{j\neq i,i}BC_{j\neq i,b}}{b_{i,j\neq i}BC_{i,b} + (cC_{i,b} + b_{j\neq i,i}B)C_{j\neq i,b}}\right]. \tag{92}$$

That is, the liquidation funds are passed from bank i to bank $j \neq i$, back to bank i, and so on. Similarly, if bank $j \neq i$ is the only successful bank and it is not bailed out, the creditors of bank i receive

$$\sum_{n=0}^{\infty} a_{j\neq i}A\delta_{i,c}\delta_{i,b}^n\delta_{j\neq i,b}^{1+n} = a_{j\neq i}A\delta_{i,c}\delta_{j\neq i,b}\frac{1}{1-\delta_{i,b}\delta_{j\neq i,b}} = a_{j\neq i}A\frac{b_{j\neq i,i}BC_{j\neq i}}{b_{i,j\neq i}BC_{i,b} + (cC_{i,b} + b_{j\neq i,i}B)C_{j\neq i}}. \tag{93}$$

Solving the binding Constraint (91) for the interest rate $C_{i,b}$ yields

$$\begin{aligned}
C_{i,b} =& \frac{1}{\rho_{1,2} + (1-\rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha} - \frac{(\lambda_a - \rho_{1,2})(1-\alpha)^2 A}{c(\rho_{1,2} + (1-\rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha)}\\
&- \frac{(1-\lambda_a)(1-\alpha)\alpha}{c(\rho_{1,2} + (1-\rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha)}\delta_{i,c}b_{i,j\neq i}B,
\end{aligned} \tag{94}$$

where we used that $a_i = a_{j\neq i} = 1$, which follows from the banks' budget constraints from Eq. (22) and $a_i \leq 1$ (see Section 9.2). The implicit derivative of $C_{i,b}$ with respect to $\rho_{1,2}$ is[40]

$$\frac{dC_{i,b}}{d\rho_{1,2}} = \frac{1}{c}\frac{(1-\alpha)^2(A - cC_{i,b})}{\rho_{1,2} + (1-\rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha + (1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b} + b_{j\neq i,i}B)^2}} > 0. \tag{95}$$

[40]Please see Section 9.19 for the derivation of Eq. (95).

Finally, incorporating the banks' budget constraints and the creditors' participation constraint and taking the derivative of $\Pi_{i,b}$ with respect to $\rho_{1,2}$ yields[41]

$$\frac{d\Pi_{i,b}}{d\rho_{1,2}} = \frac{(1-\alpha)\alpha\left[1+(1-\lambda_a)(1-\alpha)\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right](A-cC_{i,b})}{\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha+(1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}} > 0. \tag{96}$$

From Eq. (96), it follows that when the banks are protected by government guarantees, they both have an incentive to maximize their portfolio correlation (i.e., choose $\rho^*_{1,2} = \lambda_a$) and their expected returns become

$$\Pi_{i,b} = \lambda_a\left[a_iA + b_{i,j\neq i}B - \frac{c}{\lambda_a+(1-\lambda_a)\alpha} - b_{j\neq i,i}B\right] + \frac{\lambda_a(1-\lambda_a)(1-\alpha)\alpha}{\lambda_a+(1-\lambda_a)\alpha}\delta_{i,c}b_{i,j\neq i}B, \tag{97}$$

which equals Eq. (30). Hence, the banks again have the incentive to maximize their interbank exposure.

8.3 Remaining cases for Section 6.2

In this section, we derive the optimal portfolio correlation and the optimal interbank exposure for the remaining sub-cases of Cases (a) and (b) discussed in Section 6.2.

8.3.1 Case (a) - $cC_{2,b} + b_{1,2}B_{1,2} \leq g$:

First, bank 2's expected return and participation constraint for all remaining sub-cases of Case (a) (i.e., a.ii to a.iv) is given by Eq. (50). Moreover, for Cases (a.ii), (a.iii), and (a.iv), the expected returns for bank 1 and the participation constraints (PC) of the creditors of both banks become

Case (a.ii): $\Pi_{1,b} = \rho_{1,2}\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (\lambda_a-\rho_{1,2})\,\alpha\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right]$

$$+(\lambda_a-\rho_{1,2})\left[b_{1,2}B_{1,2} - cC_{1,b}\right] + (1-2\lambda_a+\rho_{1,2})\alpha\left[b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{98}$$

PC creditors 1: $\rho_{1,2}cC_{1,b} + (\lambda_a-\rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b} + (1-\alpha)^2a_1A\right]$

$$+(\lambda_a-\rho_{1,2})\,cC_{1,b} + (1-2\lambda_a+\rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b}\right] \geq c \tag{99}$$

PC creditors 2: $\rho_{1,2}cC_{2,b} + (\lambda_a-\rho_{1,2})cC_{2,b} + (\lambda_a-\rho_{1,2})\alpha cC_{2,b} + (1-2\lambda_a+\rho_{1,2})\alpha cC_{2,b} \geq c \quad (100)$

Case (a.iii): $\Pi_{1,b} = \rho_{1,2}\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (\lambda_a-\rho_{1,2})\,\alpha\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] \quad (101)$

PC creditors 1: $\rho_{1,2}cC_{1,b} + (\lambda_a-\rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b} + (1-\alpha)^2a_1A\right]$

$$+(\lambda_a-\rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)b_{1,2}B_{1,2}\right] + (1-2\lambda_a+\rho_{1,2})\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha b_{1,2}B_{1,2}\right)$$

$$+(1-2\lambda_a+\rho_{1,2})\frac{1}{2}\left(\alpha^2cC_{1,b} + \alpha(1-\alpha)b_{1,2}B_{1,2} + (1-\alpha)\alpha cC_{1,b}\right) \geq c \tag{102}$$

PC creditors 2: $\rho_{1,2}cC_{2,b} + (\lambda_a-\rho_{1,2})cC_{2,b} + (\lambda_a-\rho_{1,2})\alpha cC_{2,b}$

$$+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\alpha cC_{2,b} + \frac{1}{2}\left(\alpha^2cC_{2,b} + (1-\alpha)\alpha cC_{2,b}\right)\right] \geq c \tag{103}$$

[41] Please see Section 9.20 for the derivation of Eq. (96).

$$\textbf{Case (a.vi):}\ \Pi_{1,b} = \rho_{1,2}\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (\lambda_a - \rho_{1,2})\left[a_1 A + \alpha b_{1,2}B_{1,2} - cC_{1,b}\right]$$
$$+ (\lambda_a - \rho_{1,2})\left[b_{1,2}B_{1,2} - cC_{1,b}\right] + (1 - 2\lambda_a + \rho_{1,2})\alpha\left[b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{104}$$
$$\textbf{PC creditors 1:}\ \rho_{1,2}cC_{1,b} + 2(\lambda_a - \rho_{1,2})cC_{1,b} + (1 - 2\lambda_a + \rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b}\right] \geq c \tag{105}$$
$$\textbf{PC creditors 2:}\ \rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\alpha cC_{2,b} + (1 - 2\lambda_a + \rho_{1,2})\alpha cC_{2,b} \geq c. \tag{106}$$

In Case (a.ii), bank 1 is solvent whenever it is repaid by bank 2, either because bank 2's investment is successful (first and third term of Eq. (98)) or bank 2 is bailed out (second and fourth term in Eq. (98)). In Case (a.iii), bank 1 only stays solvent if either both banks are successful (first term in Eq. (101)) or if bank 1 is successful and bank 2 is bailed out (second term in Eq. (101)), in which case the creditors of bank 1 are fully repaid. Otherwise, they receive the respective liquidation value of bank 1. Finally, in Case (a.vi) bank 1 stays solvent as soon as one bank is successful or if bank 2 is bailed out.

Taking the derivative of bank 1's expected returns with respect to $\rho_{1,2}$ yields for the remaining three sub-cases[42]

$$\textbf{Case (a.ii):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)[\alpha + (1-\alpha)\alpha]} > 0 \tag{107}$$
$$\textbf{Case (a.iii):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)\alpha\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right]}{\alpha + (1-\alpha)\left[\rho_{1,2} + (\lambda_a - \rho_{1,2})\alpha\right]} a_1 A > 0 \tag{108}$$
$$\textbf{Case (a.iv):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)\,\alpha}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\left[\alpha + (1-\alpha)\,\alpha\right]} cC_{1,b} > 0, \tag{109}$$

which shows that in all possible sub-cases of Case (a) banks will choose perfectly positively correlated portfolios, i.e., $\rho^*_{1,2} = \lambda_a$. Plugging $\rho^*_{1,2} = \lambda_a$ and the respective binding creditors' participation constraint (Eqs. (99), (102), and (105)) into bank 1's expected return (Eqs. (98), (101), and (104)) yields, after rearranging,

$$\textbf{Case (a.iii):}\ \Pi_{1,b} = \lambda_a\left[a_1 A + b_{1,2}B_{1,2}\right] + \underbrace{(1-\lambda_a)\left[\frac{\alpha c}{\lambda_a + (1-\lambda_a)\alpha} + \frac{\lambda_a(1-\alpha)\alpha b_{1,2}B_{1,2}}{\lambda_a + (1-\lambda_a)\alpha}\right]}_{=G_{1,b}} - c \tag{110}$$

$$\textbf{Cases (a.ii) and (a.vi):}\ \Pi_{1,b} = \lambda_a\left[a_1 A + b_{1,2}B_{1,2}\right] + \underbrace{(1-\lambda_a)\left[\alpha b_{1,2}B_{1,2} + \frac{(1-\alpha)\alpha c}{\lambda_a + (1-\lambda_a)[\alpha + (1-\alpha)\alpha]}\right]}_{=G_{1,b}} - c. \tag{111}$$

Finally, taking the derivative of Eqs. (110) and (111) with respect to $b_{1,2}$ yields for the three cases

$$\textbf{Case (a.iii):}\ \frac{d\Pi_{1,b}}{db_{1,2}} = \frac{\lambda_a(1-\lambda_a)(1-\alpha)\alpha A}{\lambda_a + (1-\lambda_a)\alpha} > 0 \tag{112}$$
$$\textbf{Cases (a.ii) and (a.vi):}\ \frac{d\Pi_{1,b}}{db_{1,2}} = (1-\lambda_a)\alpha A > 0, \tag{113}$$

where we incorporated already that for all three sub-cases it holds that $B_{1,2} = A$, which follows from the binding participation constraints of the creditors of bank 2 and bank 2's binding participation constraint from Eq. (50). Taking the derivative of the value of the bailout subsidies $G_{1,b}$ from Eqs.

[42] Please see Sections 9.5, 9.6, 9.7 for the derivation of Eqs. (107), (108), and (109), respectively.

(110) and (111) with respect to $b_{1,2}$ and comparing the results to Eqs. (112) and (113) shows that for all three sub-cases it holds that $dG_{1,b}/db_{1,2} = d\Pi_{1,b}/db_{1,2} > 0$. Hence, for Case (a), the banks always have an incentive to increase their interbank exposure, that is, increase $b_{1,2}$ until either $cC_{2,b} + b_{1,2}B_{1,2} = g$ or bank 1 has invested all its fund into the interbank loan to bank 2, that is, $b_{1,2} = e + c = 1$.

If $b_{1,2} = 1$ becomes binding first (which implies Case (a.ii)), bank 1's maximum expected return becomes

$$\Pi^*_{1,b} = (\lambda_a + (1-\lambda_a)\alpha)\left[A - \frac{c}{\lambda_a + (1-\lambda_a)[\alpha + (1-\alpha)\alpha]}\right] > \Pi^*_{1,a}, \tag{114}$$

while bank 2's expected return is $\Pi^*_{2,b} = \Pi^*_{2,a}$. If, on the other hand, $cC_{2,b} + b_{1,2}B_{1,2} = g$ becomes binding first, the banks' interbank exposure becomes

$$b^*_{1,2} = \frac{1}{A}\left(g - \frac{c}{\lambda_a + (1-\lambda_a)\alpha}\right). \tag{115}$$

This yields for bank 1's expected maximum return for the different cases

$$\textbf{Case (a.iii): } \Pi^*_{1,b} = \lambda_a\left[A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha}\right] + \frac{\lambda_a(1-\lambda_a)(1-\alpha)\alpha\left(g - \frac{c}{\lambda_a+(1-\lambda_a)\alpha}\right)}{\lambda_a + (1-\lambda_a)\alpha} > \Pi^*_{1,a} \tag{116}$$

$$\begin{aligned}\textbf{Case (a.ii) and (a.vi): } \Pi^*_{1,b} =& \lambda_a\left[A - \frac{c}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\alpha\right]}\right] \\ &+ (1-\lambda_a)\alpha\left[g - \frac{c}{\lambda_a + (1-\lambda_a)\alpha} - \frac{c}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\alpha\right]}\right] > \Pi^*_{1,a},\end{aligned} \tag{117}$$

where $\Pi^*_{1,b}$ is larger than $\Pi^*_{1,a}$ for all sub-cases as for Case (a) it holds that $cC_{2,b} + cC_{1,b} < cC_{2,b} + b_{1,2}B_{1,2} \leq g$. Bank 2's expected return is again equal to the expected return in the autarky case, i.e., $\Pi^*_{2,b} = \Pi^*_{2,a}$.

Hence, for Case (a), the banks always choose a positive interbank exposure as bank 1's expected return with interbank exposure always exceeds its return in the autarky case due to the increased value of the government bailout subsidies.

8.3.2 Case (b) - $cC_{2,b} + b_{1,2}B_{1,2} > g$:

For all three remaining sub-cases of Case (b), bank 2's expected return and participation constraint is again the same as in Eq. (50). For Case (b.ii), we have to distinguish three additional sub-cases: If bank 2's risky asset fails and bank 1 is not rescued, while bank 2 is bailed out by its government, the funds that bank 1 receives can either be

- (b.ii.i) always high enough to repay bank 1's creditors (i.e., $\delta_{2,b}g \geq cC_{1,b}$),
- (b.ii.ii) sufficient to fully repay bank 1's creditors if bank 1's real investment is also successful (i.e., $a_1A + \delta_{2,b}g \geq cC_{1,b} > \delta_{2,b}g$), or
- (b.ii.iii) always insufficient to repay bank 1's creditors (i.e., $cC_{1,b} > a_1A + \delta_{2,b}g$).

The respective expected returns of bank 1 and its creditors' participation constraints for these sub-cases of Case (b.ii) are given by

$$\textbf{Case (b.ii.i): } \Pi_{1,b} = \rho_{1,2}\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] + \left(\lambda_a - \rho_{1,2}\right)\alpha\left[a_1A + \delta_{2,b}g - cC_{1,b}\right]$$
$$+ \left(\lambda_a - \rho_{1,2}\right)\left[b_{1,2}B_{1,2} - cC_{1,b}\right] + (1 - 2\lambda_a + \rho_{1,2})\alpha\left[\delta_{2,b}g - cC_{1,b}\right] \tag{118}$$
$$\textbf{PC creditors 1: } \rho_{1,2}cC_{1,b} + \left(\lambda_a - \rho_{1,2}\right)\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b} + (1-\alpha)^2 a_1A\right]$$
$$+ \left(\lambda_a - \rho_{1,2}\right)cC_{1,b} + \left(1 - 2\lambda_a + \rho_{1,2}\right)\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b}\right] \geq c \tag{119}$$
$$\textbf{PC creditors 2: } \rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\alpha\delta_{2,c}g + (1 - 2\lambda_a + \rho_{1,2})\alpha\delta_{2,c}g \geq c \tag{120}$$

$$\textbf{Case (b.ii.ii): } \Pi_{1,b} = \rho_{1,2}\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] + \left(\lambda_a - \rho_{1,2}\right)\alpha\left[a_1A + \delta_{2,b}g - cC_{1,b}\right]$$
$$+ \left(\lambda_a - \rho_{1,2}\right)\left[b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{121}$$
$$\textbf{PC creditors 1: } \rho_{1,2}cC_{1,b} + \left(\lambda_a - \rho_{1,2}\right)\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b} + (1-\alpha)^2 a_1A\right] + \left(\lambda_a - \rho_{1,2}\right)cC_{1,b}$$
$$+(1 - 2\lambda_a + \rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b}g\right) + \frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\delta_{2,b}g + (1-\alpha)\alpha cC_{1,b}\right)\right] \geq c \tag{122}$$
$$\textbf{PC creditors 2: } \rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\alpha\delta_{2,c}g$$
$$+(1 - 2\lambda_a + \rho_{1,2})\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}\left(\alpha^2\delta_{2,c}g + (1-\alpha)\alpha\delta_{2,c}g\right)\right] \geq c \tag{123}$$

$$\textbf{Case (b.ii.iii): } \Pi_{1,b} = \rho_{1,2}\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] + \left(\lambda_a - \rho_{1,2}\right)\left[b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{124}$$
$$\textbf{PC creditors 1: } \rho_{1,2}cC_{1,b} + \left(\lambda_a - \rho_{1,2}\right)\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\left[\delta_{2,b}g + a_1A\right] + (1-\alpha)^2 a_1A\right)$$
$$+ \left(\lambda_a - \rho_{1,2}\right)\frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\left[\delta_{2,b}g + a_1A\right] + (1-\alpha)\alpha cC_{1,b} + (1-\alpha)^2 a_1A\right) + \left(\lambda_a - \rho_{1,2}\right)cC_{1,b}$$
$$+(1 - 2\lambda_a + \rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b}g\right) + \frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\delta_{2,b}g + (1-\alpha)\alpha cC_{1,b}\right)\right] \geq c \tag{125}$$
$$\textbf{PC creditors 2: } \rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}\left(\alpha^2\delta_{2,c}g + (1-\alpha)\alpha\delta_{2,c}g\right)\right]$$
$$+(1 - 2\lambda_a + \rho_{1,2})\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}\left(\alpha^2\delta_{2,c}g + (1-\alpha)\alpha\delta_{2,c}g\right)\right] \geq c. \tag{126}$$

For Case (b.iii), we again have to distinguish between two additional sub-cases: Bank 1 remains solvent if its investment in the risky asset is successful and

- (b.iii.i): if the interbank is at least partially repaid (i.e., $\delta_{2,b}g + a_1A \geq cC_{1,b}$) and
- (b.iii.ii): only if the interbank loan is fully repaid (i.e, $a_1A + b_{1,2}B_{1,2} \geq cC_{1,b} > a_1A + \delta_{2,b}g$).

The expected returns for bank 1 and the creditors' participation constraints for these cases are given by

$$\textbf{Case (b.iii.i): } \Pi_{1,b} = \rho_{1,2}\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (\lambda_a - \rho_{1,2})\,\alpha\left[a_1 A + \delta_{2,b}g - cC_{1,b}\right] \tag{127}$$

$$\begin{aligned}&\textbf{PC creditors 1: } \rho_{1,2}cC_{1,b} + (\lambda_a - \rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b} + (1-\alpha)^2 a_1 A\right]\\ &+ (\lambda_a - \rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)b_{1,2}B_{1,2}\right]\\ &+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b}g\right) + \frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\delta_{2,b}g + (1-\alpha)\alpha cC_{1,b}\right)\right] \geq c\end{aligned} \tag{128}$$

$$\begin{aligned}&\textbf{PC creditors 2: } \rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\alpha\delta_{2,c}g\\ &+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}\left(\alpha^2\delta_{2,c}g + (1-\alpha)\alpha\delta_{2,c}g\right)\right] \geq c\end{aligned} \tag{129}$$

$$\textbf{Case (b.iii.ii): } \Pi_{1,b} = \rho_{1,2}\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{130}$$

$$\begin{aligned}&\textbf{PC creditors 1: } \rho_{1,2}cC_{1,b} + (\lambda_a - \rho_{1,2})\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\left[\delta_{2,b}g + a_1 A\right] + (1-\alpha)^2 a_1 A\right)\\ &+ (\lambda_a - \rho_{1,2})\frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\left[\delta_{2,b}g + a_1 A\right] + (1-\alpha)\alpha cC_{1,b} + (1-\alpha)^2 a_1 A\right)\\ &+ (\lambda_a - \rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)b_{1,2}B_{1,2}\right]\\ &+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b}g\right) + \frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\delta_{2,b}g + (1-\alpha)\alpha cC_{1,b}\right)\right] \geq c\end{aligned} \tag{131}$$

$$\begin{aligned}&\textbf{PC creditors 2: } \rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}\left(\alpha^2\delta_{2,c}g + (1-\alpha)\alpha\delta_{2,c}g\right)\right]\\ &+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}\left(\alpha^2\delta_{2,c}g + (1-\alpha)\alpha\delta_{2,c}g\right)\right] \geq c.\end{aligned} \tag{132}$$

Finally, for Case (b.iv), we also have to distinguish between two sub-cases, that is, bank 1 remains solvent

- (b.iv.i) if the interbank is at least partially repaid (i.e., $\delta_{2,b}g \geq cC_{1,b}$) and
- (b.iv.ii) only if the interbank loan is fully repaid (i.e, $\delta_{2,b}g < cC_{1,b}$).

The expected returns for bank 1 and the creditors' participation constraints for these cases are given by

$$\begin{aligned}&\textbf{Case (b.iv.i): } \Pi_{1,b} = \rho_{1,2}\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (\lambda_a - \rho_{1,2})\left[a_1 A + \alpha\delta_{2,b}g - cC_{1,b}\right]\\ &+ (\lambda_a - \rho_{1,2})\left[b_{1,2}B_{1,2} - cC_{1,b}\right] + (1-2\lambda_a+\rho_{1,2})\alpha\left[\delta_{2,b}g - cC_{1,b}\right]\end{aligned} \tag{133}$$

$$\textbf{PC creditors 1: } \rho_{1,2}cC_{1,b} + 2\left(\lambda_a - \rho_{1,2}\right)cC_{1,b} + (1-2\lambda_a+\rho_{1,2})\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b}\right] \geq c \tag{134}$$

$$\textbf{PC creditors 2: } \rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\alpha\delta_{2,c}g + (1-2\lambda_a+\rho_{1,2})\alpha\delta_{2,c}g \geq c \tag{135}$$

Case (b.iv.ii): $\Pi_{1,b} = \rho_{1,2}\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (\lambda_a - \rho_{1,2})\left[a_1A + \alpha\delta_{2,b}g - cC_{1,b}\right]$

$$+ (\lambda_a - \rho_{1,2})\left[b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{136}$$

PC creditors 1: $\rho_{1,2}cC_{1,b} + 2(\lambda_a - \rho_{1,2})cC_{1,b}$

$$+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b}g\right) + \frac{1}{2}\left(\alpha^2 cC_{1,b} + \alpha(1-\alpha)\delta_{2,b}g + (1-\alpha)\alpha cC_{1,b}\right)\right] \geq c \tag{137}$$

PC creditors 2: $\rho_{1,2}cC_{2,b} + (\lambda_a - \rho_{1,2})cC_{2,b} + (\lambda_a - \rho_{1,2})\alpha\delta_{2,c}g$

$$+(1-2\lambda_a+\rho_{1,2})\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}\left(\alpha^2\delta_{2,c}g + (1-\alpha)\alpha\delta_{2,c}g\right)\right] \geq c. \tag{138}$$

Next, we determine the banks' optimal portfolio correlation for all sub-cases of Case (b). Incorporating the binding participation constraints and taking the derivative of bank 1's expected returns with respect to $\rho_{1,2}$ yields for the different cases[43]

$$\textbf{Case (b.ii.i):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha\frac{(1-\alpha)a_1A}{\lambda_a + (1-\lambda_a)[\alpha + (1-\alpha)\alpha]} > 0 \tag{139}$$

$$\textbf{Case (b.ii.ii):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha\frac{(1-\alpha)a_1A + \alpha(cC_{1,b} - \delta_{2,b}g)}{\lambda_a + (1-\lambda_a)\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha} > 0 \tag{140}$$

$$\textbf{Case (b.ii.iii):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha\frac{a_1A}{\lambda_a + (1-\lambda_a)} > 0 \tag{141}$$

$$\textbf{Case (b.iii.i):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha\frac{a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha(a_1A + \delta_{2,b}g - cC_{1,b})}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1-2\lambda_a+\rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha} > 0 \tag{142}$$

$$\textbf{Case (b.iii.ii):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha\frac{a_1A + b_{1,2}B_{1,2} - cC_{1,b}}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1-2\lambda_a+\rho_{1,2})\alpha} > 0 \tag{143}$$

$$\textbf{Case (b.vi.i):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha\frac{(1-\alpha)cC_{1,b}}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1-2\lambda_a+\rho_{1,2})[\alpha + (1-\alpha)\alpha]} > 0 \tag{144}$$

$$\textbf{Case (b.iv.ii):}\ \frac{d\Pi_{1,b}}{d\rho_{1,2}} = \alpha\frac{cC_{1,b} - \alpha\delta_{2,b}g}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1-2\lambda_a+\rho_{1,2})\alpha} > 0. \tag{145}$$

Hence, in all cases, the banks will again choose $\rho^*_{1,2} = \lambda_a$. Next, we determine the banks' optimal interbank exposure for Case (b). Plugging $\rho^*_{1,2} = \lambda_a$ into bank 1's expected return yields for the different cases

$$\textbf{Cases (b.ii.ii), (b.ii.iii), (b.iii.i), (b.iii.ii), (b.iv.ii):}\ \Pi_{1,b} = \lambda_a\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] \tag{146}$$

$$\textbf{PC creditors 1:}\ \lambda_a cC_{1,b} + (1-\lambda_a)\left[\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b}g\right] \geq c \tag{147}$$

$$\textbf{PC creditors 2:}\ \lambda_a cC_{2,b} + (1-\lambda_a)\alpha\delta_{2,c}g \geq c \tag{148}$$

$$\textbf{Cases (b.ii.i), (b.iv.i):}\ \Pi_{1,b} = \lambda_a\left[a_1A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (1-\lambda_a)\alpha\left[\delta_{2,b}g - cC_{1,b}\right] \tag{149}$$

$$\textbf{PC creditors 1:}\ \lambda_a cC_{1,b} + (1-\lambda_a)\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b}\right] \geq c \tag{150}$$

$$\textbf{PC creditors 2:}\ \lambda_a cC_{2,b} + (1-\lambda_a)\alpha\delta_{2,c}g \geq c. \tag{151}$$

[43]Please see Sections 9.9 to 9.15 for the derivation of Eqs. (139) to (145).

Incorporating the binding participation constraints and taking the derivative of Eqs. (146) and (149) with respect to $b_{1,2}$ yields[44]

$$\textbf{Cases (b.ii.ii), (b.ii.iii), (b.iii.i), (b.iii.ii), (b.iv.ii):}\ \frac{d\Pi_{1,b}}{db_{1,2}} = -\frac{\frac{\lambda_a(1-\lambda_a)\alpha^2}{\lambda_a+(1-\lambda_a)\alpha}\frac{\delta_{2,c}gA}{cC_{2,b}+b_{1,2}B_{1,2}}}{\lambda_a+(1-\lambda_a)\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}} < 0 \quad (152)$$

$$\textbf{Cases (b.ii.i), (b.iv.i):}\ \frac{d\Pi_{1,b}}{db_{1,2}} = 0. \quad (153)$$

Hence, for Case (b), that is, the case where banks increase their interbank exposure to a level such that bank 2's total liabilities exceed the government budget, increasing the interbank exposure even further than this threshold reduces or has no effect on the banks' expected return. As in Case (a), the derivatives of the bailout subsidies values with respect to $b_{1,2}$ again equal the derivatives of the expected returns, given in Eqs. (152) and (153). That is, also for Case (b) it holds that $dG_{1,b}/db_{1,2} = d\Pi_{1,b}/db_{1,2}$, as shown in Section 9.16.

For the cases (b.ii.ii), (b.ii.iii), (b.iii.i), (b.iii.ii), and (b.iv.ii), the intuition for this result is exactly the same as for Case (b.i) described in Section 6.2. Moreover, while the effect of interbank exposure on $C_{2,b}$ is the same for Cases (b.ii.i) and (b.iv.i) as for the other cases, for these two cases an increase in the interbank exposure has no effect on $C_{1,b}$ (while $C_{1,b}$ decreases in the other cases) because bank 1 stays solvent if bank 2 is bailed out. Hence, for Cases (b.ii.i) and (b.iv.i), the size of the interbank repayment does not change the payment to the creditors of bank 1. Therefore, for these two cases, interbank exposure has no effect on the value of the government bailout subsidies provided by government 1 and 2 and thus also no effect on the banks' expected returns.

8.4 Limited government bailout budgets - One country case.

In this section, we analyze the case in which both banks are located in the same country and where the bailout funds that the country's government is able to provide are limited. Again, to analyze how the banks' incentive to become interconnected changes when increasing the interbank exposure potentially leads to a situation where the government is not able to fully bail out all banks in the economy anymore, we now assume that the government is endowed with a bailout budget of $g \geq cC_{1,a} = cC_{2,a}$.[45] For brevity, we take the banks' decision to invest in perfectly correlated portfolios as given.

For the one country case we have to distinguish between four different cases:

- *Case (a) -* $cC_{2,b} + b_{1,2}B_{1,2} \leq g$ *and* $cC_{1,b} + cC_{2,b} \leq g$*:* the bailout budget exceeds the total liabilities of bank 2 and exceeds the sum of the creditors' claims of bank 1 and bank 2,
- *Case (b) -* $cC_{2,b} + b_{1,2}B_{1,2} \leq g$ *and* $cC_{1,b} + cC_{2,b} > g$*:* the bailout budget exceeds the total liabilities of the banks individually, but the budget is lower than the sum of the creditors' claims of bank 1 and bank 2,

[44]Please see Section 9.16 for the derivation of Eqs. (152) and (153).

[45]Recall that due to the additional hedge provided by the interbank exposure for bank 1's creditors it always holds that $cC_{1,b} < cC_{1,a}$.

- *Case (c) - $cC_{2,b} + b_{1,2}B_{1,2} > g$ and $cC_{1,b} + cC_{2,b} \leq g$:* the bailout budget is lower than the total liabilities of bank 2, but exceeds the sum of the creditors' claims of bank 1 and bank 2, and
- *Case (d) - $cC_{2,b} + b_{1,2}B_{1,2} > g$ and $cC_{1,b} + cC_{2,b} > g$:* the bailout budget is lower than the total liabilities of bank 2 and lower than the sum of the creditors' claims of bank 1 and bank 2.

Case (a): $cC_{2,b} + b_{1,2}B_{1,2} \leq g$ and $cC_{1,b} + cC_{2,b} \leq g$.

In Case (a), the government's budget limit is not reached yet and thus the banks' maximization problem equals Case (a) in the two-country setting, which we discussed in Section 6.2.1. Therefore, for Case (a) banks always have an incentive to increase the interbank exposure until either $cC_{2,b} + b_{1,2}B_{1,2} = g$ or $cC_{1,b} + cC_{2,b} = g$ becomes binding.

Case (b): $cC_{2,b} + b_{1,2}B_{1,2} \leq g$ and $cC_{1,b} + cC_{2,b} > g$.

For this case, bank 1's expected return is given by[46]

$$\Pi_{1,b} = \lambda_a \left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right], \tag{154}$$

and the participation constraint of the creditors of bank 1 becomes

$$\begin{aligned} &\lambda_a cC_{1,b} + (1-\lambda_a)\frac{1}{2}\left[\alpha cC_{1,b} + (1-\alpha)\alpha b_{1,2}B_{1,2}\right] \\ &+(1-\lambda_a)\frac{1}{2}\left[\alpha^2(g - cC_{2,b}) + \alpha(1-\alpha)b_{1,2}B_{1,2} + (1-\alpha)\alpha cC_{1,b}\right] \geq c. \end{aligned} \tag{155}$$

That is, if both banks fail and both banks are rescued, but bank 2 is rescued first, the creditors of bank 1 only receive a partial repayment (i.e., $g - cC_{2,b}$) since the government's bailout budget is not sufficient to fully bail out both banks.

For all cases where both banks are located in the same country, the participation constraint of bank 2 becomes

$$\Pi_{2,b} = \lambda_a \left[(1+b_{1,2})A - b_{1,2}B_{1,2}\right] + (1-\lambda_a)\alpha cC_{2,b} - c \geq \Pi_{2,a}. \tag{156}$$

Finally, the participation constraint of the creditors of bank 2 is given by

$$\lambda_a cC_{2,b} + (1-\lambda_a)\left[\frac{1}{2}\alpha cC_{2,b} + \frac{1}{2}\left(\alpha^2(g - cC_{1,b}) + (1-\alpha)\alpha cC_{2,b}\right)\right] \geq c. \tag{157}$$

Again, if both banks fail and are both rescued, but bank 1 is bailed out first, the creditors of bank 2 are only partially repaid. Incorporating the binding creditors' participation constraints and the banks' budget constraints and taking the derivative of Eq. (154) with respect to $b_{1,2}$ yields[47]

$$\frac{d\Pi_{1,b}}{db_{1,2}} = \frac{\lambda_a\left(1-\lambda_a\right)\left(1-\alpha\right)\alpha\left(\lambda_a + \left(1-\lambda_a\right)\alpha\right)}{\left(\lambda_a + \left(1-\lambda_a\right)\frac{1}{2}\left[\alpha + \left(1-\alpha\right)\alpha\right]\right)^2 - \left(1-\lambda_a\right)^2\left(1-\frac{1}{2}\alpha\right)\frac{1}{2}\alpha^3} A > 0. \tag{158}$$

[46] Note that the conditions of Case (b) imply that $cC_{1,b} > b_{1,2}B_{1,2}$.
[47] Please see Section 9.21 for the derivation of Eq. (158).

Therefore, also in Case (b) the banks are incentivized to maximize their interbank exposure, that is, increase $b_{1,2}$ until $cC_{2,b} + b_{1,2}B_{1,2} = g$ becomes binding. The reason is that, for Case (b), increasing the interbank exposure increases the bailout injection in the case where only bank 2 is rescued. Hence, interbank exposure increases the value of the bailout subsidy and thus the banks' expected return.

Case (c): $cC_{2,b} + b_{1,2}B_{1,2} > g$ and $cC_{1,b} + cC_{2,b} \leq g$.

For Case (c), bank 1's expected return is given by[48]

$$\Pi_{1,b} = \lambda_a \left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] + (1-\lambda_a)\alpha \left[\delta_{2,b} g - cC_{1,b}\right]. \tag{159}$$

Moreover, the participation constraint of the creditors of bank 1 becomes

$$\lambda_a cC_{1,b} + (1-\lambda_a)\left[\alpha cC_{1,b} + (1-\alpha)\alpha cC_{1,b}\right] \geq c, \tag{160}$$

and the participation constraint of the creditors of bank 2 is given by

$$\lambda_a cC_{2,b} + (1-\lambda_a)\alpha\delta_{2,c} g \geq c. \tag{161}$$

Again, incorporating the binding creditors' participation constraints and the banks' budget constraints and taking the derivative of Eq. (159) with respect to $b_{1,2}$ yields[49]

$$\frac{d\Pi_{1,b}}{db_{1,2}} = 0. \tag{162}$$

Therefore, for Case (c), the banks' expected return is not affected by a change in the interbank exposure. While higher interbank exposure shifts bailout funds from bank 2 to bank 1, which leads to an increase in $C_{2,b}$, it has no effect on the bailout injection in case bank 2 is rescued, since the total liabilities of bank 2 already exceed the government's bailout budget. Moreover, for Case (c), a change in the interbank exposure has no effect on $C_{1,b}$ since for this case bank 1 always stays solvent if bank 2 is bailed out. As a result, increasing the interbank exposure does not affect the value of the government bailout subsidy and thus has no effect on the banks' expected return.

Case (d): $cC_{2,b} + b_{1,2}B_{1,2} > g$ and $cC_{1,b} + cC_{2,b} > g$.

For this case, bank 1's expected return is given by[50]

$$\Pi_{1,b} = \lambda_a \left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right], \tag{163}$$

while the participation constraint of the creditors of bank 1 becomes

$$\lambda_a cC_{1,b} + (1-\lambda_a)\left[\frac{1}{2}\left(\alpha cC_{1,b} + (1-\alpha)\alpha\delta_{2,b} g\right) + \frac{1}{2}\left(\alpha\delta_{2,b} g + (1-\alpha)\alpha cC_{1,b}\right)\right] \geq c. \tag{164}$$

[48]The conditions of Case (c) imply that $b_{1,2}B_{1,2} > cC_{1,b}$ and that, if bank 2 defaults, bank 1's pro-rata share of bank 2's liquidation value is larger than the claim of the creditors of bank 1 (i.e., $\delta_{2,b} g \geq cC_{1,b}$).

[49]Please see Section 9.22 for the derivation of Eq. (162).

[50]Note that the conditions of Case (d) imply that $\delta_{2,b} g < cC_{1,b}$.

Finally, the participation constraint of the creditors of bank 2 is given by

$$\lambda_a c C_{2,b} + (1-\lambda_a)\left[\frac{1}{2}\alpha\delta_{2,c}g + \frac{1}{2}(\alpha^2(g - cC_{1,b}) + (1-\alpha)\alpha\delta_{2,c}g)\right] \geq c. \quad (165)$$

Incorporating the binding creditors' participation constraints and the banks' budget constraints and taking the derivative of Eq. (163) with respect to $b_{1,2}$ yields[51]

$$\frac{d\Pi_{1,b}}{db_{1,2}} = -\frac{\lambda_a\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\frac{(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}\frac{g}{b_{1,2}B+cC_{2,b}}}{\lambda_a+\left(\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\right)\frac{(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}\frac{g}{b_{1,2}B+cC_{2,b}}}\delta_{2,c}A < 0. \quad (166)$$

Hence, for Case (d), that is, the case where the bailout budget is lower than the total liabilities of bank 2 and lower than the sum of the creditors' claims of bank 1 and bank 2, increasing the interbank exposure even further reduces the banks' expected return. In particular, in this case, channeling more funds through the borrower bank (i.e., bank 2) has no effect on the size of the bailout injection if this bank is rescued. However, higher interbank exposure shifts bailout funds from the borrower to the lender bank (i.e., from bank 2 to bank 1). Hence, while for Case (d) higher interbank exposure decreases the amount of bailout funds received by the creditors of bank 2 (thus increasing $C_{2,b}$), it has not effect on the value of the bailout subsidy anymore since the bailout budget is already maxed out. Moreover, as an increase in the interbank exposure increases the amount received by the creditors of bank 1, it lowers $C_{1,b}$ and thus also lowers the value of bank 1's bailout subsidy (a lower $C_{1,b}$ implies a smaller bailout injection when bank 1 is bailed out). Overall, channeling more funds through bank 2 when the government bailout budget is already maxed out thus decreases the value of the total government bailout subsidy and, in turn, reduces the banks' expected return for this case.

Taken together, the analysis of Cases (a) to (d) shows that, when banks are located in the same country and the bailout budget of the government is limited, channeling more funds through the borrower bank also increases the lender bank's return as long as the government's bailout budget is still sufficient to settle all of the borrower bank's liabilities (i.e., Cases (a) and (b)). Therefore, banks have an incentive to increase their interbank exposure until $cC_{2,b} + b_{1,2}B_{1,2} = g$ becomes binding.

8.5 Effect of deadweight bankruptcy costs on the banks' herding incentive

The mechanism discussed in this paper always shifts a bank's portfolio choice toward a higher portfolio correlation with its counterparties. However, when bankruptcy costs are introduced into the model, this can potentially mitigate banks' incentive to invest in correlated assets. By allocating sufficient funds into both investment opportunities (i.e., the real asset and the interbank market), the bank might remain solvent when at least one of the investments is successful. When the banks then choose negatively correlated portfolios, they can potentially decrease their default probability. Whether the banks choose positively or negatively correlated portfolios then depends on whether the benefits from exploiting the banks' government guarantees dominate the diversification benefits or vice versa. In particular, the higher the probability that a bank is rescued by its government in case of a default

[51] Please see Section 9.23 for the derivation of Eq. (166).

and the lower the bankruptcy costs, the stronger is the banks' incentive to increase their portfolio correlation. The reason is that, as the likelihood of being rescued decreases, the sensitivity of the bank creditors' interest rates with regard to the banks liquidation value increases. The liquidation value, in turn, depends on the bankruptcy costs of the bank, where higher bankruptcy costs imply a lower liquidation value and vice versa.[52]

9 Derivations

9.1 Derivation of Eq. (26)

First, we take the implicit derivative of Eq. (25) with respect to the banks' portfolio correlation $\rho_{1,2}$, which yields

$$c\frac{dC_{i,b}}{d\rho_{1,2}} = \frac{(1-\alpha)^2}{\lambda_a + (1-\lambda_a)\alpha}\delta_{i,c}b_{i,j\neq i}B - \frac{(1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1-2\lambda_a+\rho_{1,2})\alpha\right]}{\lambda_a + (1-\lambda_a)\alpha}\frac{d\delta_{i,c}}{d\rho_{1,2}}b_{i,j\neq i}B, \tag{167}$$

and, second, the implicit derivative of $\delta_{i,c}$ with respect to $\rho_{1,2}$:

$$\begin{aligned}\frac{d\delta_{i,c}}{d\rho_{1,2}} &= \frac{c\frac{dC_{i,b}}{d\rho_{1,2}}\left(cC_{i,b} + b_{j\neq i,i}B\right) - cC_{i,b}c\frac{dC_{i,b}}{d\rho_{1,2}}}{\left(cC_{i,b} + b_{j\neq i,i}B\right)^2} \\ &= c\frac{dC_{i,b}}{d\rho_{1,2}}\frac{b_{j\neq i,i}B}{\left(cC_{i,b} + b_{j\neq i,i}B\right)^2}.\end{aligned} \tag{168}$$

Next, we plug Eq. (168) into Eq. (167) and solve for $c\frac{dC_{i,b}}{d\rho_{1,2}}$:

$$\begin{aligned}c\frac{dC_{i,b}}{d\rho_{1,2}} =& \frac{(1-\alpha)^2}{\lambda_a + (1-\lambda_a)\alpha}\delta_{i,c}b_{i,j\neq i}B - \frac{(1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1-2\lambda_a+\rho_{1,2})\alpha\right]}{\lambda_a + (1-\lambda_a)\alpha}c\frac{dC_{i,b}}{d\rho_{1,2}}\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{\left(cC_{i,b} + b_{j\neq i,i}B\right)^2} \\ c\frac{dC_{i,b}}{d\rho_{1,2}}&\left(1 + \frac{(1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1-2\lambda_a+\rho_{1,2})\alpha\right]}{\lambda_a + (1-\lambda_a)\alpha}\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{\left(cC_{i,b} + b_{j\neq i,i}B\right)^2}\right) = \frac{(1-\alpha)^2}{\lambda_a + (1-\lambda_a)\alpha}\delta_{i,c}b_{i,j\neq i}B \\ \frac{dC_{i,b}}{d\rho_{1,2}} =& \frac{1}{c}\frac{(1-\alpha)^2\delta_{i,c}b_{i,j\neq i}B}{\lambda_a + (1-\lambda_a)\alpha + (1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1-2\lambda_a+\rho_{1,2})\alpha\right]\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{\left(cC_{i,b} + b_{j\neq i,i}B\right)^2}} > 0.\end{aligned} \tag{169}$$

9.2 Derivation of Eq. (27)

Taking the derivative of Eq. (23) with respect to $\rho_{1,2}$ yields, after simplifying,

$$\begin{aligned}\frac{d\Pi_{i,b}}{d\rho_{1,2}} &= \left(a_iA + b_{i,j\neq i}B - cC_{i,b} - b_{j\neq i,i}B\right) - \rho_{1,2}c\frac{dC_{i,b}}{d\rho_{1,2}} - \left(a_iA + \alpha b_{i,j\neq i}B + (1-\alpha)\delta_{j\neq i,b}b_{j\neq i,i}B - cC_{i,b} - b_{j\neq i,i}B\right) \\ &+ (\lambda_a - \rho_{1,2})\left((1-\alpha)\frac{d\delta_{j\neq i,b}}{d\rho_{1,2}}b_{j\neq i,i}B - c\frac{dC_{i,b}}{d\rho_{1,2}}\right) \\ &= (1-\alpha)b_{i,j\neq i}B - \rho_{1,2}c\frac{dC_{i,b}}{d\rho_{1,2}} - (1-\alpha)\delta_{j\neq i,b}b_{j\neq i,i}B + (\lambda_a - \rho_{1,2})\left((1-\alpha)\frac{d\delta_{j\neq i,b}}{d\rho_{1,2}}b_{j\neq i,i}B - c\frac{dC_{i,b}}{d\rho_{1,2}}\right) \\ &= (1-\alpha)(1-\delta_{j\neq i,b})b_{i,j\neq i}B - \lambda_a c\frac{dC_{i,b}}{d\rho_{1,2}} - (\lambda_a - \rho_{1,2})(1-\alpha)\frac{d\delta_{j\neq i,c}}{d\rho_{1,2}}b_{j\neq i,i}B.\end{aligned} \tag{170}$$

[52] This could be even reinforced when banks face higher costs in the event of joint liquidation (see Wagner, 2011).

where we used that $b_{i,j\neq i} = b_{j\neq i,i}$ to simplify the expression. This relationship between $b_{i,j\neq i}$ and $b_{j\neq i,i}$ follows from the banks' budget constraints from Eq. (22), $a_i \leq 1$, and $a_{j\neq i} \leq 1$ and can be shown as follows. Solving bank $j \neq i$'s budget constraint from Eq. (22) for $b_{i,j\neq i}$ yields

$$b_{i,j\neq i} = a_{j\neq i} + b_{j\neq i,i} - 1, \tag{171}$$

where we used that $e + c = 1$. Plugging $b_{i,j\neq i}$ into bank i's budget constraint and rearranging yields

$$\sum_{i=1}^{2} a_i = 2. \tag{172}$$

This result together with the investment limits $a_i \leq 1$ and $a_{j\neq i} \leq 1$ implies that $a_i = 1$ for all $i \in \{1, 2\}$. Finally, from plugging $a_i = a_{j\neq i} = 1$ into the banks' budget constraints it follows that $b_{i,j\neq i} = b_{j\neq i,i}$.

Next, we plug Eqs. (168) and (169) into Eq. (170), which gives the final result after simplifying

$$\begin{aligned}
\frac{d\Pi_{i,b}}{d\rho_{1,2}} &= (1-\alpha)\,\delta_{j\neq i,c} b_{i,j\neq i} B - \lambda_a c \frac{dC_{i,b}}{d\rho_{1,2}} - (\lambda_a - \rho_{1,2})\,(1-\alpha)\, c \frac{dC_{j\neq i,b}}{d\rho_{1,2}} \frac{b_{i,j\neq i} B}{\left(cC_{j\neq i,b} + b_{i,j\neq i} B\right)^2} b_{j\neq i,i} B \\
&= (1-\alpha)\,\delta_{j\neq i,c} b_{i,j\neq i} B - \left(\lambda_a + (\lambda_a - \rho_{1,2})\,(1-\alpha)\,\frac{b_{i,j\neq i} B b_{j\neq i,i} B}{\left(cC_{i,b} + b_{j\neq i,i} B\right)^2}\right) c \frac{dC_{i,b}}{d\rho_{1,2}} \\
&= (1-\alpha)\,\delta_{j\neq i,c} b_{i,j\neq i} B - \left(\lambda_a + (\lambda_a - \rho_{1,2})\,(1-\alpha)\,\frac{b_{i,j\neq i} B b_{j\neq i,i} B}{\left(cC_{i,b} + b_{j\neq i,i} B\right)^2}\right) \\
&\quad \frac{(1-\alpha)^2 \delta_{i,c} b_{i,j\neq i} B}{\lambda_a + (1-\lambda_a)\alpha + (1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha\right] \frac{b_{i,j\neq i} B b_{j\neq i,i} B}{(cC_{i,b} + b_{j\neq i,i} B)^2}} \\
&= (1-\alpha)\left[1 - \frac{(1-\alpha)\left(\lambda_a + (\lambda_a - \rho_{1,2})\,(1-\alpha)\,\frac{b_{i,j\neq i} B b_{j\neq i,i} B}{(cC_{i,b} + b_{j\neq i,i} B)^2}\right)}{\lambda_a + (1-\lambda_a)\alpha + (1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha\right] \frac{b_{i,j\neq i} B b_{j\neq i,i} B}{(cC_{i,b} + b_{j\neq i,i} B)^2}}\right] \delta_{i,c} b_{i,j\neq i} B \\
&= \frac{(1-\alpha)\,\alpha\left(1 + (1-\lambda_a)\,(1-\alpha)\,\frac{b_{i,j\neq i} B b_{j\neq i,i} B}{(cC_{i,b} + b_{j\neq i,i} B)^2}\right)}{\lambda_a + (1-\lambda_a)\alpha + (1-\alpha)\left[(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha\right] \frac{b_{i,j\neq i} B b_{j\neq i,i} B}{(cC_{i,b} + b_{j\neq i,i} B)^2}} \delta_{i,c} b_{i,j\neq i} B > 0,
\end{aligned} \tag{173}$$

where we also used that $C_{i,b} = C_{j\neq i,b}$ and $\delta_{i,c} = \delta_{j\neq i,c}$ to simplify the expression, which is true due to the symmetric model setup.

9.3 Derivation of Eq. (31)

First, we use the banks' budget constraints to substitute $b_{j\neq i,i}$ and a_i in Eq. (30). Solving bank $j \neq i$'s budget constraint from Eq. (22) for $b_{j\neq i,i}$ yields

$$b_{j\neq i,i} = 1 - a_{j\neq i} + b_{i,j\neq i}. \tag{174}$$

Moreover, solving bank i's budget constraint for a_i yields

$$a_i = 1 + b_{j\neq i,i} - b_{i,j\neq i}. \tag{175}$$

Plugging Eq. (174) into Eq. (175) yields

$$a_i = 2 - a_{j\neq i}. \tag{176}$$

After incorporating Eqs. (174) and (176) into Eq. (30), we take the derivative with respect to the banks' interbank exposure, which yields

$$\frac{d\Pi_{i,b}}{db_{i,j\neq i}} = \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha} \left(\frac{\partial \delta_{i,c}}{\partial b_{i,j\neq i}} b_{i,j\neq i} + \delta_{i,c} + \frac{\partial \delta_{i,c}}{\partial C_{i,b}} \frac{dC_{i,b}}{db_{i,j\neq i}} b_{i,j\neq i} \right) B. \tag{177}$$

Second, we take the implicit derivative of Eq. (29) with respect to the banks' interbank exposure, which yields, after simplifying,

$$\begin{aligned} \lambda_a c \frac{dC_{i,b}}{db_{i,j\neq i}} + (1-\lambda_a) \left(\alpha c \frac{dC_{i,b}}{db_{i,j\neq i}} + (1-\alpha)\alpha \left(\frac{\partial \delta_{i,c}}{\partial b_{i,j\neq i}} b_{i,j\neq i} + \delta_{i,c} + \frac{\partial \delta_{i,c}}{\partial C_{i,b}} \frac{dC_{i,b}}{db_{i,j\neq i}} b_{i,j\neq i} \right) B \right) = 0 \\ c\frac{dC_{i,b}}{db_{i,j\neq i}} = -\frac{(1-\lambda_a)(1-\alpha)\alpha \left(\frac{\partial \delta_{i,c}}{\partial b_{i,j\neq i}} b_{i,j\neq i} + \delta_{i,c} \right) B}{\lambda_a + (1-\lambda_a)\left(\alpha + (1-\alpha)\alpha \frac{1}{c} \frac{\partial \delta_{i,c}}{\partial C_{i,b}} b_{i,j\neq i} B \right)}. \end{aligned} \tag{178}$$

Third, incorporating Eq. (174) and deriving the partial derivative of $\delta_{i,c}$ with respect to $b_{i,j\neq i}$ yields

$$\frac{\partial \delta_{i,c}}{\partial b_{i,j\neq i}} = -\frac{cC_{i,b}B}{\left(cC_{i,b} + b_{j\neq i,i}B\right)^2}. \tag{179}$$

Fourth, we determine the partial derivative of $\delta_{i,c}$ with respect to $C_{i,b}$:

$$\frac{\partial \delta_{i,c}}{\partial C_{i,b}} = \frac{cb_{j\neq i,i}B}{\left(cC_{i,b} + b_{j\neq i,i}B\right)^2}. \tag{180}$$

Next, we plug Eqs. (179) and (180) into Eq. (178) and simplify, which yields

$$\begin{aligned} c\frac{dC_{i,b}}{db_{i,j\neq i}} &= -\frac{(1-\lambda_a)(1-\alpha)\alpha \left(1 - \frac{b_{i,j\neq i}B}{cC_{i,b}+b_{j\neq i,i}B}\right)\delta_{i,c}B}{\lambda_a + (1-\lambda_a)\left(\alpha + (1-\alpha)\alpha \frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right)} \\ &= -\frac{(1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\left(\alpha + (1-\alpha)\alpha \frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right)} \delta_{i,c}^2 B. \end{aligned} \tag{181}$$

where we used that $b_{i,j\neq i} = b_{j\neq i,i}$ to simplify the expression (see Section 9.2). Finally, we plug Eqs. (179) to (181) into Eq. (177) and simplify, which gives

$$\begin{aligned} \frac{d\Pi_{i,b}}{db_{i,j\neq i}} &= \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha} \\ &\left(-\frac{cC_{i,b}b_{i,j\neq i}B}{\left(cC_{i,b}+b_{j\neq i,i}B\right)^2} + \delta_{i,c} - \frac{b_{j\neq i,i}B}{\left(cC_{i,b}+b_{j\neq i,i}B\right)^2} \frac{(1-\lambda_a)(1-\alpha)\alpha\delta_{i,c}^2 b_{i,j\neq i}B}{\lambda_a + (1-\lambda_a)\left(\alpha + (1-\alpha)\alpha \frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right)} \right) B \\ &= \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha} \left(1 - \frac{b_{i,j\neq i}B}{cC_{i,b}+b_{j\neq i,i}B} - \frac{(1-\lambda_a)(1-\alpha)\alpha \frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}}{\lambda_a + (1-\lambda_a)\left(\alpha + (1-\alpha)\alpha \frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right)} \delta_{i,c} \right) \delta_{i,c}B \\ &= \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha} \left(1 - \frac{(1-\lambda_a)(1-\alpha)\alpha \frac{b_{j\neq i,i}Bb_{i,j\neq i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}}{\lambda_a + (1-\lambda_a)\left(\alpha + (1-\alpha)\alpha \frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right)} \right) \delta_{i,c}^2 B \\ &= \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a)\alpha + (1-\lambda_a)(1-\alpha)\alpha \frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}} \delta_{i,c}^2 B > 0, \end{aligned} \tag{182}$$

where we again used that $b_{i,j\neq i} = b_{j\neq i,i}$ to simplify the expression.

9.4 Derivation of Eq. (51)

First, we first take the derivative of Eq. (48) with respect to the portfolio correlation, which yields, after simplifying

$$\begin{aligned}
& cC_{1,b}+\rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}}-cC_{1,b}+\left(\lambda_a-\rho_{1,2}\right)c\frac{dC_{1,b}}{d\rho_{1,2}}-\left(\alpha cC_{1,b}+\left(1-\alpha\right)b_{1,2}B_{1,2}\right)\\
+\quad & \left(\lambda_a-\rho_{1,2}\right)\alpha c\frac{dC_{1,b}}{d\rho_{1,2}}+\left(\alpha cC_{1,b}+\left(1-\alpha\right)\alpha b_{1,2}B_{1,2}\right)+\left(1-2\lambda_a+\rho_{1,2}\right)\alpha c\frac{dC_{1,b}}{d\rho_{1,2}}=0\\
& \left(\lambda_a+\left(1-\lambda_a\right)\alpha\right)c\frac{dC_{1,b}}{d\rho_{1,2}}=\left(1-\alpha\right)^2 b_{1,2}B_{1,2}\\
& c\frac{dC_{1,b}}{d\rho_{1,2}}=\frac{\left(1-\alpha\right)^2}{\lambda_a+\left(1-\lambda_a\right)\alpha}b_{1,2}A,
\end{aligned}\tag{183}$$

where we already used that $B_{1,2}=A$, which follows from Eqs. (49) and (50). Next, we take the derivative of Eq. (47) with respect to the portfolio correlation, substitute Eq. (183), and simplify

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= \left(a_1A+b_{1,2}B_{1,2}-cC_{1,b}\right)-\rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}}-\left(a_1A+\alpha b_{1,2}B_{1,2}-cC_{1,b}\right)-\left(\lambda_a-\rho_{1,2}\right)c\frac{dC_{1,b}}{d\rho_{1,2}}\\
&= \left(1-\alpha\right)b_{1,2}B_{1,2}-\lambda_a c\frac{dC_{1,b}}{d\rho_{1,2}}\\
&= \left(1-\alpha\right)b_{1,2}A-\lambda_a\frac{\left(1-\alpha\right)^2}{\lambda_a+\left(1-\lambda_a\right)\alpha}b_{1,2}A\\
&= \left(1-\frac{\lambda_a\left(1-\alpha\right)}{\lambda_a+\left(1-\lambda_a\right)\alpha}\right)\left(1-\alpha\right)b_{1,2}A\\
&= \frac{\left(1-\alpha\right)\alpha}{\lambda_a+\left(1-\lambda_a\right)\alpha}b_{1,2}A>0.
\end{aligned}\tag{184}$$

9.5 Derivation of Eq. (107)

From Eqs. (50) and (100) it follows that $B_{1,2}=A$. First, we take the derivative of Eq. (99) with respect to the portfolio correlation, which yields, after simplifying

$$\begin{aligned}
& cC_{1,b}+\rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}}-\left(\alpha cC_{1,b}+\left(1-\alpha\right)\alpha cC_{1,b}+\left(1-\alpha\right)^2 a_1A\right)+\left(\lambda_a-\rho_{1,2}\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]c\frac{dC_{1,b}}{d\rho_{1,2}}\\
-\quad & cC_{1,b}+\left(\lambda_a-\rho_{1,2}\right)c\frac{dC_{1,b}}{d\rho_{1,2}}+\left(\alpha cC_{1,b}+\left(1-\alpha\right)\alpha cC_{1,b}\right)+\left(1-2\lambda_a+\rho_{1,2}\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]c\frac{dC_{1,b}}{d\rho_{1,2}}=0\\
& \left(\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\right)c\frac{dC_{1,b}}{d\rho_{1,2}}=\left(1-\alpha\right)^2 a_1A\\
& c\frac{dC_{1,b}}{d\rho_{1,2}}=\frac{\left(1-\alpha\right)^2}{\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]}a_1A.
\end{aligned}\tag{185}$$

Next, we take the derivative of Eq. (98) with respect to the portfolio correlation and incorporate Eq. (185)

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= \left(a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right) - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \alpha\left(a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right) - \left(\lambda_a - \rho_{1,2}\right)\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&- \left(b_{1,2}B_{1,2} - cC_{1,b}\right) - \left(\lambda_a - \rho_{1,2}\right)c\frac{dC_{1,b}}{d\rho_{1,2}} + \alpha\left(b_{1,2}B_{1,2} - cC_{1,b}\right) - \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= \left(1-\alpha\right)a_1 A - \left(\lambda_a + \left(1-\lambda_a\right)\alpha\right)c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= \left(1-\alpha\right)a_1 A - \left(\lambda_a + \left(1-\lambda_a\right)\alpha\right)\frac{\left(1-\alpha\right)^2}{\lambda_a + \left(1-\lambda_a\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]}a_1 A \\
&= \left(1 - \frac{\left(\lambda_a + \left(1-\lambda_a\right)\alpha\right)\left(1-\alpha\right)}{\lambda_a + \left(1-\lambda_a\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]}\right)\left(1-\alpha\right)a_1 A \\
&= \frac{\left(1-\alpha\right)\alpha}{\lambda_a + \left(1-\lambda_a\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]}a_1 A > 0. \qquad (186)
\end{aligned}$$

9.6 Derivation of Eq. (108)

From Eqs. (50) and (103) it follows that $B_{1,2} = A$. First, we take the derivative of Eq. (102) with respect to the portfolio correlation, which yields after rearranging

$$\begin{aligned}
& cC_{1,b} + \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \left[\alpha cC_{1,b} + \left(1-\alpha\right)\alpha C_{1,b} + \left(1-\alpha\right)^2 a_1 A\right] + \left(\lambda_a - \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]c\frac{dC_{1,b}}{d\rho_{1,2}} \\
- \quad & \left[\alpha cC_{1,b} + \left(1-\alpha\right)b_{1,2}B_{1,2}\right] + \left(\lambda_a - \rho_{1,2}\right)\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} + \left[\alpha cC_{1,b} + \left(1-\alpha\right)\alpha b_{1,2}B_{1,2}\right] + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& \left(\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right] + \left(1-\lambda_a\right)\alpha\right)c\frac{dC_{1,b}}{d\rho_{1,2}} = \left(1-\alpha\right)^2\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{\left(1-\alpha\right)^2\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right]}{\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right] + \left(1-\lambda_a\right)\alpha}. \qquad (187)
\end{aligned}$$

Next, we take the derivative of Eq. (101) with respect to the portfolio correlation and incorporate Eq. (187), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= \left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \alpha\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] - \left(\lambda_a - \rho_{1,2}\right)\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= \left(1-\alpha\right)\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] - \left(\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\alpha\right)c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= \left(1-\alpha\right)\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] - \left(\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\alpha\right)\frac{\left(1-\alpha\right)^2\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right]}{\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right] + \left(1-\lambda_a\right)\alpha} \\
&= \left(1 - \frac{\left(\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\alpha\right)\left(1-\alpha\right)}{\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right] + \left(1-\lambda_a\right)\alpha}\right)\left(1-\alpha\right)\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] \\
&= \frac{\left(1-\alpha\right)\alpha\left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right]}{\alpha + \left(1-\alpha\right)\left[\rho_{1,2} + \left(\lambda_a - \rho_{1,2}\right)\alpha\right]} > 0. \qquad (188)
\end{aligned}$$

9.7 Derivation of Eq. (109)

From Eqs. (50) and (106) it follows that $B_{1,2} = A$. In a first step, we take the derivative of Eq. (105) with respect to the portfolio correlation and rearrange

$$\begin{aligned} & cC_{1,b} + \rho_{1,2} c \frac{dC_{1,b}}{d\rho_{1,2}} - 2cC_{1,b} + 2\left(\lambda_a - \rho_{1,2}\right) c \frac{dC_{1,b}}{d\rho_{1,2}} \\ + \quad & \left[\alpha c C_{1,b} + \left(1-\alpha\right)\alpha c C_{1,b}\right] + \left(1 - 2\lambda_a + \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right] c \frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\ & \left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]\right) c \frac{dC_{1,b}}{d\rho_{1,2}} = \left(1-\alpha\right)^2 cC_{1,b} \\ & c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{\left(1-\alpha\right)^2}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]} cC_{1,b}. \end{aligned} \tag{189}$$

Next, we take the derivative of Eq. (104) with respect to the portfolio correlation, incorporate Eq. (189), and simplify

$$\begin{aligned} \frac{d\Pi_{1,b}}{d\rho_{1,2}} &= \left[a_1 A + b_{1,2} B_{1,2} - cC_{1,b}\right] - \rho_{1,2} c \frac{dC_{1,b}}{d\rho_{1,2}} - \left[a_1 A + \alpha b_{1,2} B_{1,2} - cC_{1,b}\right] - \left(\lambda_a - \rho_{1,2}\right) c \frac{dC_{1,b}}{d\rho_{1,2}} \\ &- \left[b_{1,2} B_{1,2} - cC_{1,b}\right] - \left(\lambda_a - \rho_{1,2}\right) c \frac{dC_{1,b}}{d\rho_{1,2}} + \alpha\left[b_{1,2} B_{1,2} - cC_{1,b}\right] - \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha c \frac{dC_{1,b}}{d\rho_{1,2}} \\ &= \left(1-\alpha\right) cC_{1,b} - \left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha\right) c \frac{dC_{1,b}}{d\rho_{1,2}} \\ &= \left(1-\alpha\right) cC_{1,b} - \frac{\left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha\right)\left(1-\alpha\right)^2}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]} cC_{1,b} \\ &= \left(1 - \frac{\left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha\right)\left(1-\alpha\right)}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]}\right)\left(1-\alpha\right) cC_{1,b} \\ &= \frac{\left(1-\alpha\right)\alpha}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\left[\alpha + \left(1-\alpha\right)\alpha\right]} cC_{1,b} > 0. \end{aligned} \tag{190}$$

9.8 Derivation of Eq. (58)

First, we take the derivative of Eq. (50) with respect to portfolio correlation

$$\begin{aligned} \frac{d\Pi_{2,b}}{d\rho_{1,2}} &= \lambda_a \left[-c\frac{dC_{2,b}}{d\rho_{1,2}} - b_{1,2}\frac{dB_{1,2}}{d\rho_{1,2}}\right] = 0 \\ b_{1,2}\frac{dB_{1,2}}{d\rho_{1,2}} &= -c\frac{dC_{2,b}}{d\rho_{1,2}}. \end{aligned} \tag{191}$$

Second, we take the derivative of $\delta_{2,c}$ with respect to $\rho_{1,2}$ and incorporate Eq. (191)

$$\begin{aligned} \frac{d\delta_{2,c}}{d\rho_{1,2}} &= \frac{c\frac{dC_{2,b}}{d\rho_{1,2}}\left(cC_{2,b} + b_{1,2}B_{1,2}\right) - cC_{2,b}\left(c\frac{dC_{2,b}}{d\rho_{1,2}} + b_{1,2}\frac{dB_{1,2}}{d\rho_{1,2}}\right)}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} \\ &= \frac{c\frac{dC_{2,b}}{d\rho_{1,2}}\left(cC_{2,b} + b_{1,2}B_{1,2}\right) - cC_{2,b}\left(c\frac{dC_{2,b}}{d\rho_{1,2}} - c\frac{dC_{2,b}}{d\rho_{1,2}}\right)}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} \\ &= -\frac{d\delta_{2,b}}{d\rho_{1,2}} = c\frac{dC_{2,b}}{d\rho_{1,2}}\frac{1}{cC_{2,b} + b_{1,2}B_{1,2}}. \end{aligned} \tag{192}$$

Third, we take the derivative of Eq. (57) with respect to the portfolio correlation and incorporate Eq. (192), which yields

$$\begin{aligned}
& cC_{2,b}+\rho_{1,2}c\frac{dC_{2,b}}{d\rho_{1,2}}-cC_{2,b}+(\lambda_a-\rho_{1,2})\,c\frac{dC_{2,b}}{d\rho_{1,2}}-\alpha\delta_{2,c}g \\
+ \quad & (\lambda_a-\rho_{1,2})\,\alpha\frac{d\delta_{2,c}}{d\rho_{1,2}}g+\alpha\delta_{2,c}g+(1-2\lambda_a+\rho_{1,2})\,\alpha\frac{d\delta_{2,c}}{d\rho_{1,2}}g=0 \\
& \lambda_a c\frac{dC_{2,b}}{d\rho_{1,2}}+(1-\lambda_a)\,\alpha\frac{d\delta_{2,c}}{d\rho_{1,2}}g=0 \\
& \lambda_a c\frac{dC_{2,b}}{d\rho_{1,2}}+(1-\lambda_a)\,\alpha c\frac{dC_{2,b}}{d\rho_{1,2}}\frac{1}{cC_{2,b}+b_{1,2}B_{1,2}}g=0 \\
& c\frac{dC_{2,b}}{d\rho_{1,2}}=0, \qquad (193)
\end{aligned}$$

which implies that $dB_{1,2}/d\rho_{1,2}=0$, $d\delta_{2,c}/d\rho_{1,2}=0$, and $d\delta_{2,b}/d\rho_{1,2}=0$ due to Eqs. (191) and (192), respectively. Next, take the implicit derivative of Eq. (56) with respect to the portfolio correlation

$$\begin{aligned}
& cC_{1,b}+\rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}}-cC_{1,b}+(\lambda_a-\rho_{1,2})\,c\frac{dC_{1,b}}{d\rho_{1,2}}-[\alpha cC_{1,b}+(1-\alpha)\,b_{1,2}B_{1,2}]+(\lambda_a-\rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
+ \quad & [\alpha cC_{1,b}+(1-\alpha)\,\alpha\delta_{2,b}g]+(1-2\lambda_a+\rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}}=0. \qquad (194)
\end{aligned}$$

Then, we solve Eq. (194) for $cdC_{1,b}/d\rho_{1,2}$, which yields

$$\begin{aligned}
(\lambda_a+(1-\lambda_a)\,\alpha)\,c\frac{dC_{1,b}}{d\rho_{1,2}} &= (1-\alpha)\,(b_{1,2}B_{1,2}-\alpha\delta_{2,b}g) \\
c\frac{dC_{1,b}}{d\rho_{1,2}} &= \frac{1-\alpha}{\lambda_a+(1-\lambda_a)\,\alpha}\,[b_{1,2}B_{1,2}-\alpha\delta_{2,b}g]\,. \qquad (195)
\end{aligned}$$

Finally, we take the derivative of Eq. (55) with respect to $\rho_{1,2}$ and incorporate Eq. (195), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= [a_1A+b_{1,2}B_{1,2}-cC_{1,b}]-\rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}}-[a_1A+\alpha\delta_{2,b}g-cC_{1,b}]-(\lambda_a-\rho_{1,2})\,c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= [b_{1,2}B_{1,2}-\alpha\delta_{2,b}g]-\lambda_a c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= [b_{1,2}B_{1,2}-\alpha\delta_{2,b}g]-\lambda_a\frac{1-\alpha}{\lambda_a+(1-\lambda_a)\,\alpha}\,[b_{1,2}B_{1,2}-\alpha\delta_{2,b}g] \\
&= \left(1-\frac{\lambda_a\,(1-\alpha)}{\lambda_a+(1-\lambda_a)\,\alpha}\right)[b_{1,2}B_{1,2}-\alpha\delta_{2,b}g] \\
&= \alpha\frac{b_{1,2}B_{1,2}-\alpha\delta_{2,b}g}{\lambda_a+(1-\lambda_a)\,\alpha}>0. \qquad (196)
\end{aligned}$$

9.9 Derivation of Eq. (139)

Since the derivative of Eqs. (57) and (120) with respect to the portfolio correlation are equivalent, it follows that again $cdC_{2,b}/d\rho_{1,2}=0$ and thus $dB_{1,2}/d\rho_{1,2}=d\delta_{2,b}/d\rho_{1,2}=d\delta_{2,c}/d\rho_{1,2}=0$ (see Section

9.8). Next, we take the implicit derivative of Eq. (119) with respect to the portfolio correlation

$$\begin{aligned}
& cC_{1,b} + \rho_{1,2} c\frac{dC_{1,b}}{d\rho_{1,2}} - \left[\alpha cC_{1,b} + (1-\alpha)\,\alpha cC_{1,b} + (1-\alpha)^2\, a_1 A\right] \\
+ \quad & (\lambda_a - \rho_{1,2})\left[\alpha + (1-\alpha)\,\alpha\right] c\frac{dC_{1,b}}{d\rho_{1,2}} - cC_{1,b} + (\lambda_a - \rho_{1,2})\, c\frac{dC_{1,b}}{d\rho_{1,2}} \\
+ \quad & \left[\alpha cC_{1,b} + (1-\alpha)\,\alpha cC_{1,b}\right] + (1 - 2\lambda_a + \rho_{1,2})\left[\alpha + (1-\alpha)\,\alpha\right] c\frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& (\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right])\, c\frac{dC_{1,b}}{d\rho_{1,2}} = (1-\alpha)^2\, a_1 A \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)^2}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]} a_1 A. \qquad (197)
\end{aligned}$$

Finally, we take the derivative of Eq. (118) with respect to $\rho_{1,2}$ and incorporate Eq. (197), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= \left[a_1 A + b_{1,2}B_{1,2} - cC_{1,b}\right] - \rho_{1,2} c\frac{dC_{1,b}}{d\rho_{1,2}} - \alpha\left[a_1 A + \delta_{2,b} g - cC_{1,b}\right] - (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&\quad - \left[b_{1,2}B_{1,2} - cC_{1,b}\right] - (\lambda_a - \rho_{1,2})\, c\frac{dC_{1,b}}{d\rho_{1,2}} + \alpha\left[\delta_{2,b} g - cC_{1,b}\right] - (1 - 2\lambda_a + \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= (1-\alpha)\, a_1 A - (\lambda_a + (1-\lambda_a)\,\alpha)\, c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= (1-\alpha)\, a_1 A - (\lambda_a + (1-\lambda_a)\,\alpha)\, \frac{(1-\alpha)^2}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]} a_1 A \\
&= \left(1 - \frac{(\lambda_a + (1-\lambda_a)\,\alpha)\,(1-\alpha)}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]}\right)(1-\alpha)\, a_1 A \\
&= \alpha\frac{(1-\alpha)\, a_1 A}{\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]} > 0. \qquad (198)
\end{aligned}$$

9.10 Derivation of Eq. (140)

Since the derivative of Eqs. (57) and (123) with respect to the portfolio correlation are equivalent, it follows that again $cdC_{2,b}/d\rho_{1,2} = 0$ and thus $dB_{1,2}/d\rho_{1,2} = d\delta_{2,b}/d\rho_{1,2} = d\delta_{2,c}/d\rho_{1,2} = 0$ (see Section 9.8). Next, we take the implicit derivative of Eq. (122) with respect to the portfolio correlation and simplify

$$\begin{aligned}
& cC_{1,b} + \rho_{1,2} c\frac{dC_{1,b}}{d\rho_{1,2}} - \left[\alpha cC_{1,b} + (1-\alpha)\,\alpha cC_{1,b} + (1-\alpha)^2\, a_1 A\right] + (\lambda_a - \rho_{1,2})\left[\alpha + (1-\alpha)\,\alpha\right] c\frac{dC_{1,b}}{d\rho_{1,2}} \\
- \quad & cC_{1,b} + (\lambda_a - \rho_{1,2})\, c\frac{dC_{1,b}}{d\rho_{1,2}} + \left[\alpha cC_{1,b} + (1-\alpha)\,\alpha\delta_{2,b} g\right] + (1 - 2\lambda_a + \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} = 0. \\
& (\lambda_a + (1-\lambda_a)\,\alpha + (\lambda_a - \rho_{1,2})\,(1-\alpha)\alpha)\, c\frac{dC_{1,b}}{d\rho_{1,2}} = (1-\alpha)\left((1-\alpha)\, a_1 A + \alpha\left(cC_{1,b} - \delta_{2,b} g\right)\right) \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{1-\alpha}{\lambda_a + (1-\lambda_a)\,\alpha + (\lambda_a - \rho_{1,2})\,(1-\alpha)\alpha}\left[(1-\alpha)\, a_1 A + \alpha\left(cC_{1,b} - \delta_{2,b} g\right)\right]. \qquad (199)
\end{aligned}$$

Finally, we take the derivative of Eq. (121) with respect to the portfolio correlation and incorporate Eq. (199), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= [a_1 A + b_{1,2}B_{1,2} - cC_{1,b}] - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \alpha\,[a_1 A + \delta_{2,b}g - cC_{1,b}] \\
&- (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} - [b_{1,2}B_{1,2} - cC_{1,b}] - (\lambda_a - \rho_{1,2})\,c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= [(1-\alpha)\,a_1 A + \alpha\,(cC_{1,b} - \delta_{2,b}g)] - (\lambda_a + (\lambda_a - \rho_{1,2})\,\alpha)\,c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= [(1-\alpha)\,a_1 A + \alpha\,(cC_{1,b} - \delta_{2,b}g)] - \frac{(\lambda_a + (\lambda_a - \rho_{1,2})\,\alpha)\,(1-\alpha)}{\lambda_a + (1-\lambda_a)\,\alpha + (\lambda_a - \rho_{1,2})\,(1-\alpha)\alpha}\,[(1-\alpha)\,a_1 A + \alpha\,(cC_{1,b} - \delta_{2,b}g)] \\
&= \left(1 - \frac{(\lambda_a + (\lambda_a - \rho_{1,2})\,\alpha)\,(1-\alpha)}{\lambda_a + (1-\lambda_a)\,\alpha + (\lambda_a - \rho_{1,2})\,(1-\alpha)\alpha}\right)[(1-\alpha)\,a_1 A + \alpha\,(cC_{1,b} - \delta_{2,b}g)] \\
&= \alpha\frac{(1-\alpha)\,a_1 A + \alpha\,(cC_{1,b} - \delta_{2,b}g)}{\lambda_a + (1-\lambda_a)\,\alpha + (\lambda_a - \rho_{1,2})\,(1-\alpha)\alpha} > 0.
\end{aligned} \tag{200}$$

9.11 Derivation of Eq. (141)

Since the derivative of Eqs. (57) and (126) with respect to the portfolio correlation are equivalent, it follows that again $cdC_{2,b}/d\rho_{1,2} = 0$ and thus $dB_{1,2}/d\rho_{1,2} = d\delta_{2,b}/d\rho_{1,2} = d\delta_{2,c}/d\rho_{1,2} = 0$ (see Section 9.8). Next, we take the implicit derivative of Eq. (125) with respect to the portfolio correlation and simplify

$$\begin{aligned}
& cC_{1,b} + \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \left[\alpha cC_{1,b} + (1-\alpha)\,\alpha\,(a_1 A + \delta_{2,b}g) + (1-\alpha)^2\,a_1 A\right] + (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
- \quad & cC_{1,b} + (\lambda_a - \rho_{1,2})\,c\frac{dC_{1,b}}{d\rho_{1,2}} + [\alpha cC_{1,b} + (1-\alpha)\,\alpha\delta_{2,b}g] + (1 - 2\lambda_a + \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \left((1-\alpha)\,\alpha a_1 A + (1-\alpha)^2\,a_1 A\right) + (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
+ \quad & (\lambda_a - \rho_{1,2})\,c\frac{dC_{1,b}}{d\rho_{1,2}} + (1 - 2\lambda_a + \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& (\lambda_a + (1-\lambda_a)\,\alpha)\,c\frac{dC_{1,b}}{d\rho_{1,2}} = (1-\alpha)\,a_1 A \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)}{\lambda_a + (1-\lambda_a)\,\alpha}a_1 A.
\end{aligned} \tag{201}$$

Finally, we take the derivative of Eq. (124) with respect to the portfolio correlation and incorporate Eq. (201), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= [a_1 A + b_{1,2}B_{1,2} - cC_{1,b}] - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - [b_{1,2}B_{1,2} - cC_{1,b}] - (\lambda_a - \rho_{1,2})\,c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= a_1 A - \lambda_a c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= a_1 A - \lambda_a \frac{(1-\alpha)}{\lambda_a + (1-\lambda_a)\,\alpha}a_1 A \\
&= \left(1 - \frac{\lambda_a\,(1-\alpha)}{\lambda_a + (1-\lambda_a)\,\alpha}\right)a_1 A \\
&= \alpha\frac{a_1 A}{\lambda_a + (1-\lambda_a)\,\alpha} > 0.
\end{aligned} \tag{202}$$

9.12 Derivation of Eq. (142)

Since the derivative of Eqs. (57) and (129) with respect to the portfolio correlation are equivalent, it follows that again $cdC_{2,b}/d\rho_{1,2} = 0$ and thus $dB_{1,2}/d\rho_{1,2} = d\delta_{2,b}/d\rho_{1,2} = d\delta_{2,c}/d\rho_{1,2} = 0$ (see Section 9.8). Next, we take the implicit derivative of Eq. (128) with respect to the portfolio correlation

$$\begin{aligned}
& cC_{1,b} + \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \left[\alpha cC_{1,b} + (1-\alpha)\,\alpha cC_{1,b} + (1-\alpha)^2\,a_1A\right] + (\lambda_a - \rho_{1,2})\,[\alpha + (1-\alpha)\,\alpha]\,c\frac{dC_{1,b}}{d\rho_{1,2}} \\
- \quad & [\alpha cC_{1,b} + (1-\alpha)\,b_{1,2}B_{1,2}] + (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} + [\alpha cC_{1,b} + (1-\alpha)\,\alpha\delta_{2,b}g] + (1 - 2\lambda_a + \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& (\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha)\,c\frac{dC_{1,b}}{d\rho_{1,2}} \\
= \quad & (1-\alpha)\,[a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha\,(a_1A + \delta_{2,b}g - cC_{1,b})] \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)\,[a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha\,(a_1A + \delta_{2,b}g - cC_{1,b})]}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha}.
\end{aligned} \tag{203}$$

Finally, we take the derivative of Eq. (127) with respect to the portfolio correlation and incorporate Eqs. (203), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} \quad & = [a_1A + b_{1,2}B_{1,2} - cC_{1,b}] - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \alpha\,[a_1A + \delta_{2,b}g - cC_{1,b}] - (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
& = [a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha\,(a_1A + \delta_{2,b}g - cC_{1,b})] - (\rho_{1,2} + (\lambda_a - \rho_{1,2})\,\alpha)\,c\frac{dC_{1,b}}{d\rho_{1,2}} \\
& = [a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha\,(a_1A + \delta_{2,b}g - cC_{1,b})] \\
& - (\rho_{1,2} + (\lambda_a - \rho_{1,2})\,\alpha)\,\frac{(1-\alpha)\,[a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha\,(a_1A + \delta_{2,b}g - cC_{1,b})]}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha} \\
& = \left(1 - \frac{(\rho_{1,2} + (\lambda_a - \rho_{1,2})\,\alpha)\,(1-\alpha)}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha}\right) \\
& [a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha\,(a_1A + \delta_{2,b}g - cC_{1,b})] \\
& = \alpha\frac{a_1A + b_{1,2}B_{1,2} - cC_{1,b} - \alpha\,(a_1A + \delta_{2,b}g - cC_{1,b})}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha + (\lambda_a - \rho_{1,2})(1-\alpha)\alpha} > 0.
\end{aligned} \tag{204}$$

9.13 Derivation of Eq. (143)

Since the derivative of Eqs. (57) and (132) with respect to the portfolio correlation are equivalent, it follows that again $cdC_{2,b}/d\rho_{1,2} = 0$ and thus $dB_{1,2}/d\rho_{1,2} = d\delta_{2,b}/d\rho_{1,2} = d\delta_{2,c}/d\rho_{1,2} = 0$ (see Section 9.8). Next, we take the implicit derivative of Eq. (131) with respect to the portfolio correlation and rearrange

$$\begin{aligned}
& cC_{1,b} + \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - \left[\alpha cC_{1,b} + (1-\alpha)\,\alpha\,(a_1A + \delta_{2,b}g) + (1-\alpha)^2\,a_1A\right] + (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
- \quad & [\alpha cC_{1,b} + (1-\alpha)\,b_{1,2}B_{1,2}] + (\lambda_a - \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} + [\alpha cC_{1,b} + (1-\alpha)\,\alpha\delta_{2,b}g] + (1 - 2\lambda_a + \rho_{1,2})\,\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& (\rho_{1,2} + 2\,(\lambda_a - \rho_{1,2})\,\alpha + (1 - 2\lambda_a + \rho_{1,2})\,\alpha)\,c\frac{dC_{1,b}}{d\rho_{1,2}} = (1-\alpha)\,[a_1A + b_{1,2}B_{1,2} - cC_{1,b}] \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)\,[a_1A + b_{1,2}B_{1,2} - cC_{1,b}]}{\rho_{1,2} + 2\,(\lambda_a - \rho_{1,2})\,\alpha + (1 - 2\lambda_a + \rho_{1,2})\,\alpha}.
\end{aligned} \tag{205}$$

Finally, we take the derivative of Eq. (130) with respect to the portfolio correlation and incorporate Eq. (205), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= [a_1A + b_{1,2}B_{1,2} - cC_{1,b}] - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= [a_1A + b_{1,2}B_{1,2} - cC_{1,b}] - \rho_{1,2}\frac{(1-\alpha)}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha}[a_1A + b_{1,2}B_{1,2} - cC_{1,b}] \\
&= \left(1 - \frac{\rho_{1,2}(1-\alpha)}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha}\right)[a_1A + b_{1,2}B_{1,2} - cC_{1,b}] \\
&= \alpha\frac{a_1A + b_{1,2}B_{1,2} - cC_{1,b}}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2})\alpha + (1 - 2\lambda_a + \rho_{1,2})\alpha} > 0. \qquad (206)
\end{aligned}$$

9.14 Derivation of Eq. (144)

Since the derivative of Eqs. (57) and (135) with respect to the portfolio correlation are equivalent, it follows that again $cdC_{2,b}/d\rho_{1,2} = 0$ and thus $dB_{1,2}/d\rho_{1,2} = d\delta_{2,b}/d\rho_{1,2} = d\delta_{2,c}/d\rho_{1,2} = 0$ (see Section 9.8). Next, we take the implicit derivative of Eq. (134) with respect to the portfolio correlation and simplify

$$\begin{aligned}
& cC_{1,b} + \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - 2cC_{1,b} + 2(\lambda_a - \rho_{1,2})c\frac{dC_{1,b}}{d\rho_{1,2}} \\
+ \quad & [\alpha + (1-\alpha)\alpha]cC_{1,b} + (1 - 2\lambda_a + \rho_{1,2})[\alpha + (1-\alpha)\alpha]c\frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& (\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})[\alpha + (1-\alpha)\alpha])c\frac{dC_{1,b}}{d\rho_{1,2}} = (1-\alpha)^2 cC_{1,b} \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{(1-\alpha)^2}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})[\alpha + (1-\alpha)\alpha]}cC_{1,b}. \qquad (207)
\end{aligned}$$

Next, we take the derivative of Eq. (133) with respect to the portfolio correlation and incorporate Eq. (207), which yields

$$\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= [a_1A + b_{1,2}B_{1,2} - cC_{1,b}] - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - [a_1A + \alpha\delta_{2,b}g - cC_{1,b}] - (\lambda_a - \rho_{1,2})c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&- [b_{1,2}B_{1,2} - cC_{1,b}] - (\lambda_a - \rho_{1,2})c\frac{dC_{1,b}}{d\rho_{1,2}} + \alpha(\delta_{2,b}g - cC_{1,b}) - (1 - 2\lambda_a + \rho_{1,2})\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= cC_{1,b} - \rho_{1,2}c\frac{dC_{1,b}}{d\rho_{1,2}} - (\lambda_a - \rho_{1,2})c\frac{dC_{1,b}}{d\rho_{1,2}} - (\lambda_a - \rho_{1,2})c\frac{dC_{1,b}}{d\rho_{1,2}} - \alpha cC_{1,b} - (1 - 2\lambda_a + \rho_{1,2})\alpha c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= (1-\alpha)cC_{1,b} - [\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha]c\frac{dC_{1,b}}{d\rho_{1,2}} \\
&= (1-\alpha)cC_{1,b} - \frac{(1-\alpha)^2[\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha]}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})[\alpha + (1-\alpha)\alpha]}cC_{1,b} \\
&= \left(1 - \frac{[\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})\alpha](1-\alpha)}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})[\alpha + (1-\alpha)\alpha]}\right)(1-\alpha)cC_{1,b} \\
&= \alpha\frac{(1-\alpha)cC_{1,b}}{\rho_{1,2} + 2(\lambda_a - \rho_{1,2}) + (1 - 2\lambda_a + \rho_{1,2})[\alpha + (1-\alpha)\alpha]} > 0. \qquad (208)
\end{aligned}$$

9.15 Derivation of Eq. (145)

Since the derivative of Eqs. (57) and (138) with respect to the portfolio correlation are equivalent, it follows that again $cdC_{2,b}/d\rho_{1,2} = 0$ and thus $dB_{1,2}/d\rho_{1,2} = d\delta_{2,b}/d\rho_{1,2} = d\delta_{2,c}/d\rho_{1,2} = 0$ (see Section

9.8). Next, we take the implicit derivative of Eq. (137) with respect to the portfolio correlation and rearrange

$$
\begin{aligned}
& cC_{1,b} + \rho_{1,2} c \frac{dC_{1,b}}{d\rho_{1,2}} - 2cC_{1,b} + 2\left(\lambda_a - \rho_{1,2}\right) c \frac{dC_{1,b}}{d\rho_{1,2}} + \left[\alpha c C_{1,b} + \left(1-\alpha\right)\alpha \delta_{2,b} g\right] + \left(1 - 2\lambda_a + \rho_{1,2}\right) \alpha c \frac{dC_{1,b}}{d\rho_{1,2}} = 0 \\
& \left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha\right) c \frac{dC_{1,b}}{d\rho_{1,2}} = \left(1-\alpha\right)\left[cC_{1,b} - \alpha\delta_{2,b} g\right] \\
& c\frac{dC_{1,b}}{d\rho_{1,2}} = \frac{1-\alpha}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha}\left[cC_{1,b} - \alpha\delta_{2,b} g\right]. \qquad (209)
\end{aligned}
$$

Finally, we take the derivative of Eq. (136) with respect to the portfolio correlation and incorporate Eqs. (209), which yields

$$
\begin{aligned}
\frac{d\Pi_{1,b}}{d\rho_{1,2}} &= \left[a_1 A + b_{1,2} B_{1,2} - cC_{1,b}\right] - \rho_{1,2} c \frac{dC_{1,b}}{d\rho_{1,2}} - \left[a_1 A + \alpha\delta_{2,b} g - cC_{1,b}\right] - \left(\lambda_a - \rho_{1,2}\right) c \frac{dC_{1,b}}{d\rho_{1,2}} \\
&\quad - \left[b_{1,2} B_{1,2} - cC_{1,b}\right] - \left(\lambda_a - \rho_{1,2}\right) c \frac{dC_{1,b}}{d\rho_{1,2}} \\
&= \left[cC_{1,b} - \alpha\delta_{2,b} g\right] - \left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right)\right) c \frac{dC_{1,b}}{d\rho_{1,2}} \\
&= \left[cC_{1,b} - \alpha\delta_{2,b} g\right] - \left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right)\right) \frac{1-\alpha}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha}\left[cC_{1,b} - \alpha\delta_{2,b} g\right] \\
&= \left(1 - \frac{\left(\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right)\right)\left(1-\alpha\right)}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha}\right)\left[cC_{1,b} - \alpha\delta_{2,b} g\right] \\
&= \alpha \frac{cC_{1,b} - \alpha\delta_{2,b} g}{\rho_{1,2} + 2\left(\lambda_a - \rho_{1,2}\right) + \left(1 - 2\lambda_a + \rho_{1,2}\right)\alpha} > 0. \qquad (210)
\end{aligned}
$$

9.16 Derivation of Eqs. (62), (152), and (153)

First, we derive the implicit derivative of the expected return of bank 1 with respect to the interbank loan size for Cases (b.i), (b.ii.ii), (b.ii.iii), (b.iii.i), (b.iii.ii), and (ii.iv.ii). Incorporating bank 1's budget constraint from Eq. (45) and taking the derivative of Eq. (59) with respect to $b_{1,2}$ yields

$$
\frac{d\Pi_{1,b}}{db_{1,2}} = \lambda_a \left(-A + B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}} - c\frac{dC_{1,b}}{db_{1,2}}\right). \qquad (211)
$$

Moreover, the implicit derivative of the binding participation of the creditors of bank 1 from Eq. (60) with respect to the interbank loan size is given by

$$
\begin{aligned}
& \lambda_a c \frac{dC_{1,b}}{db_{1,2}} + \left(1-\lambda_a\right)\left(\alpha \frac{dC_{1,b}}{db_{1,2}} + \left(1-\alpha\right)\alpha \frac{d\delta_{2,b}}{db_{1,2}} g\right) = 0 \\
& \left(\lambda_a + \left(1-\lambda_a\right)\alpha\right) c \frac{dC_{1,b}}{db_{1,2}} + \left(1-\lambda_a\right)\left(1-\alpha\right)\alpha \frac{d\delta_{2,b}}{db_{1,2}} g = 0, \qquad (212)
\end{aligned}
$$

and the implicit derivative of $\delta_{2,b}$ with respect to the interbank loan size becomes

$$
\begin{aligned}
\frac{d\delta_{2,b}}{db_{1,2}} &= \frac{\left(B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right)\left(b_{1,2} B_{1,2} + cC_{2,b}\right) - b_{1,2} B_{1,2}\left(c\frac{dC_{2,b}}{db_{1,2}} + B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right)}{\left(b_{1,2} B_{1,2} + cC_{2,b}\right)^2} \\
&= \frac{\left(B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right) cC_{2,b} - b_{1,2} B_{1,2} c\frac{dC_{2,b}}{db_{1,2}}}{\left(b_{1,2} B_{1,2} + cC_{2,b}\right)^2}. \qquad (213)
\end{aligned}
$$

Furthermore, the implicit derivative of the participation of the creditors of bank 2 from Eq. (61) with respect to the interbank loan size is given by

$$\begin{aligned} \lambda_a c \frac{dC_{2,b}}{db_{1,2}} - (1-\lambda_a)\,\alpha \frac{d\delta_{2,b}}{db_{1,2}} g &= 0 \\ c\frac{dC_{2,b}}{db_{1,2}} &= \frac{(1-\lambda_a)}{\lambda_a}\alpha \frac{d\delta_{2,b}}{db_{1,2}} g, \end{aligned} \tag{214}$$

where we substituted $\delta_{2,b}$ for $\delta_{2,c}$.

Next, taking the implicit derivative of the participation constraint of bank 2 from Eq. (50) with respect to the interbank loan size yields

$$\begin{aligned} \lambda_a \left(A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2} - b_{1,2}\frac{dB_{1,2}}{db_{1,2}} \right) &= 0 \\ b_{1,2}\frac{dB_{1,2}}{db_{1,2}} &= A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2}. \end{aligned} \tag{215}$$

First, we plug Eq. (215) into Eqs. (211) and (213) and rearrange:

$$\begin{aligned} \frac{d\Pi_{1,b}}{db_{1,2}} &= \lambda_a \left(-A + B_{1,2} + A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2} - c\frac{dC_{1,b}}{db_{1,2}} \right) \\ &= -\lambda_a c \left(\frac{dC_{2,b}}{db_{1,2}} + \frac{dC_{1,b}}{db_{1,2}} \right) \end{aligned} \tag{216}$$

$$\begin{aligned} \frac{d\delta_{2,b}}{db_{1,2}} &= \frac{\left(B_{1,2} + A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2} \right) cC_{2,b} - b_{1,2}B_{1,2}c\frac{dC_{2,b}}{db_{1,2}}}{\left(b_{1,2}B_{1,2} + cC_{2,b}\right)^2} \\ &= \frac{AcC_{2,b} - c\frac{dC_{2,b}}{db_{1,2}}\left(cC_{2,b} + b_{1,2}B_{1,2}\right)}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2}. \end{aligned} \tag{217}$$

Second, we plug Eq. (214) into Eq. (217) and rearrange, which yields

$$\begin{aligned} \frac{d\delta_{2,b}}{db_{1,2}} &= \frac{AcC_{2,b} - \frac{(1-\lambda_a)}{\lambda_a}\alpha\frac{d\delta_{2,b}}{db_{1,2}} g\left(cC_{2,b} + b_{1,2}B_{1,2}\right)}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} \\ \frac{d\delta_{2,b}}{db_{1,2}} \left(1 + \frac{(1-\lambda_a)\,\alpha}{\lambda_a} \frac{g}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)} \right) &= A\frac{cC_{2,b}}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} \\ \frac{d\delta_{2,b}}{db_{1,2}} &= \frac{\lambda_a A}{\lambda_a + (1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}} \frac{cC_{2,b}}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2}. \end{aligned} \tag{218}$$

Third, we plug Eq. (218) into Eq. (212) and rearrange, which gives:

$$\begin{aligned} \left(\lambda_a + (1-\lambda_a)\,\alpha\right) c\frac{dC_{1,b}}{db_{1,2}} &= -\left(1-\lambda_a\right)\left(1-\alpha\right)\alpha\frac{d\delta_{2,b}}{db_{1,2}} g \\ \left(\lambda_a + (1-\lambda_a)\,\alpha\right) c\frac{dC_{1,b}}{db_{1,2}} &= -\left(1-\lambda_a\right)\left(1-\alpha\right)\alpha\frac{\lambda_a A}{\lambda_a + (1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}} \frac{cC_{2,b}}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} g \\ c\frac{dC_{1,b}}{db_{1,2}} &= -\frac{(1-\lambda_a)(1-\alpha)\,\alpha}{\lambda_a + (1-\lambda_a)\,\alpha} \frac{\lambda_a A}{\lambda_a + (1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}} \frac{cC_{2,b}}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} g. \end{aligned} \tag{219}$$

Fourth, we plug Eq. (218) into Eq. (214) and simplify:

$$
\begin{aligned}
c\frac{dC_{2,b}}{db_{1,2}} &= \frac{(1-\lambda_a)\,\alpha}{\lambda_a}\frac{\lambda_a A}{\lambda_a+(1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}}\frac{cC_{2,b}}{(cC_{2,b}+b_{1,2}B_{1,2})^2}g \\
&= \frac{(1-\lambda_a)\,\alpha A}{\lambda_a+(1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}}\frac{cC_{2,b}}{(cC_{2,b}+b_{1,2}B_{1,2})^2}g. \qquad (220)
\end{aligned}
$$

Finally, we substitute Eqs. (219) and (220) into Eq. (216) and rearrange, which yields the final result:

$$
\begin{aligned}
\frac{d\Pi_{1,b}}{db_{1,2}} &= -\lambda_a\frac{(1-\lambda_a)\,\alpha A}{\lambda_a+(1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}}\frac{cC_{2,b}}{(cC_{2,b}+b_{1,2}B_{1,2})^2}g \\
&+\frac{\lambda_a(1-\lambda_a)(1-\alpha)\,\alpha}{\lambda_a+(1-\lambda_a)\,\alpha}\frac{\lambda_a A}{\lambda_a+(1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}}\frac{cC_{2,b}}{(cC_{2,b}+b_{1,2}B_{1,2})^2}g \\
&= -\lambda_a\alpha(1-\lambda_a)\left(1-\frac{(1-\alpha)\,\lambda_a}{\lambda_a+(1-\lambda_a)\,\alpha}\right)\frac{A}{\lambda_a+(1-\lambda_a)\,\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}}\frac{cC_{2,b}}{(cC_{2,b}+b_{1,2}B_{1,2})^2}g \\
&= -\frac{\frac{\lambda_a(1-\lambda_a)\alpha^2}{\lambda_a+(1-\lambda_a)\alpha}\frac{\delta_{2,c}gA}{cC_{2,b}+b_{1,2}B_{1,2}}}{\lambda_a+(1-\lambda_a)\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}} < 0. \qquad (221)
\end{aligned}
$$

Next, we determine the derivatives of the expected return of bank 1 with respect to the interbank loan size for the Cases (b.ii.i) and (b.iv.i). The problem is similar to the one above, but Eqs. (211) and (212) are replaced by

$$
\frac{d\Pi_{1,b}}{db_{1,2}} = \lambda_a\left(-A+B_{1,2}+b_{1,2}\frac{dB_{1,2}}{db_{1,2}}-c\frac{dC_{1,b}}{db_{1,2}}\right)+(1-\lambda_a)\,\alpha\left(\frac{d\delta_{2,b}}{db_{1,2}}g-c\frac{dC_{1,b}}{db_{1,2}}\right). \qquad (222)
$$

and

$$
\begin{aligned}
\lambda_a c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)\left[\alpha+(1-\alpha)\,\alpha\right]c\frac{dC_{1,b}}{db_{1,2}} &= 0 \\
\frac{dC_{1,b}}{db_{1,2}} &= 0. \qquad (223)
\end{aligned}
$$

After plugging Eqs. (214), (215), and (223) into Eq. (222) and rearranging, we obtain the final result:

$$
\frac{d\Pi_{1,b}}{db_{1,2}} = 0. \qquad (224)
$$

Next, we show that for all sub-cases of Case (b) it holds that $dG_{1,b}/db_{1,2} = d\Pi_{1,b}/db_{1,2}$. Using the participation constraints of the creditors of bank 1 and bank 2 from Eqs. (60) and (61), respectively, and the participation constraint of bank 2 from Eq. (50), the expected bank return for Cases (b.i), (b.ii.ii), (b.ii.iii), (b.iii.i), (b.iii.ii), and (ii.iv.ii) from Eq. (59) can be rewritten as

$$
\Pi_{1,b} = \lambda_a\left[A-\frac{c}{\lambda_a+(1-\lambda_a)\alpha}\right]+\underbrace{(1-\lambda_a)\left[\alpha cC_{1,b}+\alpha g-\alpha^2\delta_{2,b}g\right]}_{=G_{1,b}}-2c. \qquad (225)
$$

Taking the derivative of the value of the bailout subsidy $G_{1,b}$ with respect to $b_{1,2}$ yields

$$
\frac{dG_{1,b}}{db_{1,2}} = (1-\lambda_a)\left[\alpha c\frac{dC_{1,b}}{db_{1,2}}-\alpha^2\frac{d\delta_{2,b}}{db_{1,2}}g\right]. \qquad (226)
$$

Finally, plugging Eqs. (218) and (219) into Eq. (226) and simplifying shows that $dG_{1,b}/db_{1,2} =$

$d\Pi_{1,b}/db_{1,2}$ from Eq. (221).

Similarly, for Cases (b.ii.i) and (b.iv.i), the expected bank return from Eq. (149) can be rewritten as

$$\Pi_{1,b} = \lambda_a \left[A - \frac{c}{\lambda_a + (1-\lambda_a)\alpha}\right] + \underbrace{(1-\lambda_a)\alpha g + (1-\lambda_a)(1-\alpha)\alpha c C_{1,b}}_{=G_{1,b}} - 2c, \tag{227}$$

to isolate the value of the bailout subsidy. Taking the derivative of $G_{1,b}$ with respect to $b_{1,2}$ yields

$$\frac{dG_{1,b}}{db_{1,2}} = (1-\lambda_a)(1-\alpha)\alpha c \frac{dC_{1,b}}{db_{1,2}} = 0, \tag{228}$$

which is equal due to zero due to Eq. (223) and thus $dG_{1,b}/db_{1,2} = d\Pi_{1,b}/db_{1,2}$ from Eq. (224).

9.17 Derivation of Eqs. (76) and (77)

First, we determine the derivative of Eq. (72) with respect to $b_{1,2}$, which yields

$$\frac{d\Pi_{1,b}}{db_{1,2}} = \lambda_a \left[B - (c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}} - B\right] = -\lambda_a (c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}}, \tag{229}$$

where we already incorporated the banks' budget constraints and the investment constraints $a_1 \leq 1$ and $a_2 \leq 1$. Second, taking the implicit derivative of Eq. (74) with respect to $b_{1,2}$ gives

$$\lambda_a (c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}} + (1-\lambda_a)\left[\frac{\chi \frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}\delta_{1,c} b_{1,2} + \alpha\left(\frac{d\delta_{1,c}}{db_{1,2}} b_{1,2} + \delta_{1,c}\right)\right] B = 0. \tag{230}$$

Third, we determine derivative of $\delta_{1,c}$ with respect to $b_{1,2}$:

$$\begin{aligned}
\frac{d\delta_{1,c}}{db_{1,2}} &= \frac{(c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}}\left((c-\underline{\epsilon})C_{1,b} + b_{2,1}B\right) - (c-\underline{\epsilon})C_{1,b}\left((c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}} + B\right)}{\left((c-\underline{\epsilon})C_{1,b} + b_{2,1}B\right)^2} \\
&= \frac{(c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}} b_{2,1}B - (c-\underline{\epsilon})C_{1,b}B}{\left((c-\underline{\epsilon})C_{1,b} + b_{2,1}B\right)^2} \\
&= \frac{b_{2,1}B}{\left((c-\underline{\epsilon})C_{1,b} + b_{2,1}B\right)^2}(c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}} - \frac{(c-\underline{\epsilon})C_{1,b}}{\left((c-\underline{\epsilon})C_{1,b} + b_{2,1}B\right)^2}B.
\end{aligned} \tag{231}$$

Next, we substitute Eq. (231) into Eq. (230) and simplify:

$$\lambda_a(c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}}$$
$$+\left(1-\lambda_a\right)\left[\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}\delta_{1,c}+\alpha\left(\frac{b_{2,1}B(c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}}}{\left((c-\underline{\epsilon})C_{1,b}+b_{2,1}B\right)^2}-\frac{(c-\underline{\epsilon})C_{1,b}B}{\left((c-\underline{\epsilon})C_{1,b}+b_{2,1}B\right)^2}\right)\right]b_{1,2}B=-\left(1-\lambda_a\right)\alpha\delta_{1,c}B$$
$$\left(\lambda_a+\frac{\left(1-\lambda_a\right)\alpha b_{2,1}Bb_{1,2}B}{\left((c-\underline{\epsilon})C_{1,b}+b_{2,1}B\right)^2}\right)(c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}}=-\left(1-\lambda_a\right)\left[\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}\delta_{1,c}b_{1,2}+\alpha\left(\delta_{1,c}-\frac{(c-\underline{\epsilon})C_{1,b}b_{1,2}B}{\left((c-\underline{\epsilon})C_{1,b}+b_{2,1}B\right)^2}\right)\right]B$$
$$(c-\underline{\epsilon})\frac{dC_{1,b}}{db_{1,2}}=-\frac{\left(1-\lambda_a\right)\left[\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}b_{1,2}+\alpha\frac{(c-\epsilon)C_{1,b}+\epsilon B}{(c-\underline{\epsilon})C_{1,b}+b_{2,1}B}\right]}{\lambda_a+\left(1-\lambda_a\right)\alpha\frac{b_{1,2}Bb_{2,1}B}{\left((c-\underline{\epsilon})C_{1,b}+b_{2,1}B\right)^2}}\delta_{1,c}B, \tag{232}$$

where we incorporated that $b_{2,1}=b_{1,2}+\underline{\epsilon}$, which follows from the banks' budget constraints. Substituting Eq. (232) into Eq. (229) yields

$$\frac{d\Pi_{1,b}}{db_{1,2}}=\frac{\lambda_a\left(1-\lambda_a\right)\left[\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}b_{1,2}+\alpha\frac{(c-\epsilon)C_{1,b}+\epsilon B}{(c-\underline{\epsilon})C_{1,b}+b_{2,1}B}\right]}{\lambda_a+\left(1-\lambda_a\right)\alpha\frac{b_{1,2}Bb_{2,1}B}{\left((c-\underline{\epsilon})C_{1,b}+b_{2,1}B\right)^2}}\delta_{1,c}B. \tag{233}$$

Finally, plugging $b_{1,2}=0$ into Eq. (233) gives the final result

$$\frac{d\Pi_{1,b}}{db_{1,2}}(b_{1,2}=0)=\left(1-\lambda_a\right)\alpha\delta_{1,c}B>0. \tag{234}$$

In a second step, we determine the derivative of Eq. (73) with respect to $b_{1,2}$, which yields

$$\frac{d\Pi_{2,b}}{db_{1,2}}=\lambda_a\left[B-(c+\underline{\epsilon})\frac{dC_{2,b}}{db_{1,2}}-B\right]=-\lambda_a(c+\underline{\epsilon})\frac{dC_{2,b}}{db_{1,2}}, \tag{235}$$

where we again already incorporated the banks' budget constraints and the investment constraints. Taking the implicit derivative of Eq. (75) with respect to $b_{1,2}$ and rearranging yields

$$\lambda_a(c+\underline{\epsilon})\frac{dC_{2,b}}{db_{1,2}}+\left(1-\lambda_a\right)\left(\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}\left[(c+\underline{\epsilon})C_{2,b}-\left(1-\frac{\chi}{2L}\right)L\right]+\alpha(c+\underline{\epsilon})\frac{dC_{2,b}}{db_{1,2}}\right)=0$$
$$(c+\underline{\epsilon})\frac{dC_{2,b}}{db_{1,2}}=-\frac{\left(1-\lambda_a\right)\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}\left[(c+\underline{\epsilon})C_{2,b}-\left(1-\frac{\chi}{2L}\right)L\right]}{\lambda_a+\left(1-\lambda_a\right)\alpha}. \tag{236}$$

Finally, plugging Eq. (236) and $b_{1,2}=0$ into (235) yields the final result

$$\frac{d\Pi_{2,b}}{db_{1,2}}(b_{1,2}=0)=\frac{\lambda_a\left(1-\lambda_a\right)\frac{\chi\frac{d\overline{\beta}}{db_{1,2}}}{\overline{\beta}^2 L}\left[(c+\underline{\epsilon})C_{2,b}-\left(1-\frac{\chi}{2L}\right)L\right]}{\lambda_a+\left(1-\lambda_a\right)\alpha}>0. \tag{237}$$

9.18 Derivation of Eqs. (87) to (89)

As the optimization problems of the three banks are similar due to the symmetric setup, we only solve bank 1's maximization problem in the following. First, we determine the implicit derivative of the binding participation constraint of the creditors of bank 1 from Eq. (84) with respect to $b_{1,2}$:

$$\lambda_a c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)\left(\alpha c\frac{dC_{1,b}}{db_{1,2}}+(1-\alpha)\,\alpha\left(\frac{d\delta_{1,c}}{db_{1,2}}b_{1,2}+\delta_{1,c}\right)B\right)=0. \tag{238}$$

Second, we derive the derivative of $\delta_{1,c}$ with respect to $b_{1,2}$, incorporating the budget constraints of bank 1 and 3:

$$\begin{aligned}\frac{d\delta_{1,c}}{db_{1,2}} &= \frac{c\frac{dC_{1,b}}{db_{1,2}}\left(cC_{1,b}+b_{3,1}B\right)-cC_{1,b}\left(c\frac{dC_{1,b}}{db_{1,2}}+B\right)}{\left(cC_{1,b}+b_{3,1}B\right)^2}\\ &= \frac{c\frac{dC_{1,b}}{db_{1,2}}b_{3,1}B-cC_{1,b}B}{\left(cC_{1,b}+b_{3,1}B\right)^2}.\end{aligned} \tag{239}$$

Next, we plug Eq. (239) into Eq. (238) and solve for $dC_{1,b}/db_{1,2}$, incorporating the budget constraints of bank 1 and 3, which yields

$$\lambda_a c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)\left(\alpha c\frac{dC_{1,b}}{db_{1,2}}+(1-\alpha)\,\alpha\left(\frac{c\frac{dC_{1,b}}{db_{1,2}}b_{3,1}B-cC_{1,b}B}{\left(cC_{1,b}+b_{3,1}B\right)^2}b_{1,2}+\delta_{1,c}\right)B\right)=0. \tag{240}$$

Solving Eq. (240) for $cdC_{1,b}/db_{1,2}$ and simplifying yields

$$c\frac{dC_{1,b}}{db_{1,2}}=-\frac{(1-\lambda_a)(1-\alpha)\,\alpha}{\lambda_a+(1-\lambda_a)\,\alpha+(1-\lambda_a)(1-\alpha)\,\alpha\frac{b_{1,2}Bb_{3,1}B}{(cC_{1,b}+b_{3,1}B)^2}}\delta_{1,c}^2B. \tag{241}$$

Finally, taking the derivative of bank 1's expected return from Eq. (78) with respect to $b_{1,2}$ and incorporating the banks' budget constraints and Eq. (241) yields the final result:

$$\begin{aligned}\frac{d\Pi_{1,b}}{db_{1,2}} &= -\lambda_a c\frac{dC_{1,b}}{db_{1,2}}\\ &= \frac{\lambda_a(1-\lambda_a)(1-\alpha)\,\alpha}{\lambda_a+(1-\lambda_a)\,\alpha+(1-\lambda_a)(1-\alpha)\,\alpha\frac{b_{1,2}Bb_{3,1}B}{(cC_{1,b}+b_{3,1}B)^2}}\delta_{1,c}^2B>0.\end{aligned} \tag{242}$$

9.19 Derivation of Eq. (95)

First, determining the derivative of the binding participation constraint of the creditors of bank i from Eq. (91) with respect $\rho_{1,2}$ and rearranging yields

$$\begin{aligned}&\left(\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha\right)c\frac{dC_{i,b}}{d\rho_{1,2}}\\ =\;& (1-\alpha)^2\left(A-cC_{i,b}\right)-(1-\lambda_a)(1-\alpha)\,\alpha\frac{d\delta_{i,c}}{d\rho_{1,2}}b_{i,j\neq i}B,\end{aligned} \tag{243}$$

where we already used that $a_i=a_{j\neq i}=1$, which follows from the banks' budget constraints from Eq. (22) and $a_i\leq 1$ (see Section 9.2). Next, we derive the derivative of $\delta_{i,c}$ with respect to $\rho_{1,2}$

$$\begin{aligned}\frac{d\delta_{i,c}}{d\rho_{1,2}} &= \frac{c\frac{dC_{i,b}}{d\rho_{1,2}}\left(cC_{i,b}+b_{j\neq i,i}B\right)-cC_{i,b}c\frac{dC_{i,b}}{d\rho_{1,2}}}{\left(cC_{i,b}+b_{j\neq i,i}B\right)^2}\\ &= c\frac{dC_{i,b}}{d\rho_{1,2}}\frac{b_{j\neq i,i}B}{\left(cC_{i,b}+b_{j\neq i,i}B\right)^2}.\end{aligned} \tag{244}$$

Plugging Eq. (244) into Eq. (243) and simplifying yields the final result:

$$
\begin{aligned}
&(\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha)\, c\frac{dC_{i,b}}{d\rho_{1,2}}\\
&=(1-\alpha)^2(A-cC_{i,b})-(1-\lambda_a)(1-\alpha)\,\alpha c\frac{dC_{i,b}}{d\rho_{1,2}}\frac{b_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}b_{i,j\neq i}B\\
&\left(\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha+(1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right)c\frac{dC_{i,b}}{d\rho_{1,2}}=(1-\alpha)^2(A-cC_{i,b})\\
&\frac{dC_{i,b}}{d\rho_{1,2}}=\frac{1}{c}\frac{(1-\alpha)^2(A-cC_{i,b})}{\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha+(1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}}>0.
\end{aligned}
\tag{245}
$$

9.20 Derivation of Eq. (96)

First, incorporating the banks' budget constraints from Eq. (22) and taking the derivative of Eq. (90) with respect to $\rho_{1,2}$ yields

$$
\frac{d\Pi_{i,b}}{d\rho_{1,2}}=(1-\alpha)(a_iA-cC_{i,b})-(\rho_{1,2}+(\lambda_a-\rho_{1,2})\alpha)c\frac{dC_{i,b}}{d\rho_{1,2}}.
\tag{246}
$$

Next, we plug Eq. (245) into Eq. (246), which yields the final result:

$$
\begin{aligned}
\frac{d\Pi_{i,b}}{d\rho_{1,2}}&=(1-\alpha)(A-cC_{i,b})-(\rho_{1,2}+(\lambda_a-\rho_{1,2})\alpha)\\
&\quad c\frac{1}{c}\frac{(1-\alpha)^2(A-cC_{i,b})}{\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha+(1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}}\\
&=\left(1-\frac{(\rho_{1,2}+(\lambda_a-\rho_{1,2})\alpha)(1-\alpha)}{\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha+(1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}}\right)(1-\alpha)(A-cC_{i,b})\\
&=\frac{(1-\alpha)\alpha\left[1+(1-\lambda_a)(1-\alpha)\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}\right][A-cC_{i,b}]}{\rho_{1,2}+(1-\rho_{1,2})\alpha+(\lambda_a-\rho_{1,2})(1-\alpha)\alpha+(1-\lambda_a)(1-\alpha)\alpha\frac{b_{i,j\neq i}Bb_{j\neq i,i}B}{(cC_{i,b}+b_{j\neq i,i}B)^2}}>0,
\end{aligned}
\tag{247}
$$

where we used that $a_i=1$ to simplify the expression, which follows from the banks' budget constraints and $a_i\leq 1$ (see Section 9.2).

9.21 Derivation of Eq. (158)

First, we determine the implicit derivative of the participation constraint for the creditors of bank 1 from Eq. (155) with respect to $b_{1,2}$:

$$
\begin{aligned}
&\lambda_a c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)\frac{1}{2}\left[\alpha c\frac{dC_{1,b}}{db_{1,2}}+(1-\alpha)\,\alpha\left(B_{1,2}+b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right)\right]\\
&+(1-\lambda_a)\frac{1}{2}\left[-\alpha^2c\frac{dC_{2,b}}{db_{1,2}}+\alpha(1-\alpha)\left(B_{1,2}+b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right)+(1-\alpha)\,\alpha c\frac{dC_{1,b}}{db_{1,2}}\right]=0\\
&\left(\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\,\alpha\right]\right)c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)(1-\alpha)\,\alpha\left(B_{1,2}+b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right)\\
&-(1-\lambda_a)\frac{1}{2}\alpha^2c\frac{dC_{2,b}}{db_{1,2}}=0.
\end{aligned}
\tag{248}
$$

Second, we determine the implicit derivative of the participation constraint of the creditors of bank 2 from Eq. (157) with respect to the interbank loan:

$$\begin{aligned}
&\lambda_a c\frac{dC_{2,b}}{db_{1,2}}+(1-\lambda_a)\frac{1}{2}\left[\alpha c\frac{dC_{2,b}}{db_{1,2}}-\alpha^2 c\frac{dC_{1,b}}{db_{1,2}}+(1-\alpha)\alpha c\frac{dC_{2,b}}{db_{1,2}}\right]=0\\
&\left(\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]\right)c\frac{dC_{2,b}}{db_{1,2}}=(1-\lambda_a)\frac{1}{2}\alpha^2 c\frac{dC_{1,b}}{db_{1,2}}\\
&c\frac{dC_{2,b}}{db_{1,2}}=\frac{(1-\lambda_a)\frac{1}{2}\alpha^2}{\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]}c\frac{dC_{1,b}}{db_{1,2}}. \qquad (249)
\end{aligned}$$

Third, the implicit derivative bank 2's participation constraint from Eq. (156) with respect to the interbank loan is given by:

$$\begin{aligned}
\frac{d\Pi_{2,b}}{db_{1,2}} &= \lambda_a\left[A-c\frac{dC_{2,b}}{db_{1,2}}-B_{1,2}-b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right]=0\\
b_{1,2}\frac{dB_{1,2}}{db_{1,2}} &= A-c\frac{dC_{2,b}}{db_{1,2}}-B_{1,2}. \qquad (250)
\end{aligned}$$

Next, we plug Eq. (250) into Eq. (248), which yields

$$\begin{aligned}
&\left(\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]\right)c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)(1-\alpha)\alpha\left(B_{1,2}+A-c\frac{dC_{2,b}}{db_{1,2}}-B_{1,2}\right)-(1-\lambda_a)\frac{1}{2}\alpha^2 c\frac{dC_{2,b}}{db_{1,2}}=0\\
&\left(\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]\right)c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)(1-\alpha)\alpha A-(1-\lambda_a)\alpha\left(1-\frac{1}{2}\alpha\right)c\frac{dC_{2,b}}{db_{1,2}}=0. \qquad (251)
\end{aligned}$$

Moreover, we substitute Eq. (249) in (251) and rearrange, which yields

$$\begin{aligned}
&\left(\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]\right)c\frac{dC_{1,b}}{db_{1,2}}+(1-\lambda_a)(1-\alpha)\alpha A\\
-\;&(1-\lambda_a)\alpha\left(1-\frac{1}{2}\alpha\right)\frac{(1-\lambda_a)\frac{1}{2}\alpha^2}{\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]}c\frac{dC_{1,b}}{db_{1,2}}=0\\
&\left(\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]-\frac{(1-\lambda_a)^2\left(1-\frac{1}{2}\alpha\right)\frac{1}{2}\alpha^3}{\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]}\right)c\frac{dC_{1,b}}{db_{1,2}}=-(1-\lambda_a)(1-\alpha)\alpha A\\
&c\frac{dC_{1,b}}{db_{1,2}}=-\frac{(1-\lambda_a)(1-\alpha)\alpha}{\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]-\frac{(1-\lambda_a)^2\left(1-\frac{1}{2}\alpha\right)\frac{1}{2}\alpha^3}{\lambda_a+(1-\lambda_a)\frac{1}{2}\left[\alpha+(1-\alpha)\alpha\right]}}A. \qquad (252)
\end{aligned}$$

Finally, we incorporate bank 1's budget constraint and take the implicit derivative of bank 1's expected return from Eq. (154) with respect to the size of interbank loan and substitute Eqs. (252), (249) and

(250)

$$
\begin{aligned}
\frac{d\Pi_{1,b}}{db_{1,2}} &= \lambda_a \left(-A + B_{1,2} + b_{1,2} \frac{dB_{1,2}}{db_{1,2}} - c \frac{dC_{1,b}}{db_{1,2}} \right) \\
&= \lambda_a \left(-A + B_{1,2} + A - c \frac{dC_{2,b}}{db_{1,2}} - B_{1,2} - c \frac{dC_{1,b}}{db_{1,2}} \right) \\
&= -\lambda_a \left(\frac{dC_{2,b}}{db_{1,2}} + \frac{dC_{1,b}}{db_{1,2}} \right) \\
&= -\lambda_a \left(\frac{(1-\lambda_a) \frac{1}{2} \alpha^2}{\lambda_a + (1-\lambda) \frac{1}{2} [\alpha + (1-\alpha)\alpha]} c \frac{dC_{1,b}}{db_{1,2}} + c \frac{dC_{1,b}}{db_{1,2}} \right) \\
&= -\lambda_a \left(\frac{(1-\lambda_a) \frac{1}{2} \alpha^2}{\lambda_a + (1-\lambda) \frac{1}{2} [\alpha + (1-\alpha)\alpha]} + 1 \right) c \frac{dC_{1,b}}{db_{1,2}} \\
&= -\lambda_a \frac{\lambda_a + (1-\lambda_a)\alpha}{\lambda_a + (1-\lambda_a) \frac{1}{2} [\alpha + (1-\alpha)\alpha]} c \frac{dC_{1,b}}{db_{1,2}} \\
&= \lambda_a \frac{\lambda_a + (1-\lambda_a)\alpha}{\lambda_a + (1-\lambda_a) \frac{1}{2} [\alpha + (1-\alpha)\alpha]} \frac{(1-\lambda_a)(1-\alpha)\alpha}{\lambda_a + (1-\lambda_a) \frac{1}{2} [\alpha + (1-\alpha)\alpha] - \frac{(1-\lambda_a)^2 \left(1-\frac{1}{2}\alpha\right) \frac{1}{2}\alpha^3}{\lambda_a + (1-\lambda_a)\frac{1}{2}[\alpha + (1-\alpha)\alpha]}} A \\
&= \lambda_a \frac{\lambda_a + (1-\lambda_a)\alpha}{\lambda_a + (1-\lambda_a) \frac{1}{2} [\alpha + (1-\alpha)\alpha]} \frac{(1-\lambda_a)(1-\alpha)\alpha \left(\lambda_a + (1-\lambda_a) \frac{1}{2} [\alpha + (1-\alpha)\alpha]\right)}{\left(\lambda_a + (1-\lambda_a) \frac{1}{2} [\alpha + (1-\alpha)\alpha]\right)^2 - (1-\lambda_a)^2 \left(1-\frac{1}{2}\alpha\right) \frac{1}{2}\alpha^3} A \\
&= \frac{\lambda_a (1-\lambda_a)(1-\alpha)\alpha (\lambda_a + (1-\lambda_a)\alpha)}{\left(\lambda_a + (1-\lambda_a) \frac{1}{2} [\alpha + (1-\alpha)\alpha]\right)^2 - (1-\lambda_a)^2 \left(1-\frac{1}{2}\alpha\right) \frac{1}{2}\alpha^3} A > 0. \qquad (253)
\end{aligned}
$$

9.22 Derivation of Eq. (162)

First, we derive the implicit derivative of the binding participation constraint of the creditors of bank 1 from Eq. (160) with respect to the interbank loan

$$
\begin{aligned}
\lambda_a c \frac{dC_{1,b}}{db_{1,2}} + (1-\lambda_a) \left(\alpha c \frac{dC_{1,b}}{db_{1,2}} + (1-\alpha) \alpha c \frac{dC_{1,b}}{db_{1,2}} \right) &= 0 \\
c \frac{dC_{1,b}}{db_{1,2}} &= 0. \qquad (254)
\end{aligned}
$$

Second, we determine the implicit derivative of $\delta_{2,c}$ with respect to $b_{1,2}$

$$
\begin{aligned}
\frac{d\delta_{2,c}}{db_{1,2}} &= \frac{c \frac{dC_{2,b}}{db_{1,2}} (cC_{2,b} + b_{1,2} B_{1,2}) - cC_{2,b} \left(c \frac{dC_{2,b}}{db_{1,2}} + B_{1,2} + b_{1,2} \frac{dB_{1,2}}{db_{1,2}} \right)}{(cC_{2,b} + b_{1,2} B_{1,2})^2} \\
&= \frac{c \frac{dC_{2,b}}{db_{1,2}} b_{1,2} B_{1,2} - cC_{2,b} B_{1,2} - cC_{2,b} b_{1,2} \frac{dB_{1,2}}{db_{1,2}}}{(cC_{2,b} + b_{1,2} B_{1,2})^2}. \qquad (255)
\end{aligned}
$$

Third, we take the implicit derivative of the binding participation constraint for the creditors of bank 2 from Eq. (161) with respect to the interbank loan:

$$
\begin{aligned}
\lambda_a c \frac{dC_{2,b}}{db_{1,2}} + (1-\lambda_a) \alpha \frac{d\delta_{2,c}}{db_{1,2}} g &= 0 \\
c \frac{dC_{2,b}}{db_{1,2}} &= -\frac{1-\lambda_a}{\lambda_a} \alpha \frac{d\delta_{2,c}}{db_{1,2}} g. \qquad (256)
\end{aligned}
$$

Fourth, we derive the implicit derivative of the binding participation constraint of bank 2 from Eq. (156) with respect to $b_{1,2}$:

$$\begin{aligned}
\frac{d\Pi_{2,b}}{db_{1,2}} &= \lambda_a \left(A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2} - b_{1,2}\frac{dB_{1,2}}{db_{1,2}} \right) = 0 \\
b_{1,2}\frac{dB_{1,2}}{db_{1,2}} &= A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2}.
\end{aligned} \tag{257}$$

Next, we plug Eq. (257) into Eq. (255), which yields

$$\begin{aligned}
\frac{d\delta_{2,c}}{db_{1,2}} &= \frac{c\frac{dC_{2,b}}{db_{1,2}} b_{1,2}B_{1,2} - cC_{2,b}B_{1,2} - cC_{2,b}\left(A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2}\right)}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} \\
&= \frac{c\frac{dC_{2,b}}{db_{1,2}} \left(cC_{2,b} + b_{1,2}B_{1,2}\right) - cC_{2,b}A}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2}.
\end{aligned} \tag{258}$$

Moreover, we substitute Eq. (256) in Eq. (258), which yields

$$\begin{aligned}
\frac{d\delta_{2,c}}{db_{1,2}} &= \frac{-\frac{1-\lambda_a}{\lambda_a}\alpha\frac{d\delta_{2,c}}{db_{1,2}} g \left(cC_{2,b} + b_{1,2}B_{1,2}\right) - cC_{2,b}A}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2} \\
\frac{d\delta_{2,c}}{db_{1,2}}\left(1 + \frac{1-\lambda_a}{\lambda_a}\alpha\frac{g}{cC_{2,b} + b_{1,2}B_{1,2}}\right) &= -\frac{cC_{2,b}}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2}A \\
\frac{d\delta_{2,c}}{db_{1,2}} &= -\frac{\lambda_a}{\lambda_a + \left(1-\lambda_a\right)\alpha\frac{g}{cC_{2,b}+b_{1,2}B_{1,2}}}\frac{cC_{2,b}}{\left(cC_{2,b} + b_{1,2}B_{1,2}\right)^2}A.
\end{aligned} \tag{259}$$

Finally, we incorporate bank 1's budget constraint and take the implicit derivative of the expected return of bank 1 from Eq. (159) with respect to the interbank loan and substitute Eqs. (254), (256), (257), and (259) and use that $d\delta_{2,b}/db_{1,2} = -d\delta_{2,c}/db_{1,2}$, which yields, after simplifying,

$$\begin{aligned}
\frac{d\Pi_{1,b}}{db_{1,2}} &= \lambda_a\left[-A + B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right] + \left(1-\lambda_a\right)\alpha\frac{d\delta_{2,b}}{db_{1,2}}g \\
&= \lambda_a\left[-A + B_{1,2} + A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2}\right] + \left(1-\lambda_a\right)\alpha\frac{d\delta_{2,b}}{db_{1,2}}g \\
&= -\lambda_a c\frac{dC_{2,b}}{db_{1,2}} + \left(1-\lambda_a\right)\alpha\frac{d\delta_{2,b}}{db_{1,2}}g \\
&= -\lambda_a c\frac{dC_{2,b}}{db_{1,2}} - \left(1-\lambda_a\right)\alpha\frac{d\delta_{2,c}}{db_{1,2}}g \\
&= \lambda_a\frac{1-\lambda_a}{\lambda_a}\alpha\frac{d\delta_{2,c}}{db_{1,2}}g - \left(1-\lambda_a\right)\alpha\frac{d\delta_{2,c}}{db_{1,2}}g \\
&= 0.
\end{aligned} \tag{260}$$

9.23 Derivation of Eq. (166)

First, we derive the implicit derivative of the binding participation constraint of the creditors of bank 1 from Eq. (164) with respect to $b_{1,2}$:

$$\lambda_a c\frac{dC_{1,b}}{db_{1,2}} + (1-\lambda_a)\left[\frac{1}{2}\left(\alpha c\frac{dC_{1,b}}{db_{1,2}} + (1-\alpha)\,\alpha\frac{d\delta_{2,b}}{db_{1,2}}g\right) + \frac{1}{2}\left(\alpha\frac{d\delta_{2,b}}{db_{1,2}}g + (1-\alpha)\,\alpha c\frac{dC_{1,b}}{db_{1,2}}\right)\right] = 0$$

$$\left(\lambda_a + (1-\lambda_a)\frac{1}{2}\left[\alpha + (1-\alpha)\,\alpha\right]\right)c\frac{dC_{1,b}}{db_{1,2}} = -(1-\lambda_a)\frac{1}{2}\left[\alpha + (1-\alpha)\,\alpha\right]\frac{d\delta_{2,b}}{db_{1,2}}g$$

$$c\frac{dC_{1,b}}{db_{1,2}} = -\frac{(1-\lambda_a)\frac{1}{2}\left[\alpha + (1-\alpha)\,\alpha\right]}{\lambda_a + (1-\lambda_a)\frac{1}{2}\left[\alpha + (1-\alpha)\,\alpha\right]}\frac{d\delta_{2,b}}{db_{1,2}}g. \tag{261}$$

Second, we determine the implicit derivative of $\delta_{2,b}$ with respect to the interbank loan:

$$\begin{aligned}\frac{d\delta_{2,b}}{db_{1,2}} &= \frac{\left(B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right)(b_{1,2}B_{1,2} + cC_{2,b}) - b_{1,2}B_{1,2}\left(B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}} + c\frac{dC_{2,b}}{db_{1,2}}\right)}{(b_{1,2}B_{1,2} + cC_{2,b})^2}\\ &= \frac{\left(B_{1,2} + b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right)cC_{2,b} - b_{1,2}B_{1,2}c\frac{dC_{2,b}}{db_{1,2}}}{(b_{1,2}B_{1,2} + cC_{2,b})^2}.\end{aligned} \tag{262}$$

Third, we take the implicit derivative of the binding participation constraint of the creditors of bank 2 from Eq. (165) with respect to the interbank loan and substitute $\delta_{2,b}$ for $\delta_{2,c}$:

$$\lambda_a c\frac{dC_{2,b}}{db_{1,2}} + (1-\lambda_a)\left[\frac{1}{2}\alpha\frac{d\delta_{2,c}}{db_{1,2}}g + \frac{1}{2}\left(-\alpha^2 c\frac{dC_{1,b}}{db_{1,2}} + (1-\alpha)\,\alpha\frac{d\delta_{2,c}}{db_{1,2}}g\right)\right] = 0$$

$$(1-\lambda_a)\frac{1}{2}\alpha^2 c\frac{dC_{1,b}}{db_{1,2}} = \lambda_a c\frac{dC_{2,b}}{db_{1,2}} - (1-\lambda_a)\left(\frac{1}{2}\alpha + \frac{1}{2}(1-\alpha)\,\alpha\right)\frac{d\delta_{2,b}}{db_{1,2}}. \tag{263}$$

Fourth, we derive the implicit derivative of the binding participation constraint of bank 2 from Eq. (156) with respect to the interbank loan:

$$\begin{aligned}\frac{d\Pi_{2,b}}{db_{1,2}} &= \lambda_a\left(A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2} - b_{1,2}\frac{dB_{1,2}}{db_{1,2}}\right) = 0\\ b_{1,2}\frac{dB_{1,2}}{db_{1,2}} &= A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2}.\end{aligned} \tag{264}$$

Next, we substitute Eq. (264) in Eq. (262), which yields

$$\begin{aligned}\frac{d\delta_{2,b}}{db_{1,2}} &= \frac{\left(B_{1,2} + A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2}\right)cC_{2,b} - b_{1,2}B_{1,2}c\frac{dC_{2,b}}{db_{1,2}}}{(b_{1,2}B_{1,2} + cC_{2,b})^2}\\ &= \frac{AcC_{2,b} - (b_{1,2}B_{1,2} + cC_{2,b})\,c\frac{dC_{2,b}}{db_{1,2}}}{(b_{1,2}B_{1,2} + cC_{2,b})^2}\\ &= \frac{AcC_{2,b}}{(b_{1,2}B_{1,2} + cC_{2,b})^2} - \frac{1}{b_{1,2}B_{1,2} + cC_{2,b}}c\frac{dC_{2,b}}{db_{1,2}}.\end{aligned} \tag{265}$$

Moreover, we merge Eq. (261) and Eq. (263), which yields

$$\begin{aligned}
&-\left(1-\lambda_a\right)\frac{1}{2}\alpha^2\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{d\delta_{2,b}}{db_{1,2}}g=\lambda_a c\frac{dC_{2,b}}{db_{1,2}}-\left(1-\lambda_a\right)\left(\frac{1}{2}\alpha+\frac{1}{2}\left(1-\alpha\right)\alpha\right)\frac{d\delta_{2,b}}{db_{1,2}}g\\
&\lambda_a c\frac{dC_{2,b}}{db_{1,2}}=-\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]\left(\frac{\left(1-\lambda_a\right)\frac{1}{2}\alpha^2}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}-1\right)\frac{d\delta_{2,b}}{db_{1,2}}g\\
&c\frac{dC_{2,b}}{db_{1,2}}=\frac{\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a}\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{d\delta_{2,b}}{db_{1,2}}g. \qquad (266)
\end{aligned}$$

Next, we merge Eq. (265) and Eq. (266), which yields

$$\begin{aligned}
\frac{d\delta_{2,b}}{db_{1,2}}&=\frac{AcC_{2,b}}{\left(b_{1,2}B_{1,2}+cC_{2,b}\right)^2}\\
&\quad-\frac{\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a}\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}\frac{d\delta_{2,b}}{db_{1,2}}\\
\frac{d\delta_{2,b}}{db_{1,2}}&\left(1+\frac{\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a}\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}\right)=\frac{cC_{2,b}}{\left(b_{1,2}B_{1,2}+cC_{2,b}\right)^2}A\\
\frac{d\delta_{2,b}}{db_{1,2}}&=\frac{\lambda_a\frac{cC_{2,b}}{\left(b_{1,2}B_{1,2}+cC_{2,b}\right)^2}}{\lambda_a+\left(\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\right)\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}A. \qquad (267)
\end{aligned}$$

Then, we merge Eq. (266) and Eq. (267), which yields

$$\begin{aligned}
c\frac{dC_{2,b}}{db_{1,2}}&=\frac{\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a}\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\\
&\quad\frac{\lambda_a}{\lambda_a+\left(\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\right)\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}\frac{cC_{2,b}}{\left(b_{1,2}B_{1,2}+cC_{2,b}\right)^2}Ag\\
&=\frac{\left(\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\right)\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}{\lambda_a+\left(\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\right)\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}\delta_{2,c}A. \qquad (268)
\end{aligned}$$

In a next step, we merge Eq. (261) and Eq. (267), which yields

$$c\frac{dC_{1,b}}{db_{1,2}}\qquad-\frac{\frac{\lambda_a\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}\delta_{2,c}A}{\lambda_a+\left(\lambda_a+\left(1-\lambda_a\right)\left[\alpha+\left(1-\alpha\right)\alpha\right]\right)\frac{\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}{\lambda_a+\left(1-\lambda_a\right)\frac{1}{2}\left[\alpha+\left(1-\alpha\right)\alpha\right]}\frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}. \qquad (269)$$

Finally, we incorporate the budget constraint of bank 1 and take the implicit derivative of the expected return of bank 1 from Eq. (163) with respect to $b_{1,2}$, substitute Eq. (264), Eq. (268), and Eq. (269), and simplify

$$
\begin{aligned}
\frac{d\Pi_{1,b}}{db_{1,2}} \quad &= \lambda_a \left(-A + B_{1,2} + b_{1,2} \frac{dB_{1,2}}{db_{1,2}} - c\frac{dC_{1,b}}{db_{1,2}} \right) \\
&= \lambda_a \left(-A + B_{1,2} + A - c\frac{dC_{2,b}}{db_{1,2}} - B_{1,2} - c\frac{dC_{1,b}}{db_{1,2}} \right) \\
&= -\lambda_a \left(c\frac{dC_{2,b}}{db_{1,2}} + c\frac{dC_{1,b}}{db_{1,2}} \right) \\
&= -\lambda_a \frac{\left(\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]\right) \frac{(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]} \frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}{\lambda_a + \left(\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]\right) \frac{(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]} \frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}} \delta_{2,c} A \\
&+ \lambda_a \frac{\frac{\lambda_a(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]} \frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}{\lambda_a + \left(\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]\right) \frac{(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]} \frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}} \delta_{2,c} A \\
&= -\frac{\lambda_a\left(1-\lambda_a\right)\left[\alpha + (1-\alpha)\,\alpha\right] \frac{(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]} \frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}}{\lambda_a + \left(\lambda_a + (1-\lambda_a)\left[\alpha + (1-\alpha)\,\alpha\right]\right) \frac{(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]}{\lambda_a+(1-\lambda_a)\frac{1}{2}[\alpha+(1-\alpha)\alpha]} \frac{g}{b_{1,2}B_{1,2}+cC_{2,b}}} \delta_{2,c} A < 0. \qquad (270)
\end{aligned}
$$